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Generalized Grey Target Decision Method Based on Decision Makers' Indifference Attribute Value Preferences

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Abstract: In multi-attribute grey target decision making, decision makers may have indifference preferences towards some attribute values such that some superior index values are no different within a small scope or some inferior index values are indifference within a small range. So the target centre domain consisting of some superior values and the target edge domain comprising some inferior values under some attribute were proposed based on the grey target decision theory. Based on the two domains, the Hamming distance of each index value to its target centre domain can be calculated. Following this, the original Hamming distances can be normalized in a linear method individually. Then the decision can be made by the integrated target centre distances considering each attribute's weight. A case study indicated that the generalized grey target decision method improved easily and combined with other theories can address the decision makers' indifference attribute value preferences with its concise and simple technique compared with the conventional grey target method, which is superior in handling many feasible alternatives with little difference of superior values or inferior values.

Keywords: Generalized Grey Target Decision Method, Decision Makers, Indifference Attribute Value Preferences, Target Centre Domain, Target Edge Domain

1. Introduction

The grey target decision method proposed by Deng has been used in a wide range of fields [1]. Now, some scholars have made contributions to the grey target decision method. The incontinency problem of Deng's grey transformation was tested by Chen and Xie using the simulated method [2]. Song et al. improved the calculation operators of grey target [3-5]. Zhu et al. studied the weight determination [6-8]. And the grey target decision method for mixed attributes was studied by Luo et al. [9-14]. Ma and Ji proposed a generalized grey target decision method [14, 15]. Besides, some other theories and methods were also combined with the grey target decision method [16-19]. These researches advanced the grey target decision method. However, the target centre whether expressed as a set for multiple indices or singular index discussed in above research is a point which may be determinacy for indices of real numbers or uncertainty for those of fuzzy numbers. Furthermore, the conventional target centre represents the best selection of all alternatives, which is to differentiate all indices absolutely. In practice, decision makers may have indifference attribute preferences towards superior values or inferior values. Some superior index values are regarded as no difference when they are in a small scope. Similarly, some inferior index values are also thought as indifference while they are in a small range. But this issue has not been noticed by other scholars. Thus the generalized grey target decision method is proposed to cope with decision makers' indifference attribute value preferences. The conventional target centre index is expanded to a domain that there is no difference in it for some superior values with the proposed approach. Similarly, the target edge index can also be expanded to a domain that there is indifference in it for some inferior values. The proposed approach, substituting the target centre domain and the target edge domain for the target centre and the target edge respectively can in advance determine some indices contributing little to decision making. Furthermore, it has the potential function of "rewarding the good and punishing the bad" towards some indices. The proposed approach need not normalize the indices beforehand, and only normalize the singular index target centre distance for comparison when aggregating integrated target centre distances [15], as can reflect the accuracy distances of all indices to target centre indices. The generalized great target decision method can not only deal with decision makers' indifference attribute value preferences, simplify the calculation but also be easily improved combined with other theories.

The remainder of this paper is organized as follows: Section 2 discusses the proposed method, Section 3 studies the impacts of the two domains on the index values of all alternatives under some attribute, Section 4 presents a case study, and Section 5 concludes this paper.

2. Generalized Grey Target Decision Model

2.1. Preliminaries

Definition 1 Let $S=\left(S_1,\mathrm{S}_2,\cdots,\mathrm{S}_\mathrm{n}\right)$ be an alternative set, $A=\left(A_1,A_2,\cdots,A_m\right)$ be an attribute set, $S_{ij}\,(i=1,2,\cdots,n,j=1,2,\cdots,m)$ be the measure of alternative S_i under attribute A_j, J^+, J^- and J^M be benefit type attribute set, cost type attribute set and moderate type attribute set respectively[15], as are the bases of multi-attribute decision making.

Definition 2 Let $C^P=\left(C_1^P,C_2^P\cdots,C_m^P\right)$ be a target centre index set determined by the alternative measure $S_{ij}\,(i=1,2,\cdots,n,j=1,2,\cdots,m)$, and C_j^p satisfies

$$C_j^P=\begin{cases}\max\left\{S_{ij}\right\},S_{ij}\in J^+\\ \min\left\{S_{ij}\right\},S_{ij}\in J^- \ ,i=1,2,\cdots,n,j=1,2,\cdots,m\\ M_j,S_{ij}\in J^M\end{cases}\tag{1}$$

where M_j is the standard or desirable value of moderate type index value, which can be given in advance [15].

Definition 3 Let $C^N=\left(C_1^N,C_2^N\cdots,C_m^N\right)$ be a target edge index set determined by the alternative measure $S_{ij}\,(i=1,2,\cdots,n,j=1,2,\cdots,m)$, and C_j^N satisfies

$$C_j^N=\begin{cases}\min\left\{S_{ij}\right\},S_{ij}\in J^+\\ \max\left\{S_{ij}\right\},S_{ij}\in J^-\\ \left\{S_{ij}\left|\max\left(S_{ij}-M_j\right),S_{ij}\geq M_j\right.\right\},S_{ij}\in J^M\\ \left\{S_{ij}\left|\max\left(M_j-S_{ij}\right),S_{ij}\leq M_j\right.\right\},S_{ij}\in J^M\\ i=1,2,\cdots,n,j=1,2,\cdots,m\end{cases}\tag{2}$$

Definition 4 Some index values under attribute A_j reach or better than certain value are thought as superior values without difference, as is called indifference superior attribute value preference. Similarly, some index values under attribute A_j reach or worse than certain value are regarded as inferior values with indifference, as is called indifference inferior attribute value preference.

Definition 5 Given the best value C_j^P under attribute A_j, then the smaller scope of the value is called target centre domain. Given the worst value C_j^N under attribute A_j, then the smaller scope of the value is called target edge domain. The target centre domain and the target edge domain can be presented in Figure 1.

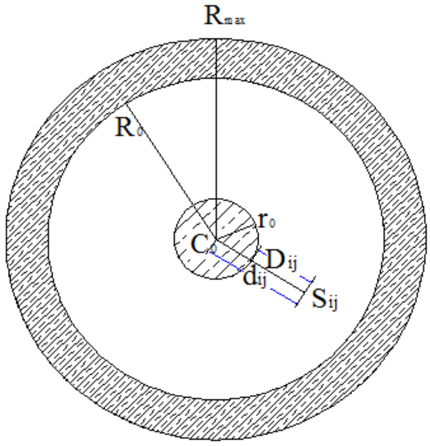

Figure 1. *Target center domain and target edge domain.*

In Figure 1, C_0, r_0, R_0 and $R_{\max}$ are the target centre, the radius of the target centre domain, the inner radius of the target edge domain, and the target edge respectively. The target centre and the target edge are expanded to two domains, as are shown by the shaded sections. While d_{ij} and D_{ij} are the distances of S_{ij} to its target centre and target centre domain respectively.

2.2. Target Centre Domain and Target Edge Domain Determination

2.2.1. Coefficient Method

(1) Target centre domain determination

The target centre domain determined by coefficient is obtained with the best index value under attribute A_j multiply the coefficient γ, where $\gamma\in(0,\ 0.1]$. Because of the small scope of target centre domain, it is better for γ no more than 0.1, which is decided by the decision makers. However, all the target centre domains determination for all attributes can employ the same coefficient or different coefficients. The following discussion limits to singular index target centre domain.

$$E_+=[a_0,b_0]=[\ x_{i0j0}(1\text{-}\gamma),x_{i0j0}]\tag{3}$$

$$E_-=[c_0,d_0]=[\ x_{i0j0},x_{i0j0}(1+\gamma)]\tag{4}$$

$$E_M=[e_0,f_0]=[\ M_0\ (1\text{-}\gamma),M_0\ (1+\gamma)]\tag{5}$$

where,

E_+ is the target centre domain of benefit type index;

a_0 and b_0 are the lower limits and upper limits of the target centre domain of benefit type index respectively;

E- is the target centre domain of cost type index;

c_0 and d_0 are the lower limits and upper limits of the target centre domain of cost type index respectively;

E_M is the target centre domain of the moderate type index;

e_0 and f_0 are the lower limits and upper limits of the target centre domain of moderate type index respectively;

x_{i0j0} is the best value of all indices;

M_0 is the desirable or standard value of moderate type index.

(2) Target edge domain determination

Let $\gamma^N \in (0, 0.1]$ be the coefficient of the target edge domain, then three types of target edge domains can be determined as follows.

$$E^N_+=[a^N_0,b^N_0]=[x^N_{i0j0},x^N_{i0j0}(1+\gamma^N)] \tag{6}$$

$$E^N_-=[c^N_0,d^N_0]=[x^N_{i0j0}(1-\gamma^N),x^N_{i0j0}] \tag{7}$$

$$E^N_M=[e^N_{0d},f^N_{0d}]\cup[e^N_{0u},f^N_{0u}]$$

$$=[x^N_{m0d},x^N_{m0d}(1+\gamma^N)]\cup[x^N_{m0u}(1-\gamma^N),x^N_{m0u}] \tag{8}$$

where,

E^N_+ is the target edge domain of benefit type index;

a^N_0 and b^N_0 are the lower limits and upper limits of the target edge domain of benefit type index respectively;

E^N_- is the target edge domain of cost type index;

c^N_0 and d^N_0 are the lower limits and upper limits of the target edge domain of cost type index respectively;

E^N_M is the target edge domain of the moderate type index;

e^N_{0d} and f^N_{0d} are the lower limits and upper limits of the downside of the target edge domain of moderate type index respectively;

e^N_{0u} and f^N_{0u} are the lower limits and upper limits of the upside of the target edge domain of moderate type index respectively;

x^N_{i0j0} is the worst value of all indices;

x^N_{m0d} and x^N_{m0u} are the worst values of the downside and the upside of the moderate value M_0 respectively.

2.2.2. Adjacent Value Method

(1) Target centre domain determination

The target centre domain can be determined according to the best value and the second best value under attribute A_j with the following equations.

$$E_+=[a_0,b_0]=[x_{i1j1},x_{i0j0}] \tag{9}$$

$$E_-=[c_0,d_0]=[x_{i0j0},x_{i1j1}] \tag{10}$$

$$E_M=[e_0,f_0]=[x_{m1d},x_{m1u}] \tag{11}$$

where,

x_{i1j1} is the second best value of all indices;

x_{m1d} and x_{m1u} are the second best values of the downside and the upside of the moderate value M_0 respectively.

(2) Target edge domain determination

The target edge domain can be determined by the worst value and the second worst value under attribute A_j use the following equations.

$$E^N_+=[a^N_0,b^N_0]=[x^N_{i0j0},x^N_{i1j1}] \tag{12}$$

$$E^N_-=[c^N_0,d^N_0]=[x^N_{i1j1},x^N_{i0j0}] \tag{13}$$

$$E^N_M=[e^N_{0d},f^N_{0d}]\cup[e^N_{0u},f^N_{0u}]$$

$$=[x^N_{m0d},x^N_{m1d}]\cup[x^N_{m1u},x^N_{m0u}] \tag{14}$$

where,

x^N_{i1j1} is the second worst value of all indices;

x^N_{m1d} and x^N_{m1u} are the second worst values of the downside and the upside of the moderate value M_0 respectively.

2.2.3. Comprehensive Method

Coefficient method and adjacent method can be combined with each other to determine the target centre domain or the target edge domain by the decision makers' intention. Furthermore, the two methods can also be used individually to determine the two domains. With respect to the two domains, decision makers can only determine the target centre domain or consider both of them for special purposes.

2.3. Singular Index Target Centre Distance Determination

Based on the theory of grey target decision, however the method of procedure and technique is different from the classical one is referred to as a generalized grey target method. Compared with the conventional model, the generalized grey target method has two differences: no need to normalize the index values $S_{ij}(i=1,2,\cdots,n, j=1,2,\cdots,m)$ and the difference of target centre distance calculation [14, 15]. Different from the conventional grey target decision method, the proposed approach does not normalize index values beforehand.

2.3.1. Determine Target Centre Distance Considering Only Target Centre Domain

Assume x is an index value under attribute A_j, and then the distance of x to its target centre distance can be obtained.

(1) Target centre distance for benefit type index

$$r_+=\begin{cases}0, & x\in[a_0,b_0]\\ a_0-x, & x\notin[a_0,b_0]\end{cases} \tag{15}$$

where $E_+=[a_0,b_0]$ is a target centre domain for benefit type index, r_+ is the distance of x to E_+.

(2) Target centre distance for cost type index

$$r_-=\begin{cases}0, & x\in[c_0,d_0]\\ x-d_0, & x\notin[c_0,d_0]\end{cases} \tag{16}$$

where $E_-=[c_0, d_0]$ is a target centre domain for cost type index, r_- is the distance of x to E_-.

(3) Target centre distance for the moderate type index

$$r_M = \begin{cases} 0, & x \in [e_0, f_0] \\ e_0 - x, & x < e_0 \\ x - f_0, & x > f_0 \end{cases} \tag{17}$$

where $E_M=[e_0,f_0]$ is a target centre domain for moderate type index, r_M is the distance of x to E_M.

2.3.2. Determine Target Centre Distance Considering Target Edge Domain

The target centre distance discussed above does not consider the target edge domain. Considering both the two domains, the target centre distances of indices outside the target edge domain are still calculated by the above equations, while the target centre distances of indices in the target edge domain can be unified with the distance of the worst index value to the target centre domain. Equations (18) to (20) can be used to solve the problem.

$$r_{+max} = a_0 - x_{min} \tag{18}$$

where r_{+max} is the distance of the worst index value of benefit type to its target centre domain, x_{min} is the worst value.

$$r_{-max} = x_{max} - d_0 \tag{19}$$

where r_{-max} is the distance of the worst index value of cost type to its target centre domain, x_{max} is the worst value.

$$r_{Mmaxd} = e_0 - x_{Mmin} \text{ or } r_{Mmaxu} = x_{Mmax} - f_0 \tag{20}$$

where r_{Mmaxd} and r_{Mmaxu} are the downside and upside of the distances of the worst index values of moderate type to its target centre domain respectively, x_{Mmin} and x_{Mmax} are the worst value of the downside and the upside of the moderate value.

2.3.3. Determine Target Centre Distance Without Considering Two Domains

The target centre distance discussed above considers only the target centre domain or both the two domains. However, Equation (21) can be used to calculate the target centre distance without considering two domains.

$$r = |x_0 - x| \tag{21}$$

where r is the target centre distance without considering two domains, x_0 and x are the target centre index and an index value under attribute A_j respectively.

2.4. Target Centre Distance Normalization

Every original singular index target centre distance can be normalized using (22).

$$z_{ij} = \frac{r_{ij}}{\sum_{i=1}^{n} r_{ij}}, j = 1 \ldots m \tag{22}$$

where r_{ij} is the distance of S_{ij} to its target centre domain under attribute A_j, z_{ij} is the normalized target centre distance.

2.5. Weights Determination

There are three types of weight determination of attributes: objective weights, subjective weights and comprehensive weights. Since weight determination has been advanced by many scholars, this paper does not repeat it, and the interested readers can see the literature [6-8].

2.6. Decision Making

The integrated target centre distances for all alternatives can be calculated using (23).

$$w_i = \sum_{j=1}^{m} \omega_j z_{ij}, \ i = 1 \ldots n \tag{23}$$

Thus, the decision can be made by the value w_i, the smaller value of it, the better of the alternative.

2.7. Steps of the Generalized Grey Target Decision Method

Step 1 Determine every attribute's target centre index and target edge index. Use Equations (1) and (2) to calculate the target centre indices and the target edge indices.

Step 2 Determine every attribute's target centre domain or meanwhile determine the target edge domain. Use Equations (3) to (5) or (9) to (11) to determine the target centre domain. Use Equations (6) to (8) or (12) to (14) to determine the target edge domains.

Step 3 Calculate the Hamming distance of every index to its target centre domain use Equations (15) to (20). Without the two domains, Equation (21) can be used to calculate the target centre distance.

Step 4 Normalize every index's target centre distance using (22).

Step 5 Determine the weights of all attributes.

Step 6 Aggregate every normalized target centre distance under all attributes using (23), then the alternative ranking can be made according to the integrated target centre distances with the smaller value the better.

3. The Impacts of Two Domains on Alternatives

Generalized grey target decision method considering either of the target centre domain and the target edge domain has the function of "rewarding good and punishing bad" towards some indices of the alternatives, as is similar to the previous study in the article [15]. In [15], the singular index target centre determined by the selection preferences can reduce the difference of superior indices and enlarge the difference of other indices; however, the singular index target centre determined by the desirable preferences can reduce the difference of the indices. Here, the impacts of both the two domains on the indices are discussed.

Assume $S_{ij}(i=1,2,\cdots,n, j=1,2,\cdots,m)$, S_{0j}, and S_{ej} are the index value, the target centre value and the target edge value under attribute A_j respectively. Let $S_{(i_0+1)j}$ and $S_{i_0 j}$ be any two indices, and $d_{(i_0+1)j}$, d_{0j} are their distances to the target centre respectively. Also, let $D_{(i_0+1)j}$ and D_{0j} be the distances of $S_{(i_0+1)j}$ and $S_{i_0 j}$ to their target centre domain respectively. Suppose that $d_{(i_0+1)j} > d_{0j}$ and $D_{(i_0+1)j} > D_{0j}$, the results will not change. The discussion only limits to under attribute A_j, thus the subscript *j* for distances will be omitted. In Figure 1, the meaning of the parameters C_0, r_0, and $R_{\max}$ are as above.

Let ΔM_0 be the normalized difference of $S_{(i_0+1)j}$ and $S_{i_0 j}$ to their target centre. So Equation (22) can be obtained without considering the two domains.

$$\Delta M_0 = \frac{d_{i_0+1}}{\sum_{i=1}^{n} d_i} - \frac{d_{i_0}}{\sum_{i=1}^{n} d_i} = \frac{d_{i_0+1} - d_{i_0}}{\sum_{i=1}^{n} d_i} \tag{24}$$

where, d_i is the distance of S_{ij} to its target centre.

Suppose that there are *p* index values in the target centre domain, *q* index values in the target edge domain, while other (*n-p-q*) index values outside the two domains, if the two domains are both considered. Thus Equation (25) can be obtained.

$$\begin{aligned}\Delta M_1 &= \frac{D_{i_0+1}}{\sum_{i=1}^{p} D_i + \sum_{j=1}^{q} D_j + \sum_{k=1}^{n-p-q} D_k} \\ &- \frac{D_{i_0}}{\sum_{i=1}^{p} D_i + \sum_{j=1}^{q} D_j + \sum_{k=1}^{n-p-q} D_k} \\ &= \frac{D_{i_0+1} - D_{i_0}}{\sum_{i=1}^{p} D_i + \sum_{j=1}^{q} D_j + \sum_{k=1}^{n-p-q} D_k}\end{aligned} \tag{25}$$

Seen from Figure 1, the distance of index value to its target centre domain will be shorter than that to its original target centre, thus Equation (26) can be obtained.

$$D_{i_0+1} = d_{i_0+1} - r_0, D_{i_0} = d_{i_0} - r_0 \tag{26}$$

The following discussions only concern three conditions: both the two indices in the target centre domain, both of them in the target edge domain and none of them in the two domains.

(1) If $S_{(i_0+1)j}$ and $S_{i_0 j}$ are both included in the target centre domain, then we can achieve Equation (27) using Equation (15).

$$D_{i_0+1} = D_{i_0} = 0 \tag{27}$$

So Equation (28) can be obtained with the comparison of Equations (24) and (25).

$$\Delta M_0 > \Delta M_1 = 0 \tag{28}$$

Equation (28) means that the two index target distances become smaller when considering target centre domain, namely both of them are superior index values.

(2) If $S_{(i_0+1)j}$ and $S_{i_0 j}$ fall within the target edge domain, then Equation (29) can be obtained using (18).

$$D_{i_0+1} = D_{i_0} = R - r_0 \tag{29}$$

Compare with Equations (24) and (25), the following equation can be obtained:

$$\Delta M_0 > \Delta M_1 = 0 \tag{30}$$

Equation (30) indicates that the difference of them becomes smaller such that both of them are inferior index values considering target edge domain.

(3) If $S_{(i_0+1)j}$ and $S_{i_0 j}$ fall outside the two domains, then Equation (31) can be obtained by Equations (18), (26) and (25).

$$\begin{aligned}\Delta M_1 &= \frac{d_{i_0+1} - d_{i_0}}{\sum_{i=1}^{p} D_i + \sum_{j=1}^{q} D_j + \sum_{k=1}^{n-p-q} (d_k - r_0)} \\ &= \frac{d_{i_0+1} - d_{i_0}}{q(R_{\max} - r_0) + \sum_{k=1}^{n-p-q} d_k - (n-q-p)r_0} \\ &= \frac{d_{i_0+1} - d_{i_0}}{qR_{\max} + \sum_{k=1}^{n-p-q} d_k - (n-p)r_0}\end{aligned} \tag{31}$$

where, $\sum_{i=1}^{p} D_i = 0$, $\sum_{j=1}^{q} D_j = q(R_{\max} - r_0)$, as calculated by Equations (4) and (5).

Compare with (24) and (31), their numerators are equal. So only the denominators are considered to determine the relationship of ΔM_0 and ΔM_1.

However the relationship of $\sum_{i=1}^{n} d_i$ and $(qR_{\max} + \sum_{k=1}^{n-p-q} d_k - (n-p)r_0)$ is unclear, as they are determined by d_i, $R_{\max}$, R_0, r_0, n, p and q. Therefore, the relationship of ΔM_0 and ΔM_1 is uncertain.

The purpose of this work is to seek for the impacts of the two domains on indices of all alternatives under some attribute.

From the discussion above, we can draw the conclusion that the superior index values and inferior index values may contribute little to make a decision, and the other index values interacted with each other may act as the main roles to determine the results.

4. Case Study

4.1. Background

To evaluate coal mines' safety performance, consider eight indices, including seam dip (°), methane emission rate (m^3/t), water inflow (m^3/h), spontaneous combustion period (month), ventilating structures qualified rate (%), equivalent orifice (m^2), mortality per million tons (person/10^6t), and accident economic loss (10^5 Yuan) [15, 20] denoted by A_1 to A_{10}, and the alternatives are denoted by S_1 to S_{10}. The data is shown in Table 1, the benefit type attributes are A_4 to A_6, and the others are cost type attributes. Decision makers have indifference superior value preferences towards all attributes, while have indifference inferior value preferences towards attributes A_2 and A_8. The target centre domain of attribute A_5 is determined by coefficient method with the value 0.05, and the others are determined by the adjacent value method. And the target edge domains of attributes A_2 and A_8 are determined by the adjacent value method.

Table 1. *Safety data for coal mines.*

S_i	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
S_1	21	6	220	12	92	1.8	0.18	381
S_2	16	3.7	200	6	90	1.4	0.712	564
S_3	26	9.2	180	10	88	2.7	1.34	1051.6
S_4	10	4	260	8	94	1.2	0	442.5
S_5	30	8.2	350	10	96	3.6	0.641	788
S_6	19	5	130	12	100	2.4	0	300
S_7	17	9.6	400	6	86	1.3	1.23	964.7
S_8	40	14	600	6	95	2.1	1.12	885.6
S_9	12	12.8	120	10	91	1.5	0.872	839.3
S_{10}	14	5.8	155	12	89	1.7	0.426	617.2

4.2. Process to Decision Making

(1) Calculate target centre indices

The target centre indices are C^P =(10, 3.7, 120, 12, 100, 3.6, 0, 300) calculated by Equation (1), and the target edge indices are C^N =(40, 14, 600, 6, 86, 1.2, 1.34, 1051.6) calculated by Equation (2).

(2) Determine the target centre domain and the target edge domain

Use Equations (3), (9) and (10), target centre domains can be obtained as E^P=([10, 12], [3.7, 4], [120, 130], [10, 12], [95, 100], [2.7, 3.6], [0, 0.18], [300, 381]). And use Equations (12) and (13), the target edge domains of attributes A_2 and A_8 are [12.8, 14] and [964.7, 1051.6] respectively.

(3) Calculate original index target centre distances

Use Equations (15), 16), (18) and (19), all index target centre distances can be calculated as shown in Table 2.

Table 2. *All index target centre distances.*

r_{ij}	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
r_{1j}	9	2	90	0	3	0.9	0	0
r_{2j}	4	0	70	4	5	1.3	0.532	183
r_{3j}	14	5.2	50	0	7	0	1.16	670.6*
r_{4j}	0	0	130	2	1	1.5	0	61.5
r_{5j}	18	4.2	220	0	0	0	0.461	407
r_{6j}	7	1	0	0	0	0.3	0	0
r_{7j}	5	5.6	270	4	9	1.4	1.05	670.6*
r_{8j}	28	10*	470	4	0	0.6	0.94	504.6
r_{9j}	0	10*	0	0	4	1.2	0.692	458.3
r_{10j}	2	1.8	25	0	6	1	0.246	236.2

Note: the value with the mark * means the value is obtained considering the target edge domain.

(4) Normalize the original target centre distances

Use Equation (22), the normalized target centre distances can be obtained shown in Table 3.

Table 3. *Every index target centre distance.*

Z_{ij}	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
Z_{1j}	0.103448	0.050251	0.067925	0	0.085714	0.109756	0	0
Z_{2j}	0.045977	0	0.05283	0.285714	0.142857	0.158537	0.104704	0.057334
Z_{3j}	0.16092	0.130653	0.037736	0	0.2	0	0.228302	0.210101
Z_{4j}	0	0	0.098113	0.142857	0.028571	0.182927	0	0.019268
Z_{5j}	0.206897	0.105528	0.166038	0	0	0	0.09073	0.127514
Z_{6j}	0.08046	0.025126	0	0	0	0.036585	0	0
Z_{7j}	0.057471	0.140704	0.203774	0.285714	0.257143	0.170732	0.206652	0.210101
Z_{8j}	0.321839	0.251256	0.354717	0.285714	0	0.073171	0.185003	0.158093
Z_{9j}	0	0.251256	0	0	0.114286	0.146341	0.136194	0.143587
Z_{10j}	0.022989	0.045226	0.018868	0	0.171429	0.121951	0.048416	0.074002

(5) Decision making

If the attribute weights given by the experts are ω=(0.06, 0.15, 0.03, 0.08, 0.12, 0.13, 0.27, 0.16), then all the integrated target centre distances can be calculated as w =(0.040336, 0.102397, 0.149643, 0.044664, 0.078123, 0.013353, 0.195989, 0.175255, 0.130173, 0.070067) using (23). So the alternative ranking is $S_6 \succ S_1 \succ S_4 \succ S_{10} \succ S_5 \succ S_2 \succ S_9 \succ S_3 \succ S_8 \succ S_7$.

(6) Discussion

Table 4 is presented to show the comparison between the two methods of considering target centre domain and target edge domain or not. If the weights given by the experts are ω =(0.06, 0.15, 0.03, 0.08, 0.12, 0.13, 0.27, 0.16), then use Equations from (21) to (23), all the integrated target centre distances without considering two domains are w =(0.046647, 0.094291, 0.154519, 0.049688, 0.089302, 0.019647, 0.171653, 0.175819, 0.129606, 0.070067, 0.068827), So the alternative ranking is $S_6 \succ S_1 \succ S_4 \succ S_{10} \succ S_5 \succ S_2 \succ S_9 \succ S_3 \succ S_7 \succ S_8$.

Table 4. *Results comparison between the two methods.*

S_i	No domains	ranking	Domains	Ranking	Changing
S_1	0.046647	2	0.040336	2	0
S_2	0.094291	6	0.102397	6	0
S_3	0.154519	8	0.149643	8	0
S_4	0.049688	3	0.044664	3	0
S_5	0.089302	5	0.078123	5	0
S_6	0.019647	1	0.013353	1	0
S_7	0.171653	9	0.195989	10	+1
S_8	0.175819	10	0.175255	9	-1
S_9	0.129606	7	0.130173	7	0
S_{10}	0.068827	4	0.070067	4	0

Seen from Table 4, except for the alternatives S_7 and S_8, the better alternatives ranking remains steady whether considering the domains or not. Through comparing the results, we draw the conclusion that some superior index values or inferior index values contributing little to decision making. Thus, some superior index values can be thought as indifferences and the same with some inferior index values, as may not affect the decision making that seeking for some better alternatives. Meanwhile, the above results also indicate that the alternative may not be an excellent alternative with only partially better index values.

5. Conclusions

The proposed grey target decision method expanding the conventional target centre to a domain and also considering target edge domain can effectively deal with decision makers' superior or inferior indifference attribute value preferences. The approach has its advantage to cope with multi-attribute alternatives with little difference of some index values especially for so many alternatives. It can not only simplify the calculation but also keep the accuracy of results, at least for some excellent alternatives. Moreover, decision makers can employ this method of decision making by their preferences with considering the two domains or either of them.

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