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On a Construction of the Optimal Trajectories by Applying the Equilibrium Mechanisms in the Discrete Dynamical Models

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Abstract: The model of economic dynamics, consisting of two units that produce, respectively, means of production and objects of commodities. It is assumed that the trajectory of epy model with a fixed budget admits characteristics and has an equilibrium state. These conditions allow us to construct effective trajectories of the model with the help of the equilibrium mechanisms. The determination of the equilibrium prices serves to determine the utility function. Formulas for determining the equilibrium coefficients are given. The conditions for constructing the efficiency of the trajectory are determined.

Keywords: Equilibrium, Effective Trajectory, Characteristics

1. Introduction

Consider the two-product model Z_t of the economic dynamics[1, 2, 7-10]. The first unit produces the means of production, and the other objects of commodities. The vector $x=(x^1,x^2)\in\left(R_+^2\right)^2$ is a model state: here $x^i=\left(K^i,L^i\right)\in R_+^2$; K^i are the main funds, L^i - labor force in the i -th division $(i=1,2)$. Production activity of the i -th sector at the time t is described by the production function $F_t^i:R_+^2\to R_+$ and safety coefficient $0\le v_t^i<1$ $(i=1,2)$ of the funds. The rate of the salary which coincides in the first and in the second divisions is assumed known. Switching from state $x_t=\left(K_t^1,L_t^1,K_t^2,L_t^2\right)$ to state $x_{t+1}=\left(K_{t+1}^1,L_{t+1}^1,K_{t+1}^2,L_{t+1}^2\right)$ is possible if

$$K_{t+1}^1+K_{t+1}^2\le v_t^1K_t^1+v_t^2K_t^2+F_t^1\left(K_t^1,L_t^1\right)\tag{1}$$

$$\omega_{t+i}\left(L_{t+1}^1+L_{t+1}^2\right)\le F_t^2\left(K_t^2,L_t^2\right)\tag{2}$$

$$F_t^1\left(K_t^1,L_t^1\right)=\min\left(\frac{K_t^1}{C_t^{11}},\frac{K_t^1}{C_t^{21}}\right),\tag{3}$$

$$F_t^2\left(K_t^2,L_t^2\right)=\min\left(\frac{K_t^2}{C_t^{12}},\frac{K_t^2}{C_t^{22}}\right),\tag{4}$$

$$C_t^{ij}>0\quad (i,j=1,2)$$

Here the coefficients C_t^{ij} stand for the product number of the i -th division that is necessary for producing the unit product of the j -th division at the moment t. Denote by a_t productive mapping of the model. The set $a_t(x_t)$ consists of the vectors x_{t+1}, for which the relation (1) and (2) are valid.

In the investigation of the model Z_t the simplest super linear mapping of the form

$$\sigma(K,L)=\left\{\left(K^1,L^1\right)\,\middle|\,K^1\ge 0,L^1\ge 0,K^1+\omega L^1\le vK+\min\left(\frac{K}{C^1},\frac{L}{C^2}\right)\right\},\tag{5}$$

Is used [3], where $C^1>0$, $C^2>0$. The Neumann-Gale model Z, defined by the mapping σ, we write in the form $Z=(\sigma,\omega,C^1,C^2)$.

We give some definitions and facts [4]. We consider a special model m with fixed budgets having the form $m = (\{y\}, u, \Lambda)$ where $y >> 0$ is an element of the cone $R_+^n, u = (u^1, ..., u^n), u^i$ is an utility function, defined by the equalities

$$u^i(\overline{f}, x^i) - [\overline{f}, B^i x^i] + \overline{f}^i F^i(x^i), \ (i = \overline{1,n})$$

Here B^i is a safety matrix, the main diagonal of which has a form $\vartheta^{i1}, ..., \vartheta^{in}$ where ϑ^{ij} is a safety coefficient.

Note that the functions $u^i (i = \overline{i,n})$ presents the volume of all funds belonging to the corresponding branch by the prices $\overline{f} = (\overline{f}^i, ..., \overline{f}^n)$.

It should be noted that the most interesting is the case, when the prices of products are the same for all branches, in other words, depend only on the products themselves. As Λ in the model m is denoted the vector $\Lambda = (\lambda_1, ..., \lambda_n)$, the coordinate λ_i of which is a given budget of the $i-th$ branch. The model m describes the distribution the produced product y between n branches.

The set of vectors $p, \overline{x}^1, ..., \overline{x}^n$ forms the equilibrium state in the model m, if the vectors $\overline{x}^i$ are the solutions of the problems

$$u^i(\overline{f}, x^i) \to \max \ (i = \overline{1,n})$$

subject to $[p, x^i] = \lambda_i, \ x^i >> 0$, and moreover the relations

$$\sum_{i=1}^{n} \overline{x}^i = \text{y,p} \geq 0$$

are fulfilled.

We construct the effective by the help of equilibrium mechanisms.

Let $x(t) = (x^1), ..., x^m(t)$ be an effective trajectory and $\sum_{i=1}^{n} x^i(0) >> 0$. (the record $x >> 0$ means that all coordinates of the vector x are positive). As is known [1, 2, 6] this trajectory admits characteristics and is valid

$$\max \frac{u^i(f(t+1), x)}{[f(t), x]} = \frac{u^i(f(t+1), x^i(t))}{[f(t), x^i(t)]} = 1, \ (i \in I)$$

where

$$I = \left\{ i : [f(t), x^i(t)] > 0 \right\},$$

$$u^i(f(t+1), x) = \max\{[f(t+1)y] : y \in a_t^i(x)\}.$$

Due to the homogeneity of the functions $u^i(f, \cdot)$ the last equality is equivalent to the following relations

$$\max_{x>0,[f(t),x]} u^i(f(t+1), x(t)) = u^i(f(t+1), x^i(t)) = 1,$$

where

$$\lambda_i = [f(t+1), x^i(t)], \ i \in I.$$

This fact shows that the effective trajectory may be obtained by the help of the equilibrium mechanisms.

2. Main Results

Now let's study the effective trajectories of the described above general model Z_t. Consider the trajectory $(x_t)_{t=0}^{\infty}$ the vectors of which $x_t = \left(K_t^1, L_t^1, K_t^2, L_t^2\right)$ are strictly positive $\left(K_t^i > 0, \ L_t^i > 0, \ i = 1,2\right)$. Let this trajectory admits characteristics $\tilde{l}_t$ [1,2,5]. Then the numbers $b_t^1 \geq 0, b_t^2 \geq 0$ may be found such that

$$\tilde{l}_t = \left(b_t^1, b_t^2, b_t^1, b_t^2\right) = l_t^{\Delta} \tag{6}$$

where $l_t = \left(b_t^1, b_t^2\right), \ l_t^{\Delta} = (l_t, l_t)$. Suppose that $b_t^1 > 0$ for all t and write the vector l_t in the form

$$l_t = b_t^1 P_t, \tag{7}$$

where

$$P_t = (1, b_t \omega_t). \tag{8}$$

Denote

$$\begin{aligned} W_t^1(x, q_{t+1}) &= v_t^1 K + \min\left(\frac{K}{C_t^{11}}, \frac{L}{C_t^{21}}\right), \\ W_t^2(x, q_{t+1}) &= v_t^2 K + b_{t+1} \min\left(\frac{K}{C_t^{12}}, \frac{L}{C_t^{22}}\right). \end{aligned} \tag{9}$$

As follows from this definition the set $\left(q_t, x_t^1, x_t^2\right)$, where $x_t^1 = \left(K_t^1, L_t^1\right), \ x_t^2 = \left(K_t^2, L_t^2\right), q_t = (1, b_t \cdot \omega_t)$ is an equilibrium state in the model with fixed budgets utility functions of the participants of which coincide with W_t^1 and W_t^2 correspondingly and the budgets $\lambda_t^i \ (i = 1,2)$ are defined by the formulas $\lambda_t^i = \left[l_t, x_t^i\right]$.

Thus, the construction of the trajectory $(x_t)_{t=0}^{\infty}$ may be accomplished using an equilibrium mechanism, the equilibrium prices at the moment $t+1$ are used to define the utility function at the time t.

$$\chi_t^2 < \chi_t^1,\ \chi_t^i = \frac{C_t^{1i}}{C_t^{2i}}(i=1,2),\ \chi_t = \frac{K_t}{L_t},\ u_t^1 = \frac{1}{v_t^1 C_t^{21}},$$

Let

$$u_t^2 = \frac{b_t}{v_t^2 C_t^{22}} \quad (t = 0,1,2,...)$$

Since, by assumption, the state trajectories and equilibrium prices are strictly positive, then we have to deal with the equilibrium of the following form

$$\chi_t^2 < \chi_t < \chi_t^1,\ \frac{\chi_t - \chi_t^2}{\chi_t^1 - \chi_t}\frac{\chi_t^1 + u_t^1}{\chi_t^2 + u_t^1} \le \mu_t < \frac{\chi_t - \chi_t^2}{\chi_t^1 - \chi_t}\frac{\chi_t^1}{\chi_t^2}$$

Appropriate equilibrium mechanisms are determined by the coefficients $\mathrm{b_t}$ and λ_t^i. Consider the relation between the coefficients b_t. For this purpose consider Z_t^1 and Z_t^2, where

$$Z_t^1 = \left(v_t^1, b_t\omega_t, C_t^{11}, C_t^{21}\right),\ Z_t^2 = \left(v_t^2, b_t\omega_t, \frac{1}{b_{t+1}}C_t^{12}, \frac{1}{b_{t+1}}C_t^{22}\right) \quad (10)$$

Take

$$\alpha_{\mathrm{t}}^{\mathrm{i}} = \alpha\left(\mathrm{Z_t^i}\right),\ \bar{\eta}_t^i = \chi_t^i,\ i = 1,2, \quad (11)$$

i.e.

$$\alpha_t^1 = \frac{1 + v_t^1 C_t^{11}}{C_t^{11} + b_t\omega_t C_t^{21}},\ \alpha_t^2 = \frac{b_{t+1} + v_t^2 C_t^{12}}{C_t^{12} + b_t\omega_t C_t^{22}} \quad (12)$$

As is shown in [3, 6] the relation

$$\max_{x\ge 0}\frac{W_t^1(x, l_{t+1})}{[l_t, x]} = \max_{x\ge 0}\frac{W_t^2(x_t, l_{t+1})}{[l_t, x]} = 1 \quad (13)$$

holds true, where l_t is defined by the formula (7). Using the formulas

$$\max_{x\ge 0}\frac{W^2(x,p)}{[q,x]} = \frac{1}{q^1}\max_{\eta>0}\frac{v^2\eta + b\min\left(\frac{\eta}{C^{12}}, \frac{1}{C^{22}}\right)}{\eta + \frac{q^2}{q^1}},$$

$$\bar{\eta}^i = x^i,\ x^i = \frac{\lambda^i}{q^1 x^i + q^2}(x^i, 1) \qquad (i = 1,2).$$

by $q = l_t$, we obtain that the relation (13) is equivalent to the following one

$$\frac{b_{t+1}^1}{b_t^1}\alpha_t^1 = \frac{b_{t+1}^1}{b_t^1}\alpha_t^2 = 1. \quad (14)$$

Take

$$A_1 = 1,\ A_2 = \alpha_1, ..., A_{t+1} = \alpha_1, ..., \alpha_t \qquad (t = 0,1,...) \quad (15)$$

As follows from (2.10)

$$\mathrm{b_t^1} = \frac{1}{A_t}b_0^1, \quad (16)$$

$$\alpha_t^1 = \alpha_t^2. \quad (17)$$

Rewrite (7) in the form

$$l_t = \frac{b_0^1}{A_t}P_t \quad (18)$$

Introduce the function

$$R_t(b) = \left(1 + v_t^1 C_t^{11}\right)\frac{C_t^{12} + b\omega_t C_t^{22}}{C_t^{11} + b\omega_t C_t^{21}} - v_t^2 C_t^{12} \quad (19)$$

a consider the equation

$$b_{t+1} = R_t(b_t) \quad (20)$$

That is equivalent to

$$\alpha_t^1 = \alpha_t^2 \quad (21)$$

From (17) follows

Теорема 1. Let $(x)_{t=0}^{\infty}$ be a trajectory in the model Z_t, with characteristics $\left(l_t^{\Delta}\right)$. Suppose that $K_t^i > 0,\ L_t^i > 0,\ i = 1,2$ and the numbers b_t are defined by (8). Then $b_{t+1} = R_t(b_t)$.

Let us define under what conditions it is possible to build an effective trajectory $(x_{\mathrm{t}})_{t=0}^{\infty}$, in which all states and characteristic prices are non-zero i.e. to find $\mathrm{x_t} \in a_{t-1}(x_{t-1})$:

$$\begin{cases} K_t^1 + K_t^2 = v_{t-1}K_{t-1}^1 + v_{t-1}^2 K_{t-1}^2 + \min\left(\dfrac{K_{t-1}^1}{C_{t-1}^{11}}, \dfrac{L_{t-1}^1}{C_{t-1}^{21}}\right), \\ \omega_t\left(L_t^1 + L_t^2\right) = \min\left(\dfrac{K_{t-1}^2}{C_{t-1}^{12}}, \dfrac{L_{t-1}^2}{C_{t-1}^{22}}\right), \\ \dfrac{K_t^1}{L_t^1} = \chi_t^1, \\ \dfrac{K_t^2}{L_t^2} = \chi_t^2. \end{cases} \quad (22)$$

To find the vector $x_t = \left(K_t^1, L_t^1, K_t^2, L_t^2\right)$. we rewrite the system (22) in the form

$$\begin{cases} K_t^1 = \chi_t^1 L_t^1, \\ K_t^2 = \chi_t^2 L_t^2, \\ \chi_t^1 L_t^1 + \chi_t^2 L_t^2 = \delta_{t-1}^1, \\ \omega_t\left(L_t^1 + L_t^2\right) = \delta_{t-1}^2. \end{cases} \quad (23)$$

where

$$\delta_{t-1}^1 = v_{t-1}^1 K_{t-1}^1 + v_{t-1}^2 K_{t-1}^2 + \min\left(\frac{K_{t-1}^1}{C_{t-1}^{12}}, \frac{L_{t-1}^1}{C_{t-1}^{21}}\right),$$
$$\delta_{t-1}^2 = \min\left(\frac{K_{t-1}^2}{C_{t-1}^{12}}, \frac{L_{t-1}^2}{C_{t-1}^{22}}\right). \quad (24)$$

Note that the system (23) is a system of balance equations of the form $\mathrm{x}_{\mathrm{t}}^1 + x_t^2 = X_t$, where $X_t = \left(K_t^1 + K_t^2, \omega_i\left(L_t^1 + L_t^2\right)\right)$.

Solving the system (23) we obtain

$$L_t^1 = \frac{\omega_t \delta_{t-1}^1 - \chi_t^2 \delta_{t-1}^2}{\left(\chi_t^1 - \chi_t^2\right)\omega_t}, L_t^2 = \frac{\chi_t^1 \delta_{t-1}^2 - \omega_t \delta_{t-1}^1}{\left(\chi_t^1 - \chi_t^2\right)\omega_t},$$
$$K_t^1 = \chi_t^1 L_t^1, \quad K_t^2 = \chi_t^2 L_t^2. \quad (25)$$

From this follows that $L_t^i > 0 \ (i = 1, 2)$ by

$$\chi_t^2 < \omega_t \frac{\delta_{t-1}^1}{\delta_{t-1}^2} < \chi_t^1 \quad (26)$$

Note 1. If to admit the variation of ω_{t} then

1) by $\omega_t \frac{\delta_{t-1}^1}{\delta_{t-1}^2} > \chi_t^1$ it is necessary decrease ω_t for fulfillment (26).

2) by $\omega_t \frac{\delta_{t-1}^1}{\delta_{t-1}^2} < \chi_t^2$ it is necessary increase ω_t for fulfillment (26).

3. Conclusions

1. The possibility of constructing effective trajectories using the equilibrium mechanism is shown.

2. The coefficients for determining the corresponding equilibrium mechanisms are found.

3. Conditions are defined under which it is possible to construct effective trajectories for which the weight of the state is nonzero and the characteristic prices are nonzero.

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