

Mixed Convective Magnetohydrodynamic Heat Transfer Flow of Williamson Fluid Over a Porous Wedge

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Abstract: The present article examines the influence of thermal radiation on two-dimensional incompressible magnetohydrodynamic (MHD) mixed convective heat transfer flow of Williamson fluid flowing past a porous wedge. An adequate similarity transformation is adopted to reduce the fundamental boundary layer partial differential equations of Williamson fluid model in to a set of non-linear ordinary differential equations. The solutions of the resulting nonlinear system are obtained numerically using the fifth order numerical scheme the Runge-Kutta-Fehlberg method. The effects of different pertinent physical parameter such as magnetic parameter, Williamson parameter, radiation parameter and Prandtl number on temperature and velocity distributions are observed through graph.

Keywords: Williamson Fluid, Boundary Layer Flow, Mixed Convection Heat Transfer, Runge-Kutta-Fehlberg Technique

1. Introduction

The theory of non-Newtonian fluids has attracted several researchers owing to its enormous applications in engineering and industrial sector. In Non-Newtonian fluids, the most frequently encountered fluids are pseudoplastic fluids, and Navier-stokes equations alone are insufficient to describe the rheological properties of these fluids, therefore, to overcome this defect, several rheological model like Ellis model, Power law model, Carreaus model and Cross model are presented, but little attention has been compensated to the Williamson fluid model and estimated to explain the rheological properties of pseudoplastic fluids. In this model both maximum viscosity μ_∞ (viscosity as shear rate tends to infinity) and minimum viscosity μ_0 (viscosity as shear rate tends to zero) are to be taken. Williamson analyzed the flow of pseudoplastic materials and presented model to described the behavior of pseudoplastic material and explain convenient importance of plastic flows, and also recognized that viscous flows are very varied from plastic

flows [1]. Nadeem et al. performed an investigation on flow of Williamson fluid in a stretching sheet [2]. Hayat et al. showed combine influence of magnetic and electric fields and thermal radiation influence over the flow pattern of two dimensional boundary layer flow of Williamson fluid past a porous stretching surface [3]. Nadeem et al. investigated flow of Williamson fluid in an inclined channel due to long wavelength assumptions [4]. Krishnamurthy et al. considered steady flow of Williamson fluid in a horizontal linearly stretching sheet with simultaneous impact of chemical reaction & melting heat transfer and by considering nanoparticle [5]. Dapra and Scarpi elaborated the perturbation solution for pulsatile motion of Williamson fluid in a rock fracture [6]. Hayat et al. attained both numerical and analytical solutions of Williamson fluid transport through stretching surface subject to joule heating, and they observed that both methods have great argument with all parameters of flow [7]. Peristaltic motion of Williamson fluid through a channel enclosed by permeable wall is significant in Biology and medicine; in this regard Vajravelu et al. studied the

peristaltic motion of non-Newtonian fluid through asymmetric channels along porous wall by means of various phase and amplitude, and also studied the manipulation of different wave structures on the fluid flow model [8]. By considering the approximation of long wave length and small Reynolds number the peristaltic pumping of Williamson fluid in a planar channel was investigated by Vasudev *et al.* and they studied the flow in wave framework which is moving with speed of wave [9]. Nadeem *et al.* developed a model for the transport of Williamson fluid in an annular region [10]. Khan *et al.* found numerically convergent solutions of two dimensional flows of non-Newtonian fluids along chemically reactive species [11]. Khan and Sultan calculated the influence of anisotropic slip [12]. Malik *et al.* studied Williamson fluid model over a stretching cylinder with homogeneous-heterogeneous reactions, and work out the problem numerically by using Keller box method [13]. In another study, Malik *et al.* discussed numerical solutions of Williamson fluid flow through stretching cylinder with variable fluid properties [14]. Malik *et al.* examined three dimensional transport of non-Newtonian fluid caused owing to stretching surface, and observed that fluid velocity decreases by increasing Williamson parameter [15]. Vittal *et al.* reported the MHD stagnation point flow and heat transfer of Williamson fluid over exponential stretching surface in existence of radiation [16]. Monica *et al.* designed an analysis for stagnation point flow of non-Newtonian fluids to stretching sheet [17]. Nagaraja and Reddy proposed the modeling of two dimensional Williamson fluids past a circular cylinder [18]. Siddiqui *et al.* found analytical solution of Blade coating investigation of a Williamson fluid by employing adomian decomposition method [19].

MHD is very useful to analyze the interaction of electrically directing fluids. Electrically directing fluid flow has received the concentration of researchers due to its several applications in technology and science like MHD pumps, MHD power generators and purification of crude oil. Hayat *et al.* examined MHD motion of nanofluid owing to rotating disk with partial slip [20]. Azimi *et al.* analytically discussed MHD flow of viscous fluid through Stretching sheet using DTM-pade approach to solve boundary layer equations of given flow problem [21]. Jabar addressed the influence of viscous dissipation and joule heating on MHD flow through a stratified sheet subjected to power Law Heat Flux having heat source [22]. Reddy discussed unsteady MHD transport of rotating fluid past a permeable surface confined by infinite vertical permeable plate and concluded that by increasing rotating parameter the velocity field is also increased [23]. Misra & Sinha observed the impact of radiation on magnetohydrodynamic flow of blood and heat transfer in a porous medium [24]. Shateyi found numerical solutions for the problem of MHD flow of Maxwell fluid past a vertical sheet owing to radiation, thermophoresis and chemical reaction [25]. El-Kabeir *et al.* explore the chemical reaction and heat absorption impact on nonlinear magnetohydrodynamic flow with heat and mass transfer characteristics of an incompressible, electrically directing fluid on a cone surface [26]. Ghara *et al.* have studied the radiation effects on MHD flow over vertical plate along ramped wall temperature and

gain two different results for fluid velocity [27]. Rasekh *et al.* obtained analytical solution of MHD stagnation point flow towards permeable stretching surface in existence of chemical reaction [28]. Nadeem *et al.* analyzed hydromagnetic motion of a micropolar nanofluid between parallel plates [29]. Ibrahima and Suneethaa investigated the effects of heat generation and thermal radiation on steady hydrodynamic flow along porous medium in presence of variable thermal conductivity and concluded that governing equations are greatly affected by involved Prandtl number [30]. Jhankal *et al.* proposed a mathematical model for MHD flow of incompressible fluid over a Darcy-Forchheimer stratified medium [31]. Koriko *et al.* presented series solution for an electrically directing transport of micropolar fluid in a vertical stratified surface [32].

Mixed convection is a coupled phantasm of two heat transfer mechanisms force convection and natural convection that act simultaneously to transfer heat in a fluid flow. It play significant role in field of technology. Srinivasacharya *et al.* analyzed mixed convection flow with irregular fluid properties through a vertical wavy plate [33]. Merkin *et al.* studied both forced and natural convection boundary layer transport by perpendicular surface in a stratified medium along connective boundary conditions [34]. Xu and Chen proposed two-layer model to simulate mixed convection flow in a room [35]. By utilizing Keller box technique numerical solutions of problem of mixed convection axisymmetric flow of air with variable physical properties was obtained by Ramarozara [36]. Bau investigated the thermal convection in a saturated stratified medium bounded between two parallel eccentric cylinders with the help of a regular perturbation expansion along Daarcy-Rayleigh number; it was observed that the appropriate preference of eccentricity values can maximize the heat transfer inside annulus of various thermal insulators [37]. Jackson *et al.* discussed combine convection in vertical tubes by using constant wall temperature and constant wall heat flux conditions [38]. Fu *et al.* investigated flow reversal of mixed convection in a three dimensional channel and concluded that an increase in Richardson number, natural convection dominates the flow and thermal field of combine convection [39]. Kaya found nonsimilar solutions of steady laminar mixed convection heat transfer flow from a perpendicular cone in a porous medium with influence of radiation, conduction, interaction and having high porosity [40]. Jafari *et al.* studied unsteady combined convection flow in a cavity in presence of nanofluid [41]. Bég *et al.* outlined mixed convection boundary layer flow influenced by thermo-diffusion [42]. Chaudhary and Jain studied the impact of mass transfer, radiation and hall on MHD mixed convection flow of viscoelastic fluid in an infinite plate [43]. Ferdows & Liu obtained the similarity solutions of mixed convection heat convey in parallel surface with internal heat production [44]. Malleswaran & Sivasankaran carried an analysis for mixed convection flow and noticed that the average heat transfer decreases with an increase in Richardson number but in general heat transfer is better at force convection mode than free convection mode

[45]. Few other interesting works about convective heat transfer can be found in [46-68].

Porous wedge is a very important characteristic in science and engineering that can be illustrated as a material having a minute opening and the opening is almost filled with fluid. The skeletal part of porous wedge is known as Frame and it is frequently a solid, but structures akin to foams. General example of porous wedge is sand, soil, sandstone and foams. Ashraf et al. studied boundary layer flow of fluid in a porous wedge subject to Newtonian heating along heat generation or absorption [69]. Deka and Sharma solved the boundary layer equations of flow over a wedge under variable temperature and chemical reaction [70]. Mukhopadhyay considered the impact of radiation and variable viscosity on the flow through wedge using lie group transformation [71]. Mukhopadhyay and Mandal numerically analyzed flow of a Casson fluid in a symmetric porous wedge along surface heat flux [72]. Hossain et al. considered the case for unsteady boundary layer flow past a wedge [73]. Dalir investigated two dimensional laminar transport of viscous fluid through a permeable wedge [74]. Rostami et al analytically studied laminar viscoelastic fluid flow past a wedge in the presence of Buoyancy forces and discussed the effects of Buoyancy parameter on velocity and temperature profiles [75]. Wedge flow problem under thermal radiation was studied by Rashidi et al. [76].

The main aim of the present paper is to elaborate electrically conducting fluid flow of Williamson fluid over a permeable wedge with thermal radiation. Similarity transformation is used to convert governing partial differential equations of the said phenomenon to couple nonlinear ordinary differential equations. The resulting non-dimensional equations are solved numerically using the Runge-Kutta-Fehlberg method. The effects of involved parameters on flow are discussed graphically.

Formulation

Consider two dimensional steady incompressible MHD mixed convective heat transfer and electrically directing Williamson fluid past a porous wedge. The Cartesian coordinate plot is assumed to be help out the solution wherein the x -axis & y -axis are together and normal to the wedge. The non-uniform magnetic field $B(x) = B_0 x^{\frac{(m-1)}{2}}$

applied to flow and perpendicular to y -axis, the induced magnetic field of flow are supposed to be negligible. It is assumed that the fluid velocity from wedge is $U_e(x) = ax^m$. Where a and m are constants. The governing partial differential equations of continuity, momentum and energy for the Williamson fluid flow using Boussinesq's and boundary layer assumptions are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + U_e \frac{\partial U_e}{\partial x} + \sqrt{2} \nu \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} (u - U_e) \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (3)$$

where u and v are velocity components in x and y directions respectively, ρ is the density, ν is Kinematic viscosity, σ is electric conductivity, T is the temperature of the fluid, α is thermal diffusivity, q_r is heat flux and C_p is specific heat.

The boundary conditions associated with the problem are at

$$y = 0, u = 0, v = v_w, T = T_w(x) \quad (4)$$

as

$$y \rightarrow \infty u = U_e(x) \frac{\partial u}{\partial y} = 0, T = T_\infty \quad (5)$$

The assumed wedge surface temperature is $T_w(x) = T_\infty + kx^m$, where k is constant and T_∞ is temperature of the moving fluid. The total wedge angle is equal to $\varphi = \pi\beta$ where $\beta = \frac{2m}{m+1}$ is wedge angle parameter. The non-dimensional form of the given system of partial differential equations is obtained by introducing the following stream function and the similarity variables [76].

$$\eta = \sqrt{\frac{u_e(x)}{ux}}, \psi = \sqrt{v x u_e(x)} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (6)$$

Where in terms of stream function $\frac{\partial \psi}{\partial x} = u$, $-\frac{\partial \psi}{\partial y} = v$

Using Eqs. (6) in Eqs. 1-3, we obtain the following non-linear system of ordinary differential equations:

$$m(f'^2(\eta) - 1) - \frac{m+1}{2} f(\eta) f''(\eta) - f'''(\eta) + M(f'(\eta) - 1) - \lambda f''(\eta) f'''(\eta) = 0, \quad (7)$$

$$(1 + N_r) \theta''(\eta) + P_r \left(\frac{m+1}{2} f(\eta) \theta'(\eta) - m f'(\eta) \theta(\eta) \right) = 0 \quad (8)$$

$$M(\eta) = \frac{\alpha B_0^2}{\rho \nu} \text{ is magnetic parameter, } \lambda = \Gamma \sqrt{\frac{2(ax^m)^3}{ux}} \text{ is}$$

Williamson parameter, $N_r = \frac{16\sigma^* T_\infty^3}{3K^* \alpha}$ is the thermal radiation parameter and $Pr = \frac{\mu c_p}{k}$ is prandtl number. The associated boundary conditions become:

$$f(0) = f_w, f'(0) = 0, \theta(0) = 1 \quad (9)$$

$$f'(\infty) = 1, \theta(\infty) = 0 \quad (10)$$

Where f_w is injection/suction parameter.

2. Solution of the Problem

The system of ordinary differential equations (7) and (8) subject to the boundary conditions (9) and (10) is first reduced to a system of first order ordinary differential equations using the substitutions $f' = Q, u' = R, \theta' = t$. This gives

$$m(Q^2(\eta) - 1) - \frac{m+1}{2} f(\eta) R(\eta) - R'(\eta) + M(u(\eta) - 1) - \lambda R(\eta) R'(\eta) = 0 \quad (11)$$

$$(1 + N_r) t'(\eta) + P_r \left(\frac{m+1}{2} f(\eta) t(\eta) - m u(\eta) \theta(\eta) \right) = 0 \quad (12)$$

With the boundary conditions

$$f(0) = f_w, u(0) = 0, \theta(0) = 1, u(\infty) = 1, \theta(\infty) = 0 \quad (13)$$

The resulting system in Eq. (11-13) is solved numerically with the help of 5th order Runge-Kutta-Fehlberg method. Further details about the obtained numerical solutions are presented in the next section.

Table 1. Comparison of $f''(0)$ for $M=0$, $\lambda=0$ and $\beta=1$.

f_w	Ishak et al. [78]	Yih [79]	Rashidi et al. [76]	Current result
-1	0.7566	0.75658	0.75658018	0.75659
-0.5	0.9692	0.96923	0.96922982	0.969227
0	1.2326	1.23259	1.23259365	1.23251
0.5	1.5418	1.541745	1.54175172	1.54163
1	1.8893	1.88931	1.88931809	1.888937

3. Results and Discussion

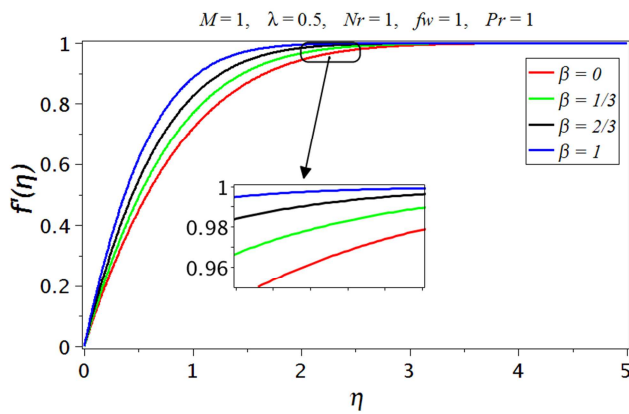


Figure 1. Variation of wedge angle parameter on velocity profile.

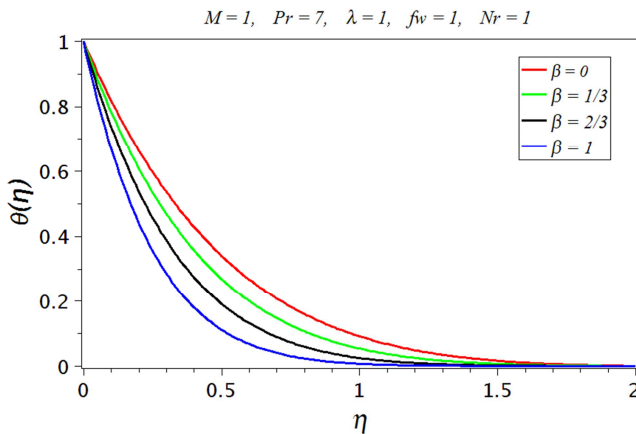


Figure 2. Variation of wedge angle parameter on temperature profile.

The transformed governing equations (11-12) subjected to boundary conditions (13) are solved numerically by employing the fifth order Runge-Kutta-Fehlberg method. The influence of all pertinent parameters on flow and heat transfer are graphed and discussed in Figures (1-8). To

examine accuracy of our work a comparison has been made with the available works of Ishak et al. [78], Yih [79] and Rashidi et al. [76] in Table 1. The agreement of our work with the prior results is stable. Figures (1-2) illustrate the influence of wedge angle parameter β with on velocity and temperature profile. It is observed that velocity increases by increasing the wedge angle parameter β , but the thermal boundary layer thickness is decreased. Since the wedge angle parameter β is a dependent over the pressure gradient, and its values may be positive or negative.

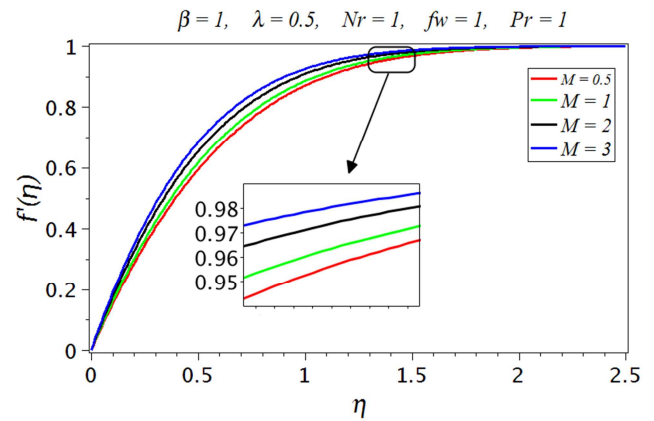


Figure 3. Variation of magnetic parameter M on velocity profile.

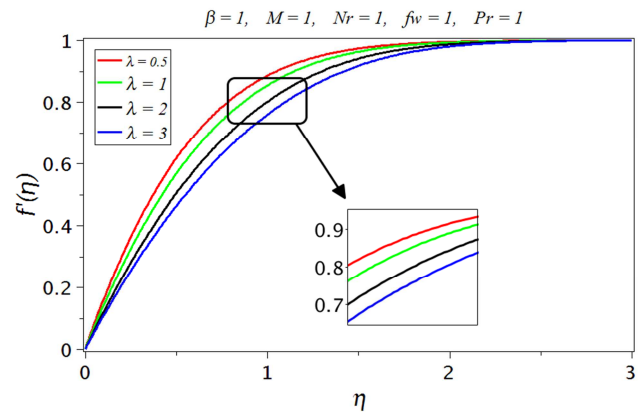


Figure 4. Variation of Williamson parameter λ on velocity profile.

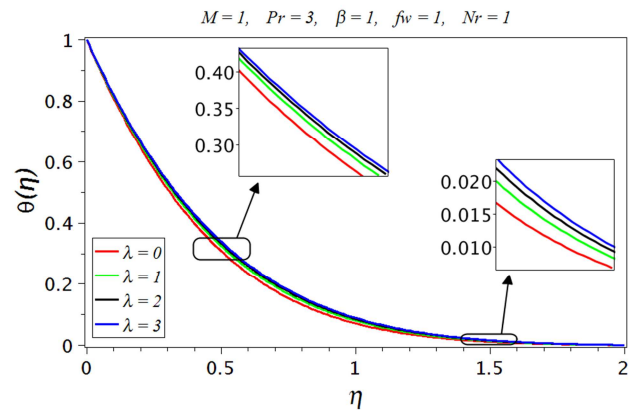


Figure 5. Variation of Williamson parameter λ on temperature profile.

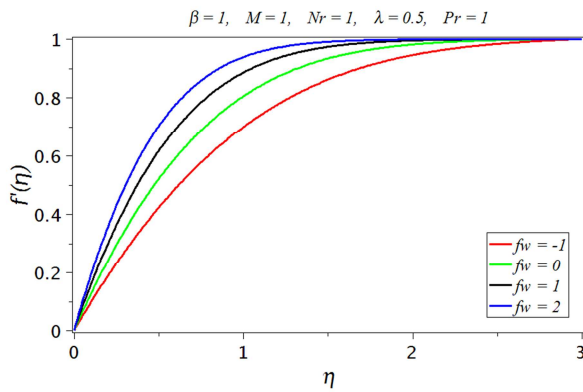


Figure 6. Variation of suction parameter on velocity profile.

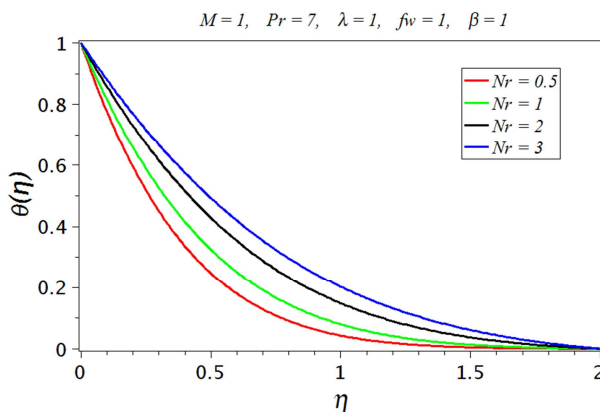


Figure 7. Variation of thermal radiation parameter Nr on temperature profile.

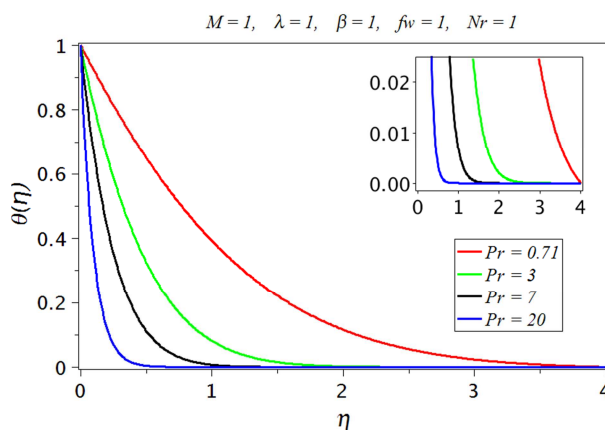


Figure 8. Variation of Prandtl number Pr on temperature profile.

Figure 3 displays the velocity profile for various values of the magnetic parameter M , the ratio of electromagnetic force to the viscous force that quantifies the intensity of applied magnetic field. It is observed from graph that with an increase in magnetic parameter M , there is a decline in the velocity distribution. This is due to the appliance of the transverse magnetic field which declines the fluid speed. This eminent phenomenon is known as Lorentz force that squeezes the momentum boundary layer. Figures 4-5 represent velocity and temperature profile for various values of Williamson parameter. It is apparent that a raise in Williamson parameter λ decreases velocity of the fluid flow.

Furthermore an increase in λ may cause increase in temperature of flow. Figure 6 drafts the non-dimensional velocity f' for different values of suction parameter f_w . From figure it is observed that an increase in the value of f_w , results in an increase in velocity. Figures 7-8 illustrate the behavior of thermal radiation and Prandtl number on fluid flow region with $M = f_w = \lambda = \beta = 1$. It is clear from graph that an increase in thermal radiation parameter leads to increase in temperature and thermal boundary layer thickness. The influence of thermal radiation is to enhance the amount of heat, while in other hand an increase in values of Prandtl number causes to decline the temperature distribution. Because the prandtl number is the relation of momentum diffusivity to thermal diffusivity, when it increases then it decreases the thermal boundary layer thickness and temperature but increases thermal capacity of fluid. Generally prandtl number is applicable in heat transform problem in order to decrease the thickness of the boundary layer and momentum.

4. Conclusion

The steady, incompressible two dimensional boundary layer flow of Williamson fluid past a porous wedge is analyzed numerically using the 5th order Fehlberg technique. The important conclusions of the analysis are

1. The non-dimensional velocity profile increases by increasing the wedge angle parameter β .
2. The non-dimensional temperature profile decreases by increasing the wedge angle parameter β .
3. The non-dimensional velocity profile decreases by increasing the magnetic parameter M .
4. The non-dimensional velocity profile decreases by increasing the Williamson parameter λ .
5. The non-dimensional temperature profile increases by increasing the Williamson parameter λ .

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