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A Dynamical Systems Model for Face Perception

Abraham Boyarsky*, Paweł Góra

Department of Mathematics and Statistics, Concordia University, 1455 de Maisonneuve Blvd. West, Montreal, Quebec H3G 1M8, Canada

Email address:

abraham.boyarsky@concordia.ca (A. Boyarsky), pawel.gora@concordia.ca (P. Góra)
*Corresponding author

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Abstract: The fusiform face area, or FFA, is a small region found on the inferior (bottom) surface of the temporal lobe. It is located in a gyrus called the fusiform gyrus.Studies in humans have shown that the FFA is sensitive to both face parts and face configurations. Recoding activity in the FFA showed that most of the neurons in the FFA are active in response to facial imagery, but not in response to images of other body parts or objects. Visual sensory neurons sensitive to a face feature and possessing a related firing rate activate an associated cluster of neurons in the FFA. This results in a partition of the FFA into clusters that respond to the various facial features. Once an entire face stimulus activates the FFA, interneurons redistribute the initial activation via the neural network. In this article a novel approach to modelling the function of the network is presented. We define by a transition matrix that describes probabilistically how one cluster, firing at a synchronous rate, affects the others in the FFA. The initial face stimulation in the FFA together with the transition matrix defines a dynamical system which possesses a stationary probability function. We claim that a stationary probability function uniquely represents a face. Among the properties of this probability function are: 1) response magnitude invariance, 2) repurposing of clusters to define new stationary probability function on the FFA partition; 3) stability of stationary probabilities under perturbations.

Keywords: Dynamical Systems, Model for Face Perception, Fusiform Face Area (FFA), Face Parts, Stationary Measure, Stability

1. Introduction

The neural correlates of human face perception are in the fusiform face area (FFA) [13]. The FFA is also implicated in "extracting the perceptual information used to distinguish between faces" [13]. The visual sensory neurons that perceive different facial features terminate in different clusters or patches in the FFA [16], from where they activate other neurons [22] via the neural network of the FFA. We assume that to each feature there corresponds a specific firing rate as has been proven in the primate brain [8]. The model in [8] formats a face in a very high dimensional space. Using 50 axes, it takes as many as 2500 numbers to characterize a face. In our model, we typically deal with clusters of the order of 10, and characterize a face by a probability function on these clusters. Hence, much less information is needed to represent a face than in [8].

We label n facial features as $F_1, \dots, F_n$. and $C = \{C_i, i = 1, \dots, n\}$ as the associated activated clusters in the FFA. This forms a partition of the FFA as depicted in Figure 1. Once an entire face stimulus - coming simultaneously and in parallel from the n facial features - arrives at the FFA, the interneuron cells in the FFA redistribute the sensory activation (Figure 2) via its neural network [1]. The network dynamics is probabilistic and is modeled in this note by a transition matrix that describes how one cluster of synchronously firing neurons affects the synchronous firing rates in other clusters.

The initial face stimulation to the FFA together with a matrix defining connections of the neural network specifies a dynamical system whose stationary probability function is supported on the FFA. The stationary probability function specifies the equilibrium firing rates throughout the FFA. It is important to note that the acquisition of this probability function in the FFA happens very quickly. Among the properties of the stationary probability function are: 1) response magnitude invariance; 2) repurposing of clusters to define new probability functions on the FFA;

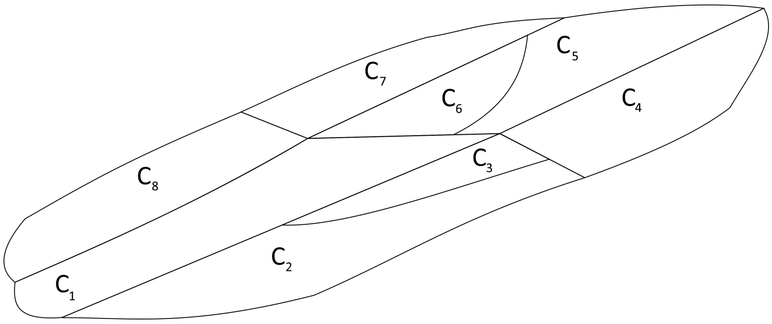

Figure 1. *Partition of FFA.*

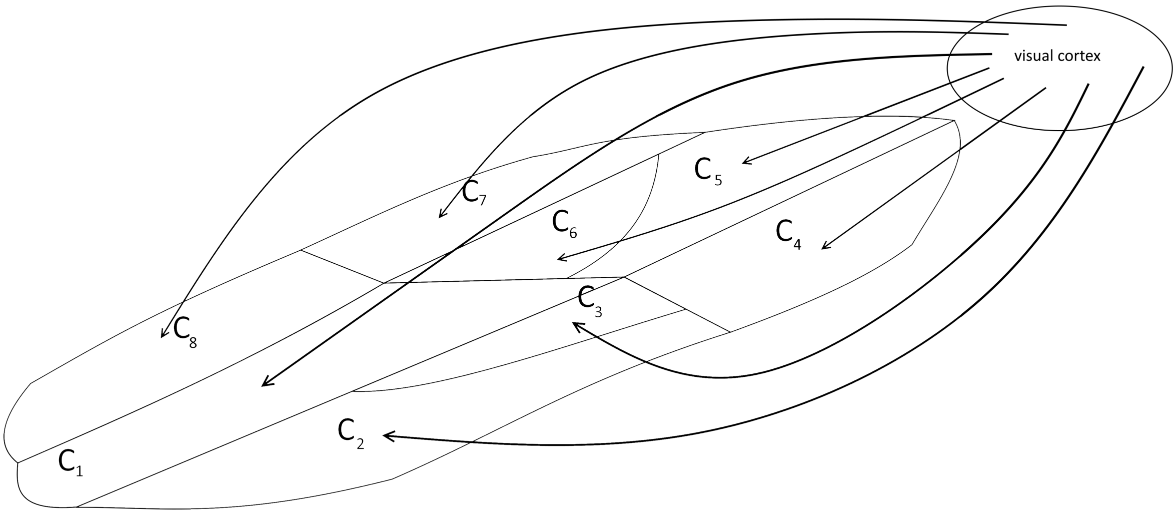

Figure 2. *Initial activation of FFA.*

3) stability of the stationary probability function under sensory perturbations.

A different model for computational facial encoding is developed in [21].

2. Dynamical Systems Model

Let V_i denote the volume of C_i, the region in the FFA corresponding to the ith facial feature as shown in Figure 1 and let M_i be the synchronous firing rate of the neurons in C_i. Let $S_i = M_i V_i$ which we refer to as the strength of (or total traffic coming from) the ith feature as represented in the FFA. Let $S = \sum_{i=1}^{n} S_i$ and define $p_i = \frac{S_i}{S}$. The collection $p_i, \ldots, p_n$ defines a probability function on the FFA since each $p_i \geq 0$ and $\sum_{i=1}^{n} p_i = 1$. An example of an activation pattern in the FFA is depicted in Figure 3.

The interneural network of the FFA determines how clusters interact with each other. This allows us to define transition probabilities P_{ij}, $i, j = 1, \ldots, n$ where for each i the positive numbers P_{ij} sum up to 1 ($\sum_{j=1}^{n} P_{ij} = 1$), and are proportional to the numbers of connections from C_i to C_j via the neuronal network [1]. A sample transition matrix is shown in Figure 4. We now associate with each region C_i an interval I_i on the unit interval $[0, 1]$, then construct a piecewise linear "semi-Markov" map [7] $\tau : [0, 1] \to [0, 1]$ on the partition $\mathcal{I} = \{I_i\}_{i=1}^{n}$ into n subintervals which realizes this transition probability function as shown in Figure 4. The precise definition of τ is given in the Appendix. The map τ belongs to a class of "piecewise expanding maps of the interval" [6] and in particular admits a unique stationary probability function, which can be identified with a vector $\mathbf{p} = [p_1, p_2, \ldots, p_n]$. The expanding property reflects the fact that each neuron activates hundreds (even thousands) of other neurons [1]. The vector $\mathbf{p}$ is the unique solution of n linear equation with n unknowns [6, Chapter 9]:

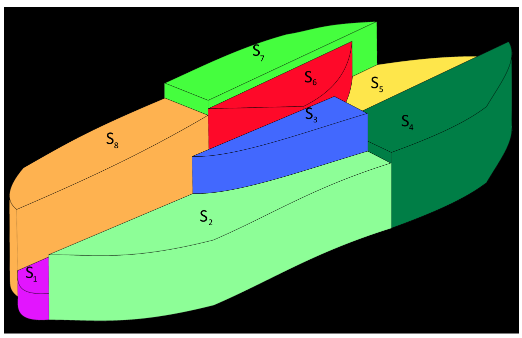

Figure 3. *Sketch of FFA activation.*

$$\begin{cases} p_1P_{11} + p_2P_{12} + \cdots + p_nP_{1n} = p_1; \\ p_1P_{21} + p_2P_{22} + \cdots + p_nP_{2n} = p_2; \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ p_nP_{n1} + p_2P_{n2} + \cdots + p_nP_{nn} = p_n. \end{cases}$$

We propose that the facial stationary probability function $\mathbf{p}$ contains the information to characterize a face, that is, $\mathbf{p}$, encodes the face identity information. This model is consistent with the work of [17, 18] which provides evidence that the FFA "responds with distinct patterns of activation to different face identities."

3. Example

Assuming the structure of the neuronal architecture in the FFA defines the 8×8 transition probability matrix in Figure 4, the stationary probability function is as follows:

$$\mathbf{p} \approx [0.167, 0.106, 0.093, 0.096, 0.134, 0.137, 0.140, 0.127].$$

The computational process of achieving the stationary measure from the initial input distribution is very fast. It takes only 10-20 steps for the process to arrive to almost the stationary measure. For matrix $\mathbb{P}$ below and a typical input it takes 7 iterations to be less than 10^{-4} close to the stationary probability function.

$$\mathbb{P} = \begin{bmatrix} 0 & 0.1 & 0 & 0.1 & 0.1 & 0.2 & 0.1 & 0.4 \\ 0.3 & 0.1 & 0 & 0.1 & 0.2 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.2 & 0 & 0.2 & 0.2 & 0 & 0.1 \\ 0.2 & 0 & 0.3 & 0.1 & 0.1 & 0 & 0.2 & 0.1 \\ 0.2 & 0.04 & 0.2 & 0.03 & 0 & 0.2 & 0.3 & 0.03 \\ 0.1 & 0.3 & 0 & 0.2 & 0.2 & 0 & 0.2 & 0 \\ 0.2 & 0.1 & 0.1 & 0.2 & 0.2 & 0.07 & 0 & 0.13 \\ 0.3 & 0 & 0.03 & 0 & 0.1 & 0.3 & 0.2 & 0.07 \end{bmatrix}$$

The Probability Transition Matrix $\mathbb{P}$ reflects neuronal architecture in the FFA, for $n = 8$ facial features.

4. Properties of the Dynamics Systems Model

4.1. Response Magnitude Invariance

Let $M_i, i = 1, \ldots, n$ denote the synchronous firing rates in the clusters $\{C_i, i = 1, \ldots, n\}$ of the FFA for a given face. Suppose the same face is now reduced in size, resulting in altered firing rates. We assume that all the synchronous firing rates are reduced uniformly by a factor $\alpha < 1$. Then, the new firing rates are: $M_i' = \alpha M_i$. Let $S_i' = \alpha M_i V_i$. Then,

$$S' = \sum_{i=1}^{n} S_i' = \alpha \sum_{i=1}^{n} M_i V_i = \alpha S.$$

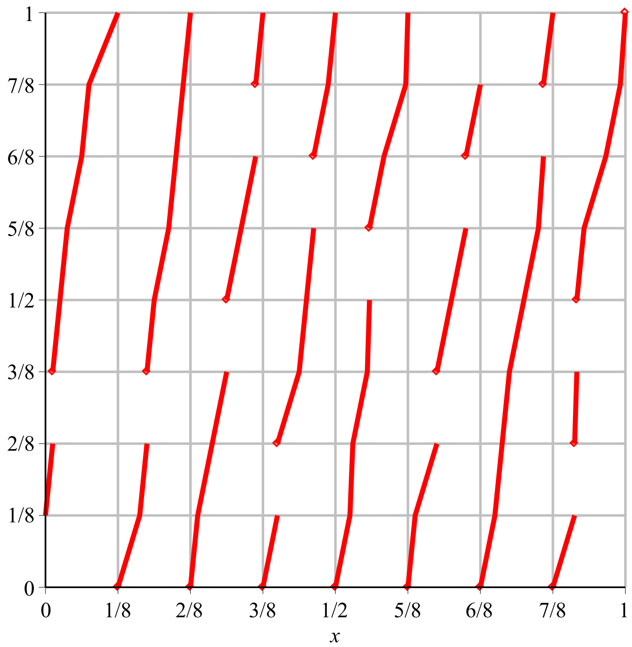

Figure 4. *Map τ constructed using the matrix $\mathbb{P}$.*

The new probability function

$$P_i' = \frac{S_i'}{S'} = \frac{\alpha S_i}{\alpha S} = P_i.$$

Hence the probability function P is invariant under a uniform size reduction (or expansion). Face-inversion [23] may result in a weakening of intensities in the clusters in the FFA. The resulting reduction in cluster activity nonetheless preserves the probability function, resulting in recognition of the original upright face. In [19], it was shown that mental images of faces activate the same FFA areas as the actual visual image. The mental image, initiated entirely by interneurons, no doubt has lower response activation than the actual visual image. Yet, both activations yield the same probability function and hence represent the same face.

The mental image in [19] can be considered as an intermediate step between sensory input and activation of the FFA. This process suggests that translation and rotation of a facial image produce the same mental image and hence activate the FFA in the same way as the original facial sensory input, implying translational and rotational invariance.

4.2. Repurposing

Repurposing can be viewed as the same clusters being activated with a set of different strength activations. It is noted in [20] that the late neuroscientist Jerry Lettvin suggested that as few as 18,000 neurons could be the bases of a conscious experience, such as seeing a face. Even so, the brain may not have enough neurons to represent all possible concepts and faces [20]. In our model, the same partition of a cortical region such as the FFA is reused to define a potentially infinite number of different patterns of activation, each one representing a different face.

4.3. Stability Under Perturbations of Facial Features

Once we associate a dynamical system to a face activation on a partition of the FFA, we can study the stability of such a system. Consider a small perturbation of a face (say a laughing face or even a caricature). We want the resulting stationary probability function to be close to the original one. Such properties are proved using stability of the stationary probability function under small perturbations of the transition matrix [6]. They hold as long as most entries in the matrix are strictly positive, which corresponds to all parts of the FFA communicating between themselves [6, 14].

To illustrate the stability of $\mathbf{p}$ under small perturbations of $\mathbb{P}$ we give an example of a transition matrix $\mathbb{P}_1 \approx \mathbb{P}$, with perturbation ± 0.01, and its stationary probability vector $\mathbf{p}_1$. Let

$$\mathbb{P}_1 = \begin{bmatrix} 0.01 & 0.09 & 0.01 & 0.1 & 0.11 & 0.21 & 0.11 & 0.36 \\ 0.29 & 0.11 & 0 & 0.1 & 0.2 & 0.09 & 0.11 & 0.10 \\ 0.1 & 0.2 & 0.19 & 0 & 0.21 & 0.2 & 0 & 0.10 \\ 0.21 & 0 & 0.3 & 0.09 & 0.1 & 0 & 0.2 & 0.10 \\ 0.19 & 0.03 & 0.21 & 0.02 & 0.01 & 0.19 & 0.3 & 0.05 \\ 0.09 & 0.29 & 0.01 & 0.19 & 0.21 & 0 & 0.2 & 0.01 \\ 0.19 & 0.11 & 0.09 & 0.2 & 0.19 & 0.07 & 0.01 & 0.14 \\ 0.29 & 0.01 & 0.02 & 0 & 0.1 & 0.3 & 0.21 & 0.07 \end{bmatrix}$$

The stationary probability function for $\mathbb{P}_1$ is given by

$$\mathbf{p}_1 \approx [0.164, 0.106, 0.0931, 0.0929, 0.138, 0.136, 0.145, 0.125] \,.$$

We see that the largest difference between the components of $\mathbf{p}_1$ and $\mathbf{p}$ is 0.004 (components number 5 and 7).

5. Conclusions

This note proposes a method for encoding facial identity using stationary probability function. To define such a function requires knowledge of which clusters in the FFA are activated by features of a face and the relative strengths of these activations in the FFA which depends on the respective firing rates. The FFA architecture [15] determines the transition probabilities and with the synaptic weights of the different clusters determines a flow of information, which defines the neural network dynamics. To define a transition matrix $\mathbb{P}$ we need to know which clusters interact with other clusters. Once $\mathbb{P}$ is known, the the map τ can be constructed and its stationary probability function and the corresponding $\mathbf{p}$ can be calculated. We claim $\mathbf{p}$ characterizes a face.

Appendix

In the Appendix we show how given transition probabilities $P_{ij},\ i,j = 1,....,n$ we construct a piecewise linear "semi-Markov" map $\tau : [0,1] \to [0,1]$ on the partition $\mathcal{I} = \{I_i\}_{i=1}^n$ into n subintervals of equal lengths.

We will construct τ on each of the subintervals $I_i = [\frac{i-1}{n}, \frac{i}{n}]$, $i = 1, 2, \ldots, n$. The graph of τ on I_i consists of segments of straight lines consecutively connecting the points:

$$\left(\frac{i-1}{n}, 0\right), \left(\frac{i-1}{n} + \frac{1}{n} P_{i,1}, \frac{1}{n}\right),$$
$$\left(\frac{i-1}{n} + \frac{1}{n}\left(P_{i,1} + P_{i,2}\right), \frac{2}{n}\right),$$
$$\ldots, \left(\frac{i-1}{n} + \frac{1}{n}\left(P_{i,1} + P_{i,2} + \cdots + P_{i,k}\right), \frac{k}{n}\right), \ldots, \left(\frac{i}{n}, 1\right) \,.$$

The vertical segments of the lines are not drawn, if two consecutive points have the same x-coordinates, then the line between them is not part of the graph.

References

[1] Abeles, M., Corticonics: Neural Circuits of the Cerebral Cortex, Cambridge University Press, 1991.

[2] Al-Am̀eri, J. H. and Saber, S. M., "IFS code as features in face recognition system," Int. J. Modern Trends in Engineering and Research, Vol. 03, Issue 08, August 2016.

[3] Bertolero, M. and Bassett D. S., "How matter becomes mind," Sci. Am., July 2019, 26–33.

[4] Boyarsky, A., "Mind: Probability functions on matter," International Journal of Brain and Cognitive Sciences, 2015, 4 (1): 1–2.

[5] Boyarsky, A. and Góra, P., "Invariant measures in brain dynamics," Physics Letters A, 358, 2000, 27–30.

[6] Boyarsky, Abraham; Góra, Paweł; Laws of chaos. Invariant measures and dynamical systems in one dimension, Probability and its Applications. Birkhäuser Boston, Inc., Boston, MA, 1997.

[7] Boyarsky, Abraham; Góra, Paweł"A matrix solution to the inverse Perron-Frobenius problem," Proceedings of American Mathematical Society 118 (1993), 409–414.

[8] Chang, Le and Tsao, D. Y., "The code for facial identity in the primate brain," Cell 169, (June 2017), 1013–1028.

[9] Herrington J. D., Taylor, J. M., Grupe, D. W., Curby, K. M. and Schultz, R. T., "Bidirectional communication between amygdala and fusiform gyrus during facial recognition," Neuroimage, (2002) June 15; 56 (4); 2348–2355.

[10] Huettel, S. A., Song, A. W., McCarthy, G., (2004). Functional magnetic resonance imaging. Sunderland, Mass., Sinauer Associates Publishers.

[11] Kanwisher, N., "Neural events and perceptual awareness," Cognition, 79 (2001) 89-113.

[12] Kanwisher, N., "What's in a face?" Neuroscience, SCIENCE, Vol. 311, 3 February (2006), 617-618.

[13] Kanwisher, N. and Yovel, G., "The fusiform face area: a cortical region specialized for the perception of faces," Phil. Trans. of the Royal Society B (2006) 361, 2109–2128.

[14] Kemeny, John G. and Snell, J. Laurie, Finite Markov chains. Reprinting of the 1960 original. Undergraduate Texts in Mathematics. Springer-Verlag, New York-Heidelberg, 1976.

[15] Klinshov, V. V., Teramae, J., Nekorkin, V. I., and Fukai, T., "Dense neuron clustering explains connectivity statistics in cortical microcircuits," PloS ONE 9 (4): e94292 (2014).

[16] Liu, J., Harrison, A. and Kanwisher, N., "Perception of Face Parts and Face Configurations: An fMRI Study," Journal of Cognitive Neuroscience 22: 1, 203-211, 2009.

[17] Nestor, A., Plaut, D. C. and Behrmann, M., "Unraveling the distributed neural code of face identity through spatiotemporal pattern analysis," PNAS, June 14, (2011), vol. 108, no. 24, 10003.

[18] Nichols, D. F., Both, L. R. and Wilson, H. R., "Decoding of faces and face components in face-sensitive human visual cortex," Frontiers in Psychology, vol. 1, article 28, July 2010.

[19] O'Craven, K. M. and Kanwisher, N., "Mental imagery of faces and places activate corresponding stimulus-specific brain regions," Journal of cognitive Neuroscience, 12: 6 1013–1023, 2000.

[20] Quiroga, R. Q., Fried, I. and Koch C., "Brain cells of Grandmother," Scientific American, February 2013.

[21] Stollhoff, R., Kennerknecht, I., Elze, T. and Jost, J., "A computational model of dysfunctional facial encoding in congenital prosopagnosia," Neural Networks 24, 652–664, 2011.

[22] Wandell, B. A, Brewer, A. A and Doughherty, "Visual field map clusters in human cortex," Philos. Trans. R. Soc. London B Biol. Sci, (2005), April 29 360 (1456), 693–707.

[23] Yovel, G. and Kanwisher, N., "The neural basis of the behavioral face-inversion effect." Current Biology, Vol. 15 (2005), 2256–2262.