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On Generalized Interval Valued Fuzzy Soft Matrices

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Abstract: Interval valued fuzzy soft set was a combination of the interval valued fuzzy set and soft set, while in generalized interval valued fuzzy soft set a degree was attached with the parameterization of fuzzy sets in defining an interval valued fuzzy soft set. In this paper we introduced the concept of generalized interval valued fuzzy soft matrices. We discussed some of its types and some operations. We also discussed about the similarity of two generalized interval valued fuzzy soft matrices.

Keywords: Interval Valued Fuzzy Soft Set (IVFSS), Generalized Fuzzy Soft Set, Generalized IVFSS, Similarity

1. Introduction

A lot of problems in our real life in economics, social sciences, medical sciences, environmental sciences and engineering etc. involve various uncertainties. Many theories have been developed to deal with these uncertainties. Some of these theories are probability theory [1], fuzzy set theory (FST) [2], rough set theory (RST) [3], interval mathematics [4] and intuitionistic fuzzy set theory (IFST) [5] etc. Molodtsov [6] pointed out that all these theories have some inherent difficulties. He proposed soft set theory (SST) to overcome these difficulties. It was a generic mathematical tool for dealing problems having uncertainty. Later Maji and Biswas [7] defined soft subset and soft super set. They also defined absolute soft set and null soft set. They introduced some operations on soft set and De Morgan's laws are also verified by them. Ali et al [8] pointed out some errors of the previous work and introduced some new operations. They further studied more and discussed some algebraic structures of soft sets. Maji et al. [9] proposed fuzzy soft set (FSS), an improvement of the SST by combining (FST) and (SST). Roy and Maji [10] gave an application of fuzzy soft set in decision making. Yang et al. [11] introduced the interval-valued fuzzy soft set (IVFSS) which was a combination of the IVFS and SST.

Majumdar and Samanta [12] introduced the concept of generalized fuzzy soft sets (GFSS). B. K. Saikia et al. [13] defined generalized fuzzy soft matrix (GFSM) and applied it to a decision making (DM) problem. Shawkat Alkhazaleh and Abdul Razak Salleh [14] introduced generalized interval valued fuzzy soft set (GIVFSS). In their generalization of FSS, they attached a degree with the parameterization of fuzzy sets in defining an IVFSS. They discussed various operations and properties of GIVFSS. Some of these are GIVFS subset, GIVFS equal set, generalized null interval valued fuzzy soft set (GNIVFS), generalized absolute interval valued fuzzy soft set (GAIVFS), compliment of GIVFSS, union of GIVFSS's and intersection of GIVFSS's. They defined AND and OR operations on GIVFSS and similarity measure of two GIVFSS's. They also give some applications of GIVFSS in DM problem and medical diagnosis.

Mi Jung Son [15] introduced interval valued fuzzy soft set and defined some of its types. P. Rajarajeswari and P. Dhanalakshmi [16] developed interval-valued fuzzy soft matrix theory. Zulqarnain. M and M. Saeed [17] defined some new types of interval valued fuzzy soft matrix and gave an application of IVFSM in a decision making problem. Anjan Mukherjee and Sadhan Sarkar [18, 19] introduced Similarity measures for interval-valued intuitionistic fuzzy soft sets and gave applications in medical diagnosis problems. B. Chetia and P. K. Das [20] used interval-valued fuzzy soft sets and Sanchez's approach for medical diagnosis. In recent years many researchers [21-25] have been worked on applications of interval valued fuzzy soft sets.

In this paper we extended the concept of GIVFSS and introduced generalized interval valued fuzzy soft matrix (GIVFSM). We defined different types of GIVFSM's and studied some properties. We also discussed some operators on the basis of weights and some of their properties.

2. Some Basic Definitions

Definition 2.1. [1] Let X be a universal set, P be set of parameters and I^X be the set of all fuzzy subsets of X. Let A⊆ P. A pair (F, A) is a fuzzy soft set over X, where

F: A → I^X.

Definition 2. 2. [11] An IVFSS X on a universe U is a mapping such that

$X: U \to Int([0,1])$, where $Int([0,1])$ represents the set of all closed subintervals of $[0,1]$, the set of all interval valued fuzzy sets on U is denoted by $P(U)$. Suppose that $X \in P(U)$, $\forall x \in U$,

$\mu_x(x) = [\mu^-{}_x(x), \mu^+{}_x(x)]$ is the degree of membership x to X, where $\mu^-{}_x(x)$ and $\mu^+{}_x(x)$ are the lower and upper degrees of membership of x to X respectively, such that

$$0 \le \mu^-{}_x(x) \le \mu^+{}_x(x) \le 1$$

Definition 2. 3. [26] Let X be the universal set and P be the set of parameters. Suppose that A ⊆ P and (F, A) be a fuzzy soft set. Then the matrix form of the fuzzy soft set (F, A) is given as

A = $[a_{ij}]_{m\times n}$, i = 1, 2, 3, …, m and j = 1, 2, 3, …, n

where

$$a_{ij} = \begin{cases} u_j(p_i), p_j \in A \\ 0, p_j \notin A \end{cases}$$

∀ i, j.

Here μ_j (p_i) denotes the membership of P_i in the fuzzy soft set F (p_j).

Definition 2. 4. [13] Let X be the universal set, P be the set of parameters and A ⊆ P. Let (F_λ, P) be a GFSS over (X, P). A subset of X × P, R_A = {(x, p), p ∈ P, x ∈ F_λ (p)} is a relation form of (F_λ, P), where

μR_A: X × P → [0,1] and λR_A : X × P → [0,1],

such that

R_A: (x, p) ∈ [0, 1], ∀ x ∈ X, p ∈ P

and

λR_A: (x, p) ∈ [0, 1], ∀ x ∈ X, p ∈ P.

If $[\mu_{ij}, \lambda_j]_{m\times n} = \mu R_A$ $((x_i, p_j), \lambda(x_i, p_j))$, then we can define an m × n generalized fuzzy soft matrix (GFSM) of GFSS over (X, P) as

$$[\mu_{ij}, \lambda_j]_{m\times n} = \begin{bmatrix} (\mu_{11}, \lambda_1) & (\mu_{12}, \lambda_2) & . & . & . & (\mu_{1n}, \lambda_n) \\ (\mu_{21}, \lambda_1) & (\mu_{22}, \lambda_{12}) & . & . & . & (\mu_{2n}, \lambda_{1n}) \\ . & . & . & & & . \\ . & . & . & . & & . \\ . & . & . & . & & . \\ (\mu_{m1}, \lambda_1) & (\mu_{m2}, \lambda_2) & . & . & . & (\mu_{mn}, \lambda_{1n}) \end{bmatrix}$$

Definition 2. 5. [16] Let U = $\{c_1, c_2, c_3 \ldots, c_m\}$ be the Universe set and E be the set of parameters given by E = $\{e_1, e_2, e_3 \ldots, e_n\}$. Let A be a subset of E and (F, A) be an interval valued fuzzy soft set over U and F is a mapping given by F: A→I^U, where I^U denotes the collection of all Interval valued fuzzy subsets of U. Then the Interval valued fuzzy soft set can expressed in matrix form as

A = $[a_{ij}]_{mxn}$ or Ã = $[a_{ij}]$ i = 1, 2, …, m, j =1, 2, …, n.

Where

$$a_{ij} = \begin{cases} \mu_{jL(C_i)}, if\ e_j \in A \\ [0,0], if\ e_j \notin A \end{cases}$$

The interval $[\mu_{jL}$ (c_i), μ_{ju} $(c_i)]$ represents the membership of c_i in the Interval valued fuzzy set F (e_j).

If μ_{jL} (c_i) = μ_{ju} (c_i) then the Interval- valued fuzzy soft matrix (IVFSM) reduces to a FSM.

Definition 2.6. [11] Let U be an initial Universe set and E be the set of parameters, let

A ⊆ E. A pair (F, A) is called Interval valued fuzzy soft set over U where F is a mapping given by F: A → I^U, where I^U denotes the collection of all Interval valued fuzzy subsets of U.

Definition 2.7. [14] Let U be the Universal set and E be the set of parameters. Let A ⊆ E and μ be a fuzzy subset of A. Let F: A → Ƥ(U) and μ: A → I = [0, 1] where Ƥ(U) is the collection of all Interval valued fuzzy subsets of U.

Define a function F_μ: A → Ƥ(U) × I, such that

$$F_\mu(e) = F(e) = (F(e), \mu(e))$$

$$F_\mu(e_i) = (F(e_i)(x), \mu(e_i)), \forall (e_i).$$

where $F(e_i)(x)$ is an interval value and is called degree of membership of an element x to F(e) and $\mu(e)$ is called the degree of possibility of this belongness. Then F_μ is a GIVFSS.

3. Generalized Interval Valued Fuzzy Soft Matrices

Definition 3.1 Let U be the Universal set and E be the set of parameters. Let A ⊆ E and μ be a fuzzy subset of A. Let F: A → Ƥ(U) and μ : A → I = [0, 1] where Ƥ(U) is the collection of all Interval valued fuzzy subsets of U. A function F_μ: A → Ƥ(U) × I defined as

$$F_\mu(e) = F(e) = (F(e), \mu(e)), \text{ where } F_\mu(e_j) = (F(e_j)(x_i), \mu(e_i)), \forall (e_j).$$

Then the generalized interval valued fuzzy soft set F_μ can be expressed in matrix form as

$$(F_\mu(e_j), \mu) = [a_{ij}]_{m\times n}$$

where

$$a_{ij} = \begin{cases} (F(e_j)(x_i), \mu(e_j)), \text{if } e_j \in A \text{ and } i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \\ (0, 0), \text{if } e_j € A \text{ and } i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \end{cases}$$

i.e. $[a_{ij}]_{m\times n}$ =

$$\begin{bmatrix} (F(e_1)(x_1),\mu(e_1)) & (F(e_1)(x_2),\mu(e_1)) & . \; . \; . & (F(e_1)(x_m),\mu(e_1)) \\ & (F(e_2)(x_1),\mu(e_2)) & & \\ . & & & . \\ . & & . \quad . & . \\ . & & . \; . \; . & . \\ (F(e_n)(x_1),\mu(e_n)) & & & (F(e_n)(x_m),\mu(e_n)) \end{bmatrix}$$

where $F(e_j)(x_i) = [F(e_{jL})(x_i), F(e_{jU})(x_i)]$ represents the membership of e_j in the GIVFSS $F_\mu(e_j)$, such that

$$0 \le F(e_{jL})(x_i) \le F(e_{jU})(x_i) \le 1.$$

If $F(e_j)(x_i) = \mu(e_j)$ then the GIVFSM reduces to a GFSM.

Definition 3.2. Let F_μ and G_λ be two GIVFSM's. Then F_μ is called GIVFS sub matrix of G_λ if

$F(e_j)(x_i) \le G(e_k)(x_i)$ and $\mu(e_j) \le ,\lambda(e_k)$, $\forall\, i = 1,2,\dots,m$ and $j,k = 1,2,\dots,n$

Example 3.3. Consider a set of three motorbikes $U = \{b_1, b_2, b_3\}$ and a set of parameters,

$E = \{e_1, e_2, e_3\}$, where $e_1, e_2 and\ e_3$ stands for cheap, expansive and comfortable respectively. The GFSS's $F_\mu$and G_λ are defined as

$$F_\mu(e_1) = \left\{\left(\frac{b_1}{[0.1,0.4]}, 0.3\right), \left(\frac{b_2}{[0.4,0.7]}, 0.3\right), \left(\frac{b_3}{[0.2,0.4]}, 0.3\right)\right\}$$

$$F_\mu(e_2) = \left\{\left(\frac{b_1}{[0.2,0.3]}, 0.4\right), \left(\frac{b_2}{[0,0.3]}, 0.4\right), \left(\frac{b_3}{[0.2,0.5]}, 0.4\right)\right\}$$

$$F_\mu(e_3) = \left\{\left(\frac{b_1}{[0.3,0.5]}, 0.1\right), \left(\frac{b_2}{[0.1,0.2]}, 0.1\right), \left(\frac{b_3}{[0,0.2]}, 0.1\right)\right\}$$

and

$$G_\lambda(e_1) = \left\{\left(\frac{b_1}{[0.3,0.5]}, 0.4\right), \left(\frac{b_2}{[0.5,0.8]}, 0.4\right), \left(\frac{b_3}{[0.3,0.6]}, 0.4\right)\right\}$$

$$G_\lambda(e_2) = \left\{\left(\frac{b_1}{[0.4,0.5]}, 0.5\right), \left(\frac{b_2}{[0.2,0.4]}, 0.5\right), \left(\frac{b_3}{[0.4,0.6]}, 0.5\right)\right\}$$

$$G_\lambda(e_3) = \left\{\left(\frac{b_1}{[0.5,0.7]}, 0.3\right), \left(\frac{b_2}{[0.3,0.5]}, 0.3\right), \left(\frac{b_3}{[0.1,0.3]}, 0.3\right)\right\}$$

Then the matrix representation of $F_\mu$and G_λ are given as

$$F_\mu = \begin{bmatrix} ([0.1,0.4],0.3) & ([0.4,0.7],0.3) & ([0.2,0.4],0.3) \\ ([0.2,0.3],0.4) & ([0,0.3],0.4) & ([0.2,0.5],0.4) \\ ([0.3,0.5],0.1) & ([0.1,0.2],0.1) & ([0,0.2],0.1) \end{bmatrix}$$

and

$$G_\lambda = \begin{bmatrix} ([0.3,0.5],0.4) & ([0.5,0.8],0.4) & ([0.3,0.6],0.4) \\ ([0.4,0.5],0.5) & ([0.2,0.4],0.5) & ([0.4,0.6],0.5) \\ ([0.5,0.7],0.3) & ([0.3,0.5],0.3) & ([0.1,0.3],0.3) \end{bmatrix}$$

It is clear that $F_\mu$is GIVFS sub matrix of G_λ.

Definition 3.4. Let F_μ and G_λ be two GIVFSM's. Then F_μ is called GIVFS equal matrix of G_λ if

$F(e_j)(x_i) = G(e_k)(x_i)$ and $\mu(e_j) = \lambda(e_k)$, $\forall\, i = 1,2,\dots,m$ and $j,k = 1,2,\dots,n.$

Definition 3.5. Let F_μ and G_λ be two GIVFSM's. Then F_μ is called GIVFS sub matrix of G_λ if

$F(e_j)(x_i) \le G(e_k)(x_i)$, $\mu(e_j) \le \lambda(e_k)$, $\forall\, e$, $F(e_j)(x_i) < G(e_k)(x_i)$ and $\mu(e_j) < \lambda(e_k)$, for at least one e.

Definition 3.6. Let F_μ and G_λ be two GIVFSM's. Then F_μ is called GIVFS sub matrix of G_λ if

$F(e_j)(x_i) < G(e_k)(x_i)$ and $\mu(e_j) < \lambda(e_k)$, $\forall\, i = 1,2,\dots,m$ and $j,k = 1,2,\dots,n.$

Definition 3.7. A GIVFSM F_μ is called GIVFS rectangular matrix if

$F_\mu = [\, F(e_j)(x_i), \mu(e_j)\,]_{m\times n}$ and $m \ne n$, $\forall$ i, j.

Definition 3.8. A GIVFSM F_μ is called GIVFS square matrix if

$F_\mu = [\, F(e_j)(x_i), \mu(e_j)\,]_{m\times n}$ and $m = n$, $\forall$ i, j.

Definition 3.9. A GIVFSM F_μ is called GIVFS diagonal matrix if

$F_\mu = [\, F(e_j)(x_i), \mu(e_j)\,]_{m\times n}$, m= n and $(F(e_j)(x_i), \mu(e_j)) = (0, 0)$, $\forall\, i \ne j$.

Definition 3.10. A GIVFSM F_μ is called GIVFS scalar matrix if

$F_\mu = [\, F(e_j)(x_i), \mu(e_j)\,]_{m\times n}$ and $(F(e_j)(x_i), \mu(e_j)) = (k, \mu)$, $\forall$ i = j.

Definition 3.11. A GIVFSM F_μ is called GIVFS row matrix if

$F_\mu = [\, F(e_j)(x_i), \mu(e_j)\,]_{m\times n}$ and $m = 1$, $\forall$ i, j.

Definition 3.12. A GIVFSM F_μ is called GIVFS column matrix if

$F_\mu = [\, F(e_j)(x_i), \mu(e_j)\,]_{m\times n}$ and $n = 1$, $\forall$ i, j.

Definition 3.13. Let F_μ be a GIVFSM, then scalar multiple of F_μ by a scalar k is defined as

$kF_\mu = [k\, F(e_j)(x_i), k\, \mu(e_j)\,]_{m\times n}$, $\forall$ i, j and $0 \le k \le 1$.

Definition 3.14. A GIVFSM A_μ is called generalized absolute IVFSM, if

$A_\mu = [\, A(e_j)(x_i)\, , \mu(e_j)\,]_{m\times n}$, where $A(e_j)(x_i) = 1\ and\ \mu(e_j) = 1, \forall$ i, j.

Definition 3.15. A GIVFSM ϕ_μ is called generalized null IVFSM, if

$\phi_\mu = [\, \phi(e_j)(x_i)\, , \mu(e_j)\,]_{m\times n}$, where $\phi(e_j)(x_i) = 0\ and\ \mu(e_j) = 0, \forall$ i, j.

4. Some Operations on GIVFSM's

Definition 4.1. Let $F_\mu = [\, F(e_j)(x_i)\, , \mu(e_j)\,]_{m\times n}$ be a GIVFSM, where

$F(e_j)(x_i) = [F(e_{jL})(x_i), F(e_{jU})(x_i)]$, then compliment of $F_\mu$is denoted by ${F_\mu}^c = G_\lambda$ and is given by

$${F_\mu}^c = G_\lambda = \left([1 - F(e_{jU})(x_i), 1 - F(e_{jL})(x_i)], 1 - \mu(e_j) \right), \forall \text{ i, j.}$$

Example 4.2. Consider a GIVFSM F_μ as in example 3.3,

$$F_\mu = \begin{bmatrix} ([0.1,0.4],0.3) & ([0.4,0.7],0.3) & ([0.2,0.4],0.3) \\ ([0.2,0.3],0.4) & ([0,0.3],0.4) & ([0.2,0.5],0.4) \\ ([0.3,0.5],0.1) & ([0.1,0.2],0.1) & ([0,0.2],0.1) \end{bmatrix}$$

Then compliment of F_μ is given by

$$F_\mu{}^c = \begin{bmatrix} ([0.6,0.9],0.7) & ([0.3,0.6],0.7) & ([0.6,0.8],0.7) \\ ([0.7,0.8],0.6) & ([0.7,1],0.6) & ([0.5,0.8],0.6) \\ ([0.5,0.7],0.9) & ([0.8,0.9],0.9) & ([0.8,1],0.1) \end{bmatrix}$$

Proposition 4.3. Let F_μ be a GIVFSM, then $(F_\mu{}^c)^c = F_\mu$

Proof:

Since

$$F_\mu{}^c = G_\lambda = \left([F(e_{jU})(x_i), F(e_{jL})(x_i)], 1-\mu(e_j)\right)$$

then $(F_\mu{}^c)^c = G_\lambda{}^c$

but from definition, $G_\lambda = \left([F(e_{jU})(x_i), F(e_{jL})(x_i)], 1-\mu(e_j)\right)$, then

$$G_\lambda{}^c = \left([F(e_{jL})(x_i), F(e_{jU})(x_i)], (1-(1-\mu(e_j)))\right) = ([F(e_{jL})(x_i), F(e_{jU})(x_i)], \mu(e_j)) = [F(e_j)(x_i), \mu(e_j)]_{m\times n} = F_\mu$$

Definition 4.4. The union of two GIVFSM's $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{m\times n}$ and

$G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{m\times n}$, denoted by $F_\mu \cup G_\lambda$ is a GIVFSM

$[H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n}$, such that

$$[H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n} = [\sup(F(e_{jL}), G(e_{kL})), \sup(F(e_{jU}), G(e_{kU}))]_{m\times n}$$

$\gamma(e_\ell) = s(\mu(e_j), \lambda(e_k))$, where s is an $s-norm$.

Example 4.5. Consider two GIVFSM's F_μ and G_λ as in example 3.3,

$$F_\mu = \begin{bmatrix} ([0.1,0.4],0.3) & ([0.4,0.7],0.3) & ([0.2,0.4],0.3) \\ ([0.2,0.3],0.4) & ([0,0.3],0.4) & ([0.2,0.5],0.4) \\ ([0.3,0.5],0.1) & ([0.1,0.2],0.1) & ([0,0.2],0.1) \end{bmatrix}$$

and

$$G_\lambda = \begin{bmatrix} ([0.3,0.5],0.4) & ([0.5,0.8],0.4) & ([0.3,0.6],0.4) \\ ([0.4,0.5],0.5) & ([0.2,0.4],0.5) & ([0.4,0.6],0.5) \\ ([0.5,0.7],0.3) & ([0.3,0.5],0.3) & ([0.1,0.3],0.3) \end{bmatrix}$$

Then the union of F_μ and G_λ is given by

$$F_\mu \cup G_\lambda = \begin{bmatrix} ([0.3,0.5],0.4) & ([0.5,0.8],0.4) & ([0.3,0.6],0.4) \\ ([0.4,0.5],0.5) & ([0.2,0.4],0.5) & ([0.4,0.6],0.5) \\ ([0.5,0.7],0.3) & ([0.3,0.5],0.3) & ([0.1,0.3],0.3) \end{bmatrix}$$

Proposition 4.6. Let F_μ be a GIVFSM, then

$$F_\mu \cup F_\mu = F_\mu$$

Proof: From Definition, we have

$$[F(e_j)(x_i), \mu(e_j)]_{m\times n} \cup [F(e_j)(x_i), \mu(e_j)]_{m\times n} = [H(e_j)(x_i), \gamma(e_j)]_{m\times n}$$

Such that

$$[H(e_j)(x_i), \gamma(e_j)]_{m\times n} = [\sup(F(e_{jL}), F(e_{jL})), \sup(F(e_{jU}), F(e_{jU}))]_{m\times n}$$

$$= [F(e_j)(x_i), \mu(e_j)]_{m\times n}$$

$$= F_\mu$$

Proposition 4.7. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{m\times n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{m\times n}$ be two GIVFSM's, then

a) $F_\mu \cup G_\lambda = G_\lambda \cup F_\mu$

b) $F_\mu \cup A_\mu = A_\mu$

c) $F_\mu \cup \phi_\mu = F_\mu$

Proof:

(a) From Definition, we have

$$[F(e_j)(x_i), \mu(e_j)]_{m\times n} \cup [G(e_k)(x_i), \lambda(e_k)]_{m\times n} = [H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n}$$

Such that

$$[H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n} = [\sup(F(e_{jL}), G(e_{kL})), \sup(F(e_{jU}), G(e_{kU}))]_{m\times n}$$

But $H_\gamma = F_\mu \cup G_\lambda = G_\lambda \cup F_\mu$ (since union of GIVFSM's is commutative)

and $\gamma(e_\ell) = s\left(\mu(e_j), \lambda(e_k)\right) = s\left(\lambda(e_k), \mu(e_j)\right)$, (since $s-norm$ is commutative)

Then,

$$F_\mu \cup G_\lambda = G_\lambda \cup F_\mu$$

The proof of (b) and (c) are straight forward from definition.

Definition 4.8. The intersection of two GIVFSM's $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{m\times n}$ and

$G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{m\times n}$, denoted by $F_\mu \cap G_\lambda$ is a GIVFSM $[H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n}$, such that

$$[H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n} = [\inf(F(e_{jL}), G(e_{kL})), \inf(F(e_{jU}), G(e_{kU}))]_{m\times n}$$

$\gamma(e_\ell) = t(\mu(e_j), \lambda(e_k))$, where t is a $t-norm$.

Example 4.9. Consider two GIVFSM's F_μ and G_λ as in example 3.3,

$$F_\mu = \begin{bmatrix} ([0.1,0.4],0.3) & ([0.4,0.7],0.3) & ([0.2,0.4],0.3) \\ ([0.2,0.3],0.4) & ([0,0.3],0.4) & ([0.2,0.5],0.4) \\ ([0.3,0.5],0.1) & ([0.1,0.2],0.1) & ([0,0.2],0.1) \end{bmatrix}$$

and

$$G_\lambda = \begin{bmatrix} ([0.3,0.5], 0.4) & ([0.5,0.8], 0.4) & ([0.3,0.6], 0.4) \\ ([0.4,0.5], 0.5) & ([0.2,0.4], 0.5) & ([0.4,0.6], 0.5) \\ ([0.5,0.7], 0.3) & ([0.3,0.5], 0.3) & ([0.1,0.3], 0.3) \end{bmatrix}$$

Then the intersection of F_μ and G_λ is given by

$$F_\mu \cap G_\lambda = \begin{bmatrix} ([0.1,0.4], 0.3) & ([0.4,0.7], 0.3) & ([0.2,0.4], 0.3) \\ ([0.2,0.3], 0.4) & ([0,0.3], 0.4) & ([0.2,0.5], 0.4) \\ ([0.3,0.5], 0.1) & ([0.1,0.2], 0.1) & ([0,0.2], 0.1) \end{bmatrix}$$

Proposition 4.10. Let F_μ be a GIVFSM, then

$$F_\mu \cap F_\mu = F_\mu$$

Proof: From Definition, we have

$$[F(e_j)(x_i), \mu(e_j)]_{m\times n} \cap [F(e_j)(x_i), \mu(e_j)]_{m\times n} = [H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n}$$

such that

$$[H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n} = [\inf(F(e_{jL}), F(e_{jL})), \inf(F(e_{jU}), F(e_{jU}))]_{m\times n}$$

$$= [F(e_j)(x_i), \mu(e_j)]_{m\times n}$$

$$= F_\mu$$

Proposition 4.11. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{m\times n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{m\times n}$ be two GIVFSM's, then

(a) $F_\mu \cap G_\lambda = G_\lambda \cap F_\mu$

(b) $F_\mu \cap A_\mu = F_\mu$

(c) $F_\mu \cap \phi_\mu = \phi_\mu$

Proof:

(a) From Definition, we have

$$[F(e_j)(x_i), \mu(e_j)]_{m\times n} \cap [G(e_k)(x_i), \lambda(e_k)]_{m\times n} = [H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n}$$

Such that

$$[H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n} = [\inf(F(e_{jL}), G(e_{kL})), \inf(F(e_{jU}), G(e_{kU}))]_{m\times n}$$

But $H_\gamma = F_\mu \cap G_\lambda = G_\lambda \cap F_\mu$ (since intersection of GIVFSM's is commutative)

and $\gamma(e_\ell) = t\left(\mu(e_j), \lambda(e_k)\right) = t\left(\lambda(e_j), \mu(e_k)\right)$, (since $t-norm$ is commutative)

Then, $F_\mu \cap G_\lambda = G_\lambda \cap F_\mu$

The proof of (b) and (c) are straight forward from definition.

Proposition 4.12. *Let* $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{m\times n}$, $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{m\times n}$ *and* $H_\gamma = [H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n}$ *be three GIVFSM's, then*

(a) $F_\mu \cap (G_\lambda \cap H_\gamma) = (F_\mu \cap G_\lambda) \cap H_\gamma$

(b) $F_\mu \cup (G_\lambda \cup H_\gamma) = (F_\mu \cup G_\lambda) \cup H_\gamma$

Proposition 4.13. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{m\times n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{m\times n}$ be two GIVFSM's, then

(a) $(F_\mu \, U \, G_\lambda)^c = F_\mu{}^c \cap G_\lambda{}^c$

(b) $(F_\mu \cap G_\lambda)^c = F_\mu{}^c U \, G_\lambda{}^c$

Proof:

(a) Consider $F_\mu{}^c \cap G_\lambda{}^c = \left([F(e_{jU})(x_i), F(e_{jL})(x_i)], 1-\mu(e_j)\right) \cap \left([G(e_{kU})(x_i), G(e_{kL})(x_i)], 1-\lambda(e_k)\right)$

$$= ([\inf(F(e_{jL}), G(e_{kL})), \inf(F(e_{jU}), G(e_{kU}))], (1-\mu(e_j)) \cap (1-\lambda(e_k)))$$

$$= (([F(e_{jU})(x_i), F(e_{jL})(x_i)] \cup [G(e_{kU})(x_i), G(e_{kL})(x_i)])^c, ((1-\mu(e_j)) \cup (1-\lambda(e_k)))^c)$$

$$= ([F(e_j)(x_i), \mu(e_j)] \cup [G(e_k)(x_i), \lambda(e_k)])^c$$

$$= (F_\mu \cup G_\lambda)^c$$

(b) The proof is similar to proof of (a).

Proposition 4.14. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{m\times n}$, $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{m\times n}$ and $H_\gamma = [H(e_\ell)(x_i), \gamma(e_\ell)]_{m\times n}$ be three GIVFSM's, then

(a) $F_\mu \cup (G_\lambda \cap H_\gamma) = (F_\mu \cup G_\lambda) \cap (F_\mu \cup H_\gamma)$

(b) $F_\mu \cap (G_\lambda \cup H_\gamma) = (F_\mu \cap G_\lambda) \cup (F_\mu \cap H_\gamma)$

Proof:

(a) $F_\mu \cup (G_\lambda \cap H_\gamma) = [\sup\left(F(e_{jL}), (G(e_{kL}) \cap H(e_{\ell L}))\right), \sup\left(F(e_{jU}), (G(e_{kU}) \cap H(e_{\ell U}))\right)]$

$$= [\sup(F(e_{jL}), \inf(G(e_{jL}), H(e_{\ell L}))), \sup(F(e_{jU}), \inf(G(e_{jU}), H(e_{\ell U})))]$$

$$= [\inf(\sup(F(e_{jL}), (G(e_{kL})), \sup((F(e_{jL}), H(e_{\ell L}))), \inf(\sup(F(e_{jU}), (G(e_{jU})), \sup((F(e_{jU}), H(e_{\ell U})))]$$

$$= \left(F\left(e_{jL}\right), F\left(e_{jU}\right)\right) \cup \left(G(e_{kL}), G(e_{kU})\right) \cap \left(\left(F\left(e_{jL}\right), F\left(e_{jU}\right)\right) \cup \left(H(e_{\ell L}), H(e_{\ell U})\right)\right)$$

and

$$\left(\mu(e_j) \cup (\lambda(e_k) \cap \gamma(e_\ell)\right) = max\{\left(\mu(e_j), (\lambda(e_k) \cap \gamma(e_\ell)\right)\}$$
$$= max\{\mu(e_j), min(\lambda(e_k) \cap \gamma(e_\ell))\}$$
$$= min\{max\left(\mu(e_j), \lambda(e_k)\right), max\left(\mu(e_j), \gamma(e_\ell)\right)\}$$
$$= min\{\mu(e_j) \cup \lambda(e_k), (\mu(e_j) \cup (\gamma(e_\ell)\}$$
$$= (\mu(e_j) \cup \lambda(e_k)) \cap ((\mu(e_j) \cup (\gamma(e_\ell))$$

(b) The proof is similar to proof of (a).

Definition 4.15. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)]_{m\times n}$ be two GIVFSM's, then AND product of F_μ and G_λ is denoted by $F_\mu \wedge G_\lambda$ and is defined as

$$F_\mu \wedge G_\lambda = H_\gamma$$

Such that $H_\gamma = [H(e_\ell)(x_i)\,, \gamma(e_\ell)]_{m\times n}$
where $H(e_\ell)(x_i) = F(e_j)(x_i) \cap G(e_k)(x_i),\ F(e_j), G(e_k) \in F(e_j) \times G(e_k)\ \forall\, x_i, e_j, e_k$.
and $\gamma(e_\ell) = t(\mu(e_j), \lambda(e_k))$, where $\mu(e_j), \lambda(e_k)), \in \mu(e_j) \times \lambda(e_k)\ \forall$ j, k and t is a $t - norm$.

Example 4.16. Consider two GIVFSM's F_μ and G_λ as in example 3.3,

$$F_\mu = \begin{bmatrix} ([0.1,0.4], 0.3) & ([0.4,0.7], 0.3) & ([0.2,0.4], 0.3) \\ ([0.2,0.3], 0.4) & ([0,0.3], 0.4) & ([0.2,0.5], 0.4) \\ ([0.3,0.5], 0.1) & ([0.1,0.2], 0.1) & ([0,0.2], 0.1) \end{bmatrix}$$

and

$$G_\lambda = \begin{bmatrix} ([0.3,0.5], 0.4) & ([0.5,0.8], 0.4) & ([0.3,0.6], 0.4) \\ ([0.4,0.5], 0.5) & ([0.2,0.4], 0.5) & ([0.4,0.6], 0.5) \\ ([0.5,0.7], 0.3) & ([0.3,0.5], 0.3) & ([0.1,0.3], 0.3) \end{bmatrix}$$

Then $F_\mu \wedge G_\lambda$ is given by

$$F_\mu \wedge G_\lambda = \begin{bmatrix} ([0.1,0.4], 0.3) & ([0.4,0.7], 0.3) & ([0.2,0.4], 0.3) \\ ([0.1,0.4], 0.3) & ([0.2,0.4], 0.3) & ([0.2,0.4], 0.3) \\ ([0.1,0.4], 0.3) & ([0.3,0.5], 0.3) & ([0.1,0.3], 0.3) \\ ([0.2,0.3], 0.4) & ([0,0.3], 0.4) & ([0.2,0.5], 0.4) \\ ([0.2,0.3], 0.4) & ([0,0.3], 0.4) & ([0.2,0.5], 0.4) \\ ([0.2,0.3], 0.3) & ([0,0.3], 0.3) & ([0.1,0.3], 0.3) \\ ([0.3,0.5], 0.1) & ([0.1,0.2], 0.1) & ([0,0.2], 0.1) \\ ([0.3,0.5], 0.1) & ([0.1,0.2], 0.1) & ([0,0.2], 0.1) \\ ([0.3,0.5], 0.1) & ([0.1,0.2], 0.1) & ([0,0.2], 0.1) \end{bmatrix}$$

Definition 4.17. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)]_{m\times n}$ be two GIVFSM's, then OR product of F_μ and G_λ is denoted by $F_\mu \vee G_\lambda$ and is defined as

$$F_\mu \vee G_\lambda = H_\gamma$$

Such that $H_\gamma = [\, H(e_\ell)(x_i)\,, \gamma(e_\ell)]_{m\times n}$

where

$$H(e_\ell)(x_i) = F(e_j)(x_i) \cap G(e_j)(x_i),\ F(e_j), G(e_k) \in F(e_j) \times G(e_k) \forall\, x_i, e_j, e_k.$$

and

$\gamma(e_\ell) = s(\mu(e_j), \lambda(e_k))$, where $\mu(e_j), \lambda(e_k)), \in \mu(e_j) \times \lambda(e_k)\ \forall$ j, k and s is an $s - norm$.

Example 4.18. Consider two GIVFSM's F_μ and G_λ as in example 3.3,

$$F_\mu = \begin{bmatrix} ([0.1,0.4], 0.3) & ([0.4,0.7], 0.3) & ([0.2,0.4], 0.3) \\ ([0.2,0.3], 0.4) & ([0,0.3], 0.4) & ([0.2,0.5], 0.4) \\ ([0.3,0.5], 0.1) & ([0.1,0.2], 0.1) & ([0,0.2], 0.1) \end{bmatrix}$$

and

$$G_\lambda = \begin{bmatrix} ([0.3,0.5], 0.4) & ([0.5,0.8], 0.4) & ([0.3,0.6], 0.4) \\ ([0.4,0.5], 0.5) & ([0.2,0.4], 0.5) & ([0.4,0.6], 0.5) \\ ([0.5,0.7], 0.3) & ([0.3,0.5], 0.3) & ([0.1,0.3], 0.3) \end{bmatrix}$$

Then $F_\mu \vee G_\lambda$ is given by

$$F_\mu \vee G_\lambda = \begin{bmatrix} ([0.1,0.4], 0.4) & ([0.4,0.7], 0.4) & ([0.2,0.4], 0.4) \\ ([0.1,0.4], 0.5) & ([0.2,0.4], 0.5) & ([0.2,0.4], 0.5) \\ ([0.1,0.4], 0.3) & ([0.3,0.5], 0.3) & ([0.1,0.3], 0.3) \\ ([0.2,0.3], 0.4) & ([0,0.3], 0.4) & ([0.2,0.5], 0.4) \\ ([0.2,0.3], 0.5) & ([0,0.3], 0.5) & ([0.2,0.5], 0.5) \\ ([0.2,0.3], 0.3) & ([0,0.3], 0.3) & ([0.1,0.3], 0.3) \\ ([0.3,0.5], 0.4) & ([0.1,0.2], 0.4) & ([0,0.2], 0.4) \\ ([0.3,0.5], 0.5) & ([0.1,0.2], 0.5) & ([0,0.2], 0.5) \\ ([0.3,0.5], 0.3) & ([0.1,0.2], 0.3) & ([0,0.2], 0.3) \end{bmatrix}$$

Definition 4.19. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)\,]_{m\times n}$ be two GIVFSM's, then arithmetic mean of F_μ and G_λ, denoted by $F_\mu @ G_\lambda$ is defined by

$$F_\mu @ G_\lambda = H_\gamma,$$

where

$$H_\gamma = [H(e_\ell)(x_i)\,, \gamma(e_\ell)]$$

such that

$$H(e_\ell)(x_i) = [H(e_{\ell L})(x_i), H(e_{\ell U})(x_i)]$$

$$H(e_{\ell L})(x_i) = \frac{F(e_{jL})(x_i) + G(e_{kL})(x_i)}{2},\ H(e_{\ell U})(x_i) = \frac{F(e_{jU})(x_i) + G(e_{kU})(x_i)}{2}$$

and

$$\gamma(e_\ell) = \frac{\mu(e_j) + \lambda(e_k)}{2}$$

Definition 4.20. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)\,]_{m\times n}$ be two GIVFSM's, then weighted arithmetic mean of F_μ and G_λ, denoted by $F_\mu @^w G_\lambda$ is defined by

$$F_\mu @^w G_\lambda = H_\gamma{}^w,$$

Where

$$H_\gamma{}^w = [H^w(e_\ell)(x_i)\,, \gamma^w(e_\ell)]$$

such that

$$H^w(e_\ell)(x_i) = [H^w(e_{\ell L})(x_i), H^w(e_{\ell U})(x_i)]$$

$$H^w(e_{\ell L})(x_i) = \frac{w_1 F(e_{jL})(x_i) + w_2 G(e_{kL})(x_i)}{w_1+w_2},\ H^w(e_{\ell U})(x_i) = \frac{w_1 F(e_{jU})(x_i) + w_2 G(e_{kU})(x_i)}{w_1+w_2}$$

and

$$\gamma^w(e_\ell) = \frac{w_1\mu(e_j) + w_2\lambda(e_k)}{w_1 + w_2}$$

Definition 4.21. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)]_{m\times n}$ be two GIVFSM's, then geometric mean of F_μ and G_λ, denoted by $F_\mu \$ G_\lambda$ is defined by

$$F_\mu \$ G_\lambda = \left(\left[\sqrt{F(e_{jL})(x_i)\,.\,G(e_{kL})(x_i)}\,, \sqrt{F(e_{jU})(x_i)\,.\,G(e_{kU})(x_i)}\right], \sqrt{\mu(e_j).\lambda(e_k)}\right)$$

Definition 4.22. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)\,]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)\,]_{m\times n}$ be two GIVFSM's, then weighted geometric mean of F_μ and G_λ, denoted by $F_\mu \$^w G_\lambda$ is defined by

$$F_\mu \$^w G_\lambda = \left(\left[\left(F(e_{jL})(x_i)^{\,w_1}.\,G(e_{kL})(x_i)^{\,w_2}\right)^{\frac{1}{w_1+w_2}}, \left(F(e_{jU})(x_i)^{\,w_1}.\,G(e_{kU})(x_i)^{\,w_2}\right)^{\frac{1}{w_1+w_2}}\right], \left(\mu(e_j)^{w_1}.\,\lambda(e_k)^{w_2}\right)^{\frac{1}{w_1+w_2}}\right)$$

Definition 4.23. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)]_{m\times n}$ be two GIVFSM's, then harmonic mean of F_μ and G_λ, denoted by $F_\mu \& G_\lambda$ is defined by

$$F_\mu \& G_\lambda = \left(\left[2.\frac{F(e_{jL})(x_i)\,.\,G(e_{kL})(x_i)}{F(e_{jL})(x_i) + G(e_{kL})(x_i)}, 2.\frac{F(e_{jU})(x_i)\,.\,G(e_{kU})(x_i)}{F(e_{jU})(x_i) + G(e_{kU})(x_i)}\right], 2.\frac{\mu(e_j).\lambda(e_k)}{\mu(e_j) + \lambda(e_k)}\right)$$

Definition 4.24. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)\,]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)\,]_{m\times n}$ be two GIVFSM's, then weighted harmonic mean of F_μ and G_λ, denoted by $F_\mu \&^w G_\lambda$ is defined by

$$F_\mu \&^w G_\lambda = \left(\left[\frac{w_1 + w_2}{\frac{w_1}{F(e_{jL})(x_i)} + \frac{w_2}{G(e_{kL})(x_i)}}, \frac{w_1 + w_2}{\frac{w_1}{F(e_{jU})(x_i)} + \frac{w_2}{G(e_{kU})(x_i)}}\right], \frac{w_1 + w_2}{\frac{w_1}{\mu(e_j)} + \frac{w_2}{\lambda(e_k)}}\right)$$

Proposition 4.25. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)]_{m\times n}$ be two GIVFSM's, then the following holds.

(a) $F_\mu @ G_\lambda = G_\lambda @ F_\mu$

(b) $F_\mu \$ G_\lambda = \$ G_\lambda F_\mu$

(c) $F_\mu \& G_\lambda = G_\lambda \& F_\mu$

5. Similarity Between Two GIVESM's

Definition 5.1. Let $F_\mu = [\, F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ and $G_\lambda = [\, G(e_k)(x_i)\,, \lambda(e_k)\,]_{m\times n}$ be two GIVFSM's, then similarity between F_μ and G_λ, denoted by $S(F_\mu, G_\lambda)$, is defined by

$S(F_\mu, G_\lambda) = H_\gamma$, where $H_\gamma = [H(e_{\ell L})(x_i).\gamma, H(e_{\ell U})(x_i).\gamma]$,

$$H(e_{\ell L})(x_i) = \min\left(\varphi\left(F(e_{jL}), G(e_{kL})\right), \phi\left(F(e_{jU}), G(e_{kU})\right)\right)$$

$$H(e_{\ell U})(x_i) = \max\left(\varphi\left(F(e_{jL}), G(e_{kL})\right), \phi\left(F(e_{jU}), G(e_{kU})\right)\right)$$

Such that

$$\varphi\left(F(e_{jL}), G(e_{kL})\right) = \begin{cases} 0, if\ F(e_{jL}) = 0, \forall\, e_j \\ \dfrac{\sum_i^n max\left\{min\left(F(e_{jL}), G(e_{kL})\right)\right\}}{\sum_i^n max\ F(e_{jL})}, otherwise \end{cases}$$

$$\phi\left(F(e_{jU}), G(e_{kU})\right) = \frac{\sum_i^n max\left\{min\left(F(e_{jU}), G(e_{kU})\right)\right\}}{\sum_i^n max\ F(e_{jU})}$$

and

$$\gamma = 1 - \frac{\sum|\mu(e_j)-\lambda(e_k)|}{\sum|\mu(e_j)-\lambda(e_k)|}, j, k = 1,2,\dots,n.$$

Definition 5.2. Two GIVFSM's F_μ and G_λ are called significantly similar if $S(F_\mu, G_\lambda) \geq 1/2$.

Theorem 5.3. *Let* $F_\mu = [\,F(e_j)(x_i)\,, \mu(e_j)]_{m\times n}$ *and* $G_\lambda = [\,G(e_k)(x_i)\,, \lambda(e_k)]_{m\times n}$ *be two GIVFSM's, then*

(a) $S(F_\mu, G_\lambda) \neq S(G_\lambda, F_\mu)$

(b) $H(e_{\ell L})(x_i) \geq 0$ *and* $H(e_{\ell U})(x_i) \leq 1$

(c) If $F_\mu = G_\lambda$, then $S(F_\mu, G_\lambda) = 1$

Proof:

(a) The proof is straightforward and follows from definition.

(b) From definition, we have

$$\varphi\left(F(e_{jL}), G(e_{kL})\right) = \begin{cases} 0, if\ F(e_{jL}) = 0, \forall\, e_j \\ \dfrac{\sum_i^n max\left\{min\left(F(e_{jL}), G(e_{kL})\right)\right\}}{\sum_i^n max\ F(e_{jL})}, otherwise \end{cases}$$

If $F(e_{jL}) = 0$, $\forall\, e_j$, then $H(e_{\ell L})(x_i) = 0$ and if $F(e_{jL}) \neq 0$, for some e_j, then it is clear that $H(e_{\ell L})(x_i) \geq 0$.

Since

$$H(e_{\ell U})(x_i) = \max(\varphi\left(F(e_{jL}), G(e_{kL})\right), \phi\left(F(e_{jU}), G(e_{kU})\right))$$

Assume that $\varphi\left(F(e_{jL}), G(e_{kL})\right) = 1$ and $\phi\left(F(e_{jU}), G(e_{kU})\right) = 1$, then $H(e_{\ell U})(x_i) = 1$

If $\varphi\left(F(e_{jL}), G(e_{kL})\right) < 1$ and $\phi\left(F(e_{jU}), G(e_{kU})\right) < 1$, then $H(e_{\ell U})(x_i) \leq 1$.

(a) The proof is straightforward and follows from definition.

Theorem 5.4. Let $F_\mu = [\,F(e_j)(x_i)\,, \mu(e_j)\,]_{m\times n}$, $G_\lambda = [\,G(e_k)(x_i)\,, \lambda(e_k)]_{m\times n}$ and $H_\gamma = [\,H(e_\ell)(x_i)\,, \gamma(e_\ell)]_{m\times n}$ be three GIVFSM's, then the following hold:

$$F_\mu \subseteq G_\lambda \subseteq H_\gamma \Rightarrow S(F_\mu, H_\gamma) \leq S(G_\lambda, H_\gamma)$$

Proof: The proof is straightforward and follows from definition.

6. Conclusion

We have introduced the concept of GIVFSM's in this paper. Some of its types are defined. Some basic operations like union, intersection, compliment, AND operation and OR operation have been defined and exemplified. Arithmetic mean, geometric mean, harmonic mean and their weighted means are also defined and some properties of these operators are discussed. Furthermore, similarity between two GIVFSM's is discussed. To future concern, GIVFSM's can be used to solve decision making problems in situations where uncertainty involved.

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