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An Examination of Different Types of Transportation Problems and Mathematical Models

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Abstract: Optimization processes in mathematics, computer science, and economics solve problems effectively by selecting the best element from a set of available alternatives. One of the most important and successful applications of optimization is the transportation problem (TP), which is a subclass of linear programming (LP) in operations research (OR). Its goal is to find shipping routes between supply and demand centers that will meet the demand for a given quantity of goods or services at each destination center while incurring the fewest transportation costs. Various transportation-related problems involving constraints, mixed constraints, intervals, bottlenecks, and uncertain quantities have recently received a great deal of attention. This relates to the transportation problem. In order to solve the TP, numerous researchers have proposed various exact, heuristic, and meta-heuristic strategies in the literature. Some strategies seek an initial, basic, feasible solution, whereas others seek the optimal way to solve the TP. Because it promotes economic and social activity, the transportation problem is important in operations research and management science. This research paper provides a high-level overview of various transportation-related issues and mathematical models. This can be used successfully to solve various business problems relating to the distribution of products, which are commonly referred to as transportation problems.

Keywords: Classical Transportation Problems, Bottleneck Transportation Problems, Multi-objective Transportation Problems, Interval and Fuzzy Transportation Problems

1. Introduction

Introduction TPs are a major focus of Operations Research (OR), with a wide range of applications including inventory control, communication networks, production planning, scheduling, and personal allocation, to name a few. The TP is another well-known optimization problem in which the goal is to minimize the total transportation cost of distributing resources from a number of sources to a number of destinations. In today's highly competitive market, organizations are under increased pressure to find better ways to create and deliver products and services to customers. It becomes more difficult to determine how and when to send products to customers in the quantities they require while remaining cost-effective. Transportation models offer a strong framework for addressing this challenge. As evidenced by the literature, different techniques have been developed in the past to solve the TP. Some techniques focus on determining an initial feasible solution (IFS), while others focus on determining the optimal solution (OS) to the TP. The Northwest, Least Cost, and Vogel's Approximation techniques are used to find an initial basic feasible solution, whereas the Modified Distribution (MODI) Method and the Stepping Stone Method are used to find an optimal solution to the TP. However, in some cases, particularly when dealing with large-scale TPs, these methods fail to produce an optimal or nearoptimal solution in a reasonable amount of time.

2. History of Transportation Problem

Leornado DaVinci (1503–1517) participated in the war against Prisa because he knew how to conduct bombardments and construct ships, armored vehicles, cannons, catapults, and other warlike machines. F. W. Lanchester, who conducted a mathematical study on opponents' ballistic potency and thus developed, from a system of differential equations, Lanchester's Square Law, which can be used to predict the outcome of a military battle, was another forerunner of the use of OR. Thomas Edison made use of OR by contributing to the anti-submarine war with great ideas such as ship-mounted torpedo shields. Many mathematicians, such as Newton, Leibnitz, Bernoulli, and Lagrange, worked on determining the maximum and minimum conditions for specific functions. Jean Baptiste and Joseph Fourier, two French mathematicians, developed the methods of modern linear programming (LP). In the late 18th century, Gaspar Monge [82] developed Descriptive Geometry, which laid the groundwork for the Graphical Method. Janos Von Neumann [85] published "The Theory of Games," which introduced mathematicians to the basic concept of LP. THE "FATHER OF LINEAR PROGRAMMING," GEORGE B. DANTZIG, was a founding member of The Institute of Management Sciences. The mathematical theory known as "linear programming" was developed by the Russian mathematician Kantorovich [60] in collaboration. The Nobel Prize was awarded to them as a result of their investigation. In the late 1990s, George Joseph Stigler presented a specific problem known as "special diet optimal," or more colloquially known as "diet problem," that occurred as a result of the US army's concern to guarantee some four nutritional requests at a lower cost for his troops. It was solved with a heuristic method whose solutions only differed in some centimes from the solution contributed years later by the Simplex Method. Kantorovich and Koopmans [72] studied the TP independently for the first time in 1941 and 1942. Initially, this type of problem for solving the TP was known as the Koopmans-Kantorovich problem. They used geometric methods based on Minkowski's theory of convexity to solve his problem. However, it is not considered that a new science known as OR was born until the Second World War, during the Battle of England, when the Deutsche Air Force, or Luftwaffe, subjected the British to a heavy air raid due to their lack of aerial capability, despite their combat experience. The British government, looking for a way to defend their country, convened a group of scientists from various disciplines to try to solve the problem so that they could get the most out of the radars they had. Thanks to his efforts in determining the best antenna localization and signal distribution, they were able to double the effectiveness of the aerial defense system. In order to recognize the scope of this new discipline, England formed additional groups of the same type in order to achieve the best results in the dispute. Similarly, when the United States (USA) entered the war in 1942, it established the project SCOOP (Scientific Computation of Optimal Programs), where George Bernard Dantzig [25] developed the Simplex algorithm.

During the Cold War, the old Soviet Union (USSR) Plan Marshall aimed to control terrestrial communications, including river routes from Berlin. To avoid the city's rendition and submission to the German communist zone, England and the United States decided to supply the city, either through escorted convoys (which could spark new confrontations) or through airlift, breaking or avoiding the blockage from Berlin. The second option was to begin the Luftbrücke (airlift) on June 25, 1948. This followed on from the problems solved by the SCOOP group, which could carry 4500 daily tons in December of that same year and 5, after studies of Research Operations optimized the supply to get to the 800,000 daily tons in March of 1949. Because this cipher was the same as that used for terrestrial transport, the Soviet Union decided to lift the blockade on May 12, 1949. Following the Second World War, the management of the United States' resources (USA) (energy, armaments, and all kinds of supplies) took advantage of the opportunity to accomplish it through optimization models, resolved by intervening LP. At the same time that the principle of OR is being developed, computation techniques and computers are also being developed, which has resulted in a reduction in the time required to solve problems. The first result of these techniques was given in 1952, when the National Bureau of Standards used a SEAC computer to solve a problem. The success at the resolution time was so encouraging that it was immediately used for all kinds of military problems, such as determining the optimal height at which the planes should fly to locate the enemy submarines, monetary foundation management for logistics and armament, and determining the depth at which the charges should be sent to reach the enemy submarines in order to cause the higher casualties, which resulted in a five-fold increase in Air Force efficacy. Because of its application in commerce and industry, OR grew in popularity and development during the 1950s and 1960s.

Consider the problem of calculating the best construction sand transportation plan for the city of Moscow's edification works, with 10 origin points and 230 destinations. To resolve it, a Strena computer was used, which took 10 days in June 1958, and such a solution contributed to an 11 percent reduction in expenses compared to the original costs. Previously, these issues were presented in a discipline known as "research companies" or "analysis companies," which did not have as effective methods as those developed during the Second World War (for example, the Método Smplex). There are numerous applications of OR in war that we can imagine, such as cattle nutrition, agricultural field distribution, goods transportation, location, personnel distribution, networking problems, queue problems, graphics, and so on. The following topics in OR are used to solve various types of problems.

3. Different Types of Transportation

3.1. Classical Transportation Problem (CTP)

Many scientific disciplines, including operations research, economics, engineering, geographic information science, and geography, have contributed to the analysis of TPs. To proceed with a minimal total cost solution technique to the TP, an IFS is required. As a result, IFS serves as a foundation for a minimum total cost solution technique to this problem. When the cost coefficients, as well as the demand and supply quantities, are known, efficient algorithms for solving TPs have been developed. The mathematical model of the TP was provided by Hitchcock [49]. The stepping stone method, developed by Charnes and Cooper [23], provided an alternative method for determining the simplex method information. As for the primal simplex transportation method, Dantzig [26] applied the simplex method to TPs. In his book LP, Hadley [45] also included the transportaUpper bounded transportationtion problem. Several heuristic solutions approaches, such as Goyal's [41], looked into the IFS and degeneracy resolution in the TP. Arsha [13] studied a general TP algorithm of the Simplex type. Krzysztof Goczyla [73], a transportation network expert, spoke about optimal routing. For a specific TP, Adlakha and Kowalski [1] proposed an alternative solution algorithm based on absolute point theory. Sharma and Sharma [123] proposed a new dual-based procedure based on heuristics for the TP. In their Determination of Degeneracy in TPs, Sultan [133] and Goyal [41] Ekanayake [33] investigated the TP and maximum flows. Okunbor [86] employed goal programming to address TPs. Putcha [101] devised a method for arriving at an initial basic feasible solution for engineering optimization problems. Adlakha and Kowalski [2] proposed an analysis of alternate solutions for TPs. Immam et al. [52] used an Object-Oriented Model to solve the TP. For example, Klibi et al. [66], for example, looked into the stochastic multi-period location TP. Pandian and Natarajan [89] proposed a novel method for dealing with TPs and Ahamed et al. [5, 6] proposed a new approach to solve TPs.

Many heuristic solution techniques have been presented in the literature to obtain an IFS for the TP. The Northwest Corner Method and the Minimum Cost Method Taha [134] are well-known. Furthermore, Sharma and Prasad [122] presented a heuristic that provided a very efficient initial feasible solution to the proposed VAM-TOC approach. Because of the impracticality of performing enormous calculations in the northwest corner method, minimum cost method, row minimum cost method, column minimum cost method, and VAM for finding an IFS to the TP, Imam et al. [52] and Sen et al. [120] implemented them in C++. Kulkarni and Datar [74] created a heuristic-based algorithm to arrive at an initial feasible solution in order to obtain the modified unbalanced TP with a low total cost. Vasko and Storozhyshina [140] investigated the role of the dummy column (row) in the VAM, the Greedy heuristic [126], the Northwest Corner method, Pargar et al. [98] proposed a heuristic for obtaining an initial solution for the TP with experimental analysis, and Hillier and Lieberman [50] in solving unbalanced TPs. Shimshak et al. [125] and Balakrishnan [15] proposed changes to VAM in order to obtain preliminary solutions to the unbalanced TP. Schrenk et al. [117] investigated degeneracy characterizations for two classical problems: the transportation paradox in linear TPs and pure constant fixed charge TPs (there is no variable cost and the fixed charge is the same on all routes). In 2013 and 2014, Juman et al. proposed a sensitivity analysis and an implementation of the well-known Vogel's approximation method for solving an unbalanced transportation problem and a heuristic solution technique to achieve the minimal total cost bounds of transporting a homogeneous product with varying demands and supplies. Liu [77] investigated the TP with varying demands and supplies within their respective ranges. Following these variations, the minimal total cost was also varied within an interval. So he created a pair of mathematical programs in which at least one of the supply or demand variables changed to calculate the lower and upper bounds of the total transportation cost. Korukolu and Balli [70] proposed an improvement to the well-known VAM by accounting for total opportunity cost. Using computational experiments, they claimed that this improved VAM provided a more efficient and feasible initial solution to a large scale TP. Singh et al. [126] improved optimization and analysis of some variants through Vogel’s approximation method [VAM]. To provide an initial feasible solution to the TP, Deshmukh [31] proposed a new method called the innovative method. However, among the existing heuristics for obtaining an IFS, VAM is one of the most efficient heuristics for TPs because it allows for a very good IFS (often an optimal solution). Furthermore, Sudhakar et al. [131] recently proposed a new approach for finding an optimal solution for TPs. Winston [142], Operations Research: Applications and Algorithms, Mathirajan, [80], Experimental analysis of some variants of Vogel’s approximation method [129], Srinivasan and Thompson [129], The Red-Blue TP, and Cost Operator Algorithms for the TP. Adlakha [3] and Das et al. [27] investigated the Logical Development of Vogel's Approximation Method (LD-VAM): a method for determining a basic viable solution to the TP. Gen, at el. [39], Samuel [114], and Zangiabadi [143] proposed improvements and a new model for TPs with qualitative data, respectively. Gupta [42] investigated paradoxical situations in TPs, as well as the identification of vanishing variables in TPs and their potential applications. Mathirajan [80] also proposed an experimental analysis of some variants. Ramadan [105] and Ramadoss [106] proposed a hybrid two-stage algorithm for solving the TP and an evolutionary heuristic algorithm for solving the assignment problem, which Kowalski et al. [71] also investigated. Pradipkundu [100] investigated some solid transportation models with crisp and rough costs, while Aizemberg's [7] Formulations for a Problem of Petroleum Transportation investigated the initial basic feasible solution and the resolution of degeneracy in TPs. Arsham and Khan [13] investigated a Simplex-type algorithm for the general TP, while Kirca and Statir [67] proposed obtaining an initial solution to the TP. Bertsekas and Castanon [19] worked on an auction algorithm for the transportation problem. Kleinschmidt [68] suggested a strongly polynomial algorithm for the transportation problem and Reinfeld and Vogel, Math [109], proposed mathematical programming. Ekanayake at el. [34, 35] recently proposed using the Ant Colony algorithm (ACA) in the first stage to find an improved initial basic solution and an Effective Alternative New Approach in Solving TPs.

Mathematical Model of the Transportation Problem: The TP can be formulated as an LP model and usually represented in a tabular form. Let us assume that in general that a particular product is manufactured in m production plants known as supply denoted by $S_1, S_2, \dots, S_m$ with respective capacities $a_1, a_2, \dots, a_m$, and distributed to n distribution centers known as demands denoted by $D_1, D_2, \dots, D_m$ with respective demands $b_1, b_2, \dots, b_n$. Also, assume that the transportation cost from i^{th} - supply to the j^{th} - demand is c_{ij} (unit transportation cost) and the amount of product shipped is x_{ij}, where *i = 1, 2,..., m* and *j = 1, 2,..., n*. The following table is known as the transportation cost:

Table 1. *Transportation cost table.*

Supply / Demand	D_1	D_2	D_3	…	D_n	Supply
S_1	c_{11}	c_{12}	c_{13}	$\cdots$	c_{1n}	a_1
S_2	c_{21}	c_{22}	c_{23}	$\cdots$	c_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
S_m	c_{m1}	c_{m2}	c_{m3}	$\cdots$	c_{mn}	a_m
Demand	b_1	b_2	b_3	$\cdots$	b_n	

The mathematical model of TP can be formulated as given below:

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij} \text{ (Total transportation cost)}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,\dots,m \text{(Supply constraints)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, = 1,2,\dots,n \text{ (Demand constraints), and}$$

$$x_{ij} \geq 0 \text{ for all } i = 1,2,\dots,m \text{ and } j = 1,2,\dots,n$$

In the above model, if the total supply is equal to total demand, then the TP is known as a balanced TP and otherwise, it is known as unbalanced TP.

These balanced and unbalanced TPs can be mathematically stated as below respectively: $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ and $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j \ or\ \sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$.

3.2. Mixed Constraint Transportation Problems

Pandian [90, 91] proposed the Fourier Method for Solving TPs with Mixed Constraints and the Solving TPs with Mixed Constraints in Rough Environment. Mondal and colleagues [81], Klingman [69] worked on an innovative method for unraveling TPs with mixed constraints, titled "Solving Transshipment Problems with Mixed Constraints and the TP with Mixed Constraints." Various methods of Solving the Transshipment and TPs with Mixed Constraints." Heinz [47] and Bielefeld [20] propose various methods for solving the TP with mixed constraints. When shipping the same amount or more from each origin and to each destination while keeping all transportation costs non-negative, the More-For-Less (MFL) paradox occurs. MFL occurs in distribution problems in nature. The mixed constraints of TP have been extensively studied in previous years. Another method was proposed by Rabindra et al. [103], and Akilbasha et al. [9] proposed a heuristic method for solving TP with Mixed Constraints in Rough Environment [8]. Gupta et al. [43] and Arora [65] obtained the more-for-less solution for the TP with mixed constraints by relaxing the constraints and introducing new slack variables. Pandian [88, 96, 97] proposed a Fourier method for solving TPs with mixed constraints in rough environments.

Mathematical Model of the Mixed Constraint Transportation Problems: If m is the number of origins or sources and n is the number of destinations, the cost of carrying one unit of the commodity from origin i to the destination j is c_{ij}. Let a_i be the quantity of the commodity available at origin i and b_j be the quantity required at destination j. Thus $a_i \geq 0$ for i and $b_j \geq 0$ for each j, The general formulation of the TP with mixed constraints proposed by Pandian and Natarajan [92] can now be written as follows:

Table 2. *Transportation cost table.*

Destination→ source↓	D_1	D_2	…	D_n	Supply
S_1	c_{11}	c_{12}	$\cdots$	c_{1n}	$=, \leq, or\ \geq a_1$
S_2	c_{21}	c_{22}	$\cdots$	c_{2n}	$=, \leq, or\ \geq a_2$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
S_m	c_{m1}	c_{m2}	$\cdots$	c_{mn}	$=, \leq, or\ \geq a_m$
Demand	$=, \leq, or\ \geq b_1$	$=, \leq, or\ \geq b_2$	$\cdots$	$=, \leq, or\ \geq b_n$	

If x_{ij} is the quantity transported from source i to destination j then the TP with mixed constraints is written with the help of Adlakha et al. [3] and Pandian and Natarajan [92] as

$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=}^{n} c_{ij}x_{ij}$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} \geq a_i, or \sum_{j=1}^{n} x_{ij} \leq a_i, or \sum_{j=1}^{n} x_{ij} = a_i$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, or \sum_{i=1}^{m} x_{ij} \leq b_j, or \sum_{i=1}^{m} x_{ij} = b_j$$

$x_{ij} \geq 0, i = 1,2,\dots,m,$ and $j = 1,2,\dots,n$ and integers;

c_{ij}, is the cost of shipping one unit from supply point i to the demand point j;

a_i, is the supply at supply point i;

b_j, is the demand at demand point j and

x_{ij}, is the number of units shipped from supply point i to demand point j.

3.3. Interval Transportation Problems with Mixed Constraints

Several methods for solving interval TPs with precisely defined cost coefficients and source and destination parameters have been developed in recent years, but in many practical situations, this is not always sufficient to meet the main objective. Akilbasha et al [10] applied the split and separation method to a rough integer interval TP. Pandian [89] solved the TP with mixed constraints in a rough environment using the rough slice sum method and developed a new method for finding an optimal solution to fully intervaled integer transportation problems. Ramesh et al. [107] proposed a method for solving interval LP problems that does not require converting them to classical LP problems. Furthermore, Purushothkumar et al. [102] developed a diagonal optimal algorithm to solve interval integer TPs beautifully. Ganesan [38] and Natarajan [83, 84] proposed some properties of interval matrices and a new method for finding an optimal solution to fully interval integer TPs. Roy and Mahapatra [111], Multi-Objective Interval-Valued Transportation Probabilistic Problem Involving Log-Normal. [104] proposed a new heuristic technique for solving the Integer Interval TP with Mixed Constraints as excellent. Das et al. [28] solved interval TPs using the right bound and the interval's midpoint. Sengupta and others. [118, 119] Theory and Methodology Regarding the comparison of interval numbers. Safi et al. [113] used interval parameters to solve a fixed charge TP by converting interval fuzzy constraints into multiobjective fuzzy constraints. Ummey [138] also proposed a new method to solve interval TPs and a multiobjective stochastic interval TP involving a general form of distribution. Panda and Das [87] determined that the best interval TP was a cost-varying interval TP with two vehicles.

3.3.1. Mathematical Model of the Interval Transportation Problems

Let us assume that in general that a particular product is manufactured in m production plants known as sources denoted by $S_1, S_2, \dots, S_m$ with respective capacities $[a_i, r_i]$, and total distributed to n distribution centers known as sinks denoted by $D_1, D_2, \dots, D_m$ with respective demands $[b_j, s_j]$. Also, assume that the transportation cost from ith - source to the jth - sink is c_{ij} and the amount shipped is x_{ij}, where $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. To assign the supply and demand units, use the table below.

Table 3. *Transportation Table.*

Destination→ source↓	D_1	D_2	…	D_n	supply (ai)
S_1	$[x_{11}, y_{11}]$	$[x_{12}, y_{12}]$	…	$[x_{1n}, y_{1n}]$	$[a_1, r_1]$
S_2	$[x_{21}, y_{21}]$	$[x_{22}, y_{22}]$	…	$[x_{2n}, y_{2n}]$	$[a_2, r_2]$
⋮	⋮	⋮	…	⋮	
S_m	$[x_{m1}, y_{m1}]$	$[x_{m2}, y_{m2}]$	…	$[x_{mn}, y_{mn}]$	$[a_m, r_m]$
Demand (bj)	$[b_1, s_1]$	$[b_2, s_2]$	…	$[b_n, s_n]$	

3.3.2. Lower Bounded Transportation Problem

The total transportation cost is

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2,\dots,n$$

Where, $x_{ij} \geq 0\ \forall\ i,\ j$

3.3.3. Upper Bounded Transportation Problem

The total transportation cost is

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} y_{ij} = r_i,\ i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} y_{ij} = s_j,\ j = 1,2,\dots,n$$

where, $y_{ij} \geq 0\ \forall\ i,\ j.$

3.4. Bottleneck Transportation Problem (BTP)

A time-minimizing TP, also known as a bottleneck TP, is a type of TP in which each shipping route is assigned a time frame. The goal of this process is to reduce the time it takes to transport all supplies to their destinations rather than to reduce costs. When transporting perishable goods, delivering emergency supplies, providing fire services, or dispatching military units from their bases to the front lines, the BTP is encountered. A BTP minimizes the time spent transporting items from origins to destinations while meeting certain conditions such as source availability and destination requirements. According to Hammer [46], the time-minimizing version of this classic problem is very nice research. Hammer's problem was described using the terms "time TP" and "bottleneck TP." The goal of this problem is to shorten the time it takes to transport goods from supply sources to various demand destinations. Hammer's work distinguished itself from previous studies of the TP by focusing on time minimization. Garfinkel and Rao [40], Szwarc [132], Sharma [124], A Minimax Method for Time Minimization, and Agarwal and Sharma's TP with Mixed Constraints [4]. Seshan and Tikekar [121], Khanna, Bakhsi, and Arora [14], and Isserman [56] looked into Hammer's problem further. The best algorithm for solving 2 x n bottleneck TPs was developed by Ravi Varadarajan [108]. Sonia and Puri [128] studied a two-level hierarchical balanced time-minimization TP. Issermann [54] used Peerayuth Charnsethikul and SaereeSvetasreni [99] and The Transportation Constrained Bottleneck Problem. A Novel Approach to Solving Transportation Bottlenecks and Cost Issues Ilija Nikolic [53] and Pandian and Natarajan [95] both presented the total transportation time problem in terms of active transportation routes. On Sharma-Swarup algorithm for time minimizing TPs, Seshan, C. R., and Tikekar [121] proposed an algorithm for solving it. Pandian and Natarajan [91] developed two algorithms: one for locating the best bottleneck-cost TP solution and the other for locating all efficient bottleneck-cost TP solutions. Alhazov and Tkacenko [136] described a method for generating total transportation schedules by minimizing TP with impurities in the commodity using linear fractional time in the multiobjective transportation bottleneck problem. Using a transportation algorithm, Sharif Uddin [137] calculated the shortest possible travel time. Jain and Saksena [55] looked into time-minimizing TP with a fractional bottleneck objective function.

Mathematical Statement of BTP: Bottleneck transportation problem can be stated mathematically as follows:

$$\text{Minimize} = \ [Maximize_{(i,j)}\ t_{ij} /\ x_{ij} > 0]$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,2,\dots,n \text{ and}$$

$$x_{ij} \geq 0 \text{ for all } i \text{and } j.$$

where m is the number of supply points; n is the number of demand points; x_{ij} is the number of units shipped from supply point i to demand point j; t_{ij} is the time of transporting goods from supply point i to demand point j; a_i is the supply at supply point iand b_j is the demand at demand point j. In a BTP, time matrix $[t_{ij}]$ is given where $t_{ij}$is the time of transporting goods from the i^{th} origin to the j^{th} destination. For any given feasible solution $X = \{x_{ij}: i = 1,2,\dots,m \ and\ j = 1,2,\dots,n\}$ of the problem (P), the time transportation is the maximum of $t_{ij}'s$ among the cells in which there are positive allocations. This time of the transportation remains independent of the amount of commodity sent so long as $x_{ij} > 0$.

3.5. Multi-objective Transportation Problem (MOTP)

Numerous analysts have created efficient methods for unraveling at least two destinations at the same time, which are optimizing TPs with multiple objectives, Lee et al. [76], Linear multiobjective programming Zeleny [145] developed different algorithms for all the non-dominated solutions for linear multi-objective TPs, Das and Isermann [28, 54], and Ummey [139] worked Multiobjective stochastic TP involving general forms of distributions, Li [79] proposed a neural network approach for multicriteria solid TP. Simplex multi-criteria method for a linear multiple objective TP, Gupta et al. [44], A simplified interactive multiple objective LP procedure, Kaur [62] and Reeves et al. [110] investigated "A New Approach to Solving Multi-Objective Transportation Problems." Applications and Applied Jimenez [57] worked Interval multiobjective solid transportation problem via genetic algorithms Aneja and Nair [11] worked bi-criteria transportation problem. An efficient algorithm for multiobjective transportation problems, Kasana et al. [61], Revised multi-choice goal programming, Chang [22], A simple algorithm for a multi-objective transportation model, Bai et al. [17], A new method for solving the bi-objective transportation problem, Pandian et al. [93], Banderet al. [16], Solving multi-objective transportation problems, Diaz, J. A. [29], A super non-dominated point for multi-objective transportation problems Evans et al. [36] and Henriques, C. O., and Coelho, D. [48] are proposing Graphic Matroids and the Multicommodity Transportation Problem and Multiobjective Interval Transportation Problems: A Short Review. in optimization and decision support systems for supply chains. A multi-objective solid transportation problem with interval costs in source and demand parameters is proposed by Nagarajan [83, 84] etc.

Mathematical Formulation: In real life situations, usually every organizer wants to achieve multiple goals simultaneously while making In real life situations, normally every coordinator needs to accomplish multiple objectives at the same time while making transportation of products. So MOTP developed by analysts to achieve various objectives. Like classical transportation problem, in MOTP, Quantity (x_{ij}) is to be transported from sources i (i = 1, 2,..., m) to destinations *j (j = 1,2,..., n)* with cost ${C_{ij}}^k$, where ${C_{ij}}^k$ can be transportation cost, total delivery time, energy consumption or minimizing transportation risk etc.

The reality, anyway all transportation problems are not single objective. The transportation problem was described by multiple objective functions. The decision maker would like to minimize set of p objectives simultaneously. Quantity (x_{ij}) is to be transported from sources *i (i = 1, 2,..., m)* to destinations *j (j = 1, 2,..., n)* with cost ${C_{ij}}^k$, where ${C_{ij}}^k$ can be transportation cost, cost of damage or total delivery time costs, energy consumption or minimizing transportation risk, etc. The p objectives $f^1(x), f^2(x), \dots, f^p(x)$ are to minimize the total cost of transportation. It is always assumed that the balance condition holds (i.e. that the total demand is equal to the total supply). With these assumptions, the MOTP can be written as follows:

$$f^1(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} {C_{ij}}^1 x_{ij}$$

$$f^2(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} {C_{ij}}^2 x_{ij}$$

$$\vdots$$

$$f^p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} {C_{ij}}^p x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i \ \text{i} = 1, \ldots, \text{m}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ \text{j} = 1, \ldots, \text{n}$$

$$x_{ij} \geq 0, i = 1,2, \ldots, m \ and \ j = 1,2, \ldots n$$

Where,

${C_{ij}}^k$ Co-efficient of the k-th objective; a_i supply amount of the product at source i *(S$_i$)*; b_j demand of the product at destination j *(D$_j$)*, and $a_i > 0$ for all i, $b_j > 0$ for all j. and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (balanced condition). The balanced condition is both necessary and sufficient for solving the transportation problem in both the cases single and multiple objectives.

Table 4. *Multi-objective Transportation Transportation cost Table.*

Destination→ source↓	***D₁***	***D₂***	**…**	***Dₙ***	***supply (ai)***
S_1	${C_{11}}^1$ ${C_{11}}^2$ ⋮ ${C_{11}}^p$	${C_{12}}^1$ ${C_{12}}^2$ ⋮ ${C_{12}}^p$	… … ⋮ …	${C_{1n}}^1$ ${C_{1n}}^2$ ⋮ ${C_{1n}}^p$	a_1
S_2	${C_{21}}^1$ ${C_{21}}^2$ ⋮ ${C_{21}}^p$	${C_{22}}^1$ ${C_{22}}^2$ ⋮ ${C_{22}}^p$	… … ⋮ …	${C_{2n}}^1$ ${C_{2n}}^2$ ⋮ ${C_{2n}}^p$	a_2
⋮	⋮	⋮	…	⋮	
S_m	${C_{m1}}^1$ ${C_{m1}}^2$ ⋮ ${C_{m1}}^p$	${C_{m2}}^1$ ${C_{m2}}^2$ ⋮ ${C_{m2}}^p$	… ⋮ …	${C_{mn}}^1$ ${C_{mn}}^2$ ⋮ ${C_{mn}}^p$	a_m
Demand (bj)	b_1	b_2	…	b_n	

3.6. Fuzzy Transportation Problem

Hitchcock and Koopmans [49, 72] first established the transportation problem, and they discussed it in detail in their paper, Optimal Utilization of the Transportation System. Dantzig created efficient methods for discovering arrangements, and Charnes and Cooper [21] later developed the stepping stone method. There are also numerous specialists [12] assigned to this field. If a few or all of the parameters of a transportation problem are fuzzy numbers, the problem is an FTP. The value of the fuzzy number has been addressed in numerous research papers on transportation costs, supply, and demand. Many scientists investigate various approaches to solving a balanced and unbalanced transportation problem using fuzzy numbers and various algorithms. The fuzzy set hypothesis has been applied in a lot of fields, for example, operation research, management science, and control theory, and soon. In the literature, a few strategies are proposed for solving transportation problems in a fuzzy environment, such as the possibility of a fluffy set that was introduced by Zadeh [144] in 1965. Bellman and Zadeh [18] discussed the concept of decision-making in a fuzzy domain. For example, after this initiating work, numerous authors have studied fuzzy LP problem techniques. For example, Zimmermann [146] showed that solutions obtained by fuzzy LP are reliably effective, Fang et al. [37], Rommelfanger et al. [112], and Tanaka et al. [135], Wakas [141] and Liu [78] worked by Solving Fuzzy Transportation Problems (FTPs) using a New Algorithm and Solving FTPs based on extension principle. Kumar [75], proposed A Simple Method for Solving Type-2 and Type-4 FTPs, and Samuel, and Raja [115], developed A New Approach for Solving Unbalanced FTPs. In addition, Samuel and Venkatachalapthy [116] worked on IZPM for Unbalanced FTPs; Sobha [127] proposed Profit Maximization of Unbalanced FTPs; and Srinivas [130] developed an Optimal Solution for Degeneracy FTP Using Zero Termination and Robust Ranking Methods., Pandian [94] by A new algorithm for finding a fuzzy optimal solution for FTPs and Jimenez [59] Uncertain solid transportation problems, and Jimenez [58] proposed solving fuzzy solid transportation problems by an evolutionary algorithm based parametric approach. Also, a trisectional fuzzy trapezoidal approach to optimize interval data based transportation problems is presented [30]. A simplified new approach for solving FTPs with generalized trapezoidal fuzzy numbers, by Ebrahimnejad [32] and Fang et al. [37] proposed LP with fuzzy coefficients in constraints. Hunwisai [51] and Kaur [63] proposed A method for solving a fuzzy transportation problem via Robust ranking technique and ATM and A new method for solving FTPs using a ranking function and a new method for solving FTPs using generalized trapezoidal fuzzy numbers, by A. Kaur and A. Kumar [64] and so on. The FTP is a transportation problem whose decision parameters are fuzzy numbers. Christi [24] examined solutions to FTPs using the Best Candidates Method and different ranking techniques. The objective of the FTP is to transport some products from various sources to different destinations with a minimum cost of transportation and satisfaction of the fuzzy supply and demand constraints.

The FTP is a TP in which the transportation expenditures, supply, and demand quantities are fuzzy quantities. The objective function is also regarded as a FN because the goal is to minimize total cost or maximize profit. The objective of this section is to find the minimum FTC using the ranking technique and concept of the ACO algorithm, and provides an analysis of its applications to solving the FTP.

The fuzzy transportation cost table describes the general FTP in a tabular format [Table 3]. The table exhibits the representative value of the fuzzy unit cost, fuzzy supply, and fuzzy demand. Here, all $\widehat{s_i}$ and $\widehat{d_j}$ are assumed to be positive, and $\widehat{s_i}$ are called supplies and $\widehat{d_j}$ are called demands, $\hat{c}_{ij}$ the cost of transshipment of one unit from i th source to j th destination.

The Fuzzy Transportation Problem as a Mathematical Formula:

Minimize $\tilde{Z} = \sum_{i=0}^{m} \sum_{j=0}^{n} \hat{c}_{ij} \tilde{X}_{ij}$

Subject to

$\sum_{j=1}^{n} \tilde{X}_{ij} \leq \widehat{s_i}$ for $i = 1,2,3,\ldots.,m$

$\sum_{i=1}^{m} \tilde{X}_{ij} \leq \widehat{d_j}$ for $j = 1,2,3,\ldots.,n$

$\tilde{X}_{ij\ \geq 0}$ for $i = 1,2,3,\ldots.,m$ and $j = 1,2,3,\ldots.,n$

Here, all $\widehat{s_i}$ and $\widehat{d_j}$ are assumed to be positive, and $\widehat{s_i}$ are typically called supplies and $\widehat{d_j}$ are called demands, as shown in the beneath table. The fuzzy cost $\hat{c}_{ij}$ are all non-negative. If $\sum_{i=1}^{m} \widehat{s_i} = \sum_{j=1}^{n} \widehat{d_j}$, it is a balanced TP. If this condition isn't met, a dummy origin or destination is generally introduced to make the problem balanced.

4. Conclusion

This paper provides a brief overview of some types of transportation problems, including classical transportation problems, transportation problems with mixed constraints, interval transportation problems, bottleneck transportation problems, multi-objective transportation problems, and FTPs. This chapter also discussed the work done so far by many scientists and statisticians on the transportation problem, as well as mathematical models for each.

Competing Interests

The authors declare that they have no competing interests.

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