American Journal of Chemical Engineering
2018; 6(1): 12-18
http://www.sciencepublishinggroup.com/j/ajche
doi: 10.11648/j.ajche.20180601.13
ISSN: 2330-8605 (Print); ISSN: 2330-8613 (Online)

Mass Transfer Influence on Entropy Generation Fluctuation on Saturated Porous Channel Poiseuille Benard Flow

Mounir Bouabid*, Rahma Bouabda, Mourad Magherbi

Chemical and Process Engineering Department, Gabès University, National School of Engineers of Gabès, Applied Thermodynamics Unit, Gabès, Tunisia

Email address:
bouabid.mpcshun@yahoo.fr (M. Bouabid)
*Corresponding author

To cite this article:
Mounir Bouabid, Rahma Bouabda, Mourad Magherbi. Mass Transfer Influence on Entropy Generation Fluctuation on Saturated Porous Channel Poiseuille Benard Flow. *American Journal of Chemical Engineering*. Vol. 6, No. 1, 2018, pp. 12-18.
doi: 10.11648/j.ajche.20180601.13

Received: December 27, 2017; **Accepted**: January 10, 2018; **Published**: April 16, 2018

Abstract: This paper reports a transient state numerical investigation of irreversibility in a saturated porous channel, of an aspect ratio A= 5, under vertical thermal and mass gradients. The governing equations, using the Darcy-Brinkman formulation, have been solved numerically by using Control Volume Finite Element Method (CVFEM). Only two variables are taken into account, the Schmidt number and the floatability ratio. The other parameters values are fixed related to the Poiseuille–Benard flow (at zero mass gradients). Results reveal that the flow tends towards the steady state with different regimes, which depends on both the Schmidt number and the buoyancy ratio.

Keywords: Mixed Convection, Porous Medium, Entropy Generation, Prigogine's Theorem

1. Introduction

In the last few decades, interest in heat and/or mass transfer in mixed convection in a saturated porous media have increases significantly due to the divers applications of the porous media in the engineers domain such as oil reservoir, groundwater, nuclear waste disposal, membranes and regenerative heat exchangers.

Nakayama et al. [1] have studied theoretically the onset of instability when both lower and upper plates were subjected to uniform temperature gradients. Ostrach and Karrsotani. [2] have conducted an experimental investigation of fully developed forced convection between two horizontal plates. They showed an appreciable heat transfer augmentation obtained by superposing a fully developed flow on the cellular flow, the first type of vortex rolls create periodic spanwise temperature distributions whereas the second type of vortex rolls distort the temperature distribution. A proposed benchmark solution for open boundary flows has been given by Evans and Paolucci. [3]. Hasnaoui *et al.* [4] have been investigated the mixed convective heat transfer in a horizontal channel heated periodically from below. They observed, for a fixed geometry and a given Rayleigh number, a complicated solution structure upon increasing the Reynolds number. Recent studies related to the convective transport processes in porous media are studied by Nield and Bejan. [5, 6] and by Vafai. [7].

Despite the various topics investigated about mixed convection in porous channel, the effect of mass transfer on entropy generation fluctuation in Poiseuille–Benard porous channel flow was not yet be encountered. Thus, our investigation is principally focalised on the influence of the solute buoyancy force on the thermodynamics approach towards the steady state of porous mixed convection.

2. Problem Statement

The present paper reports a numerical study of entropy generation on 2D porous channel flow, filled with a fluid (binary mixture of pollutant species and air) considered as ideal gas and submitted to vertical thermal and concentration gradients as seen in Figure. 1. The considered flow fluid is assumed to be laminar, Newtonian and incompressible. The bottom wall is kept at constants high temperature (T_h) and low concentration (C_l) whereas the top wall is kept at constant Low temperature (T_l) and high concentration (C_h). The physical properties of the fluid are supposed constants except the density, which satisfies the Boussinesq approximation:

$$\rho = \rho_0\left[1-\beta_\theta(\theta-\theta_0)-\beta_c(\theta-\theta_0)\right] \quad (1)$$

ρ_0 is the fluid density at average temperature (θ_0). β_θ β_C are the thermal and solutal expansions coefficients.

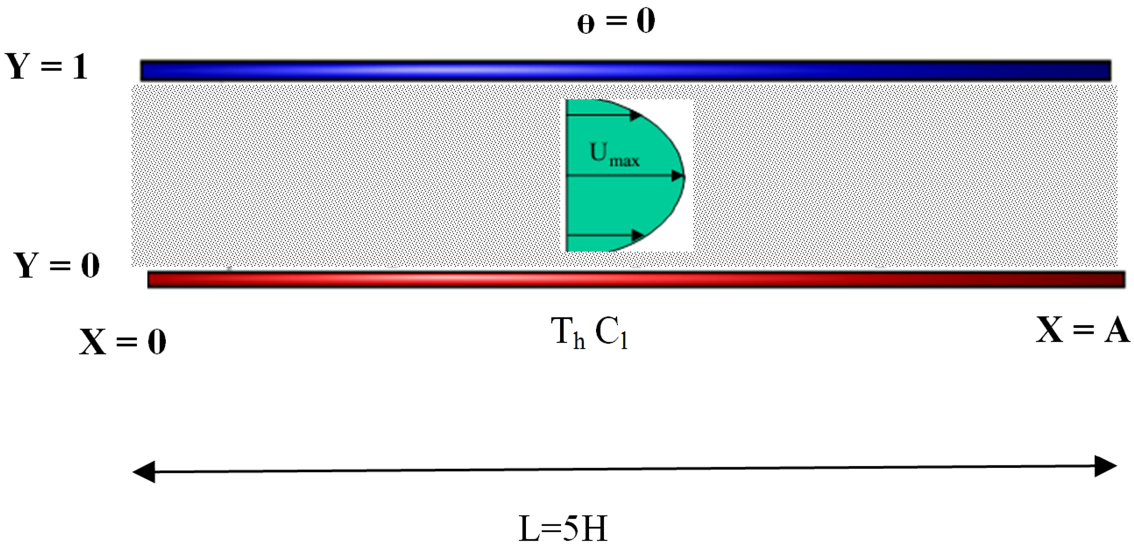


Figure 1. Physical model's schematic view at the dimensionless coordinate system.

3. Mathematical Formulation

3.1. Governing Equations

Under the above assumptions and using the Darcy-Brickman formulation, the set of dimensionless governing equations of continuity, momentum conservation and energy in laminar incompressible flow, is given by:

$$\frac{\partial \mathbf{U}}{\partial \mathrm{X}}+\frac{\partial \mathbf{V}}{\partial \mathrm{Y}}=0 \quad (2)$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{U}}{\partial \tau}+\frac{1}{\varepsilon^2}\left[U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right]=-\frac{\partial \mathbf{P}}{\partial \mathrm{X}}-\frac{1}{\mathrm{ReDa}}\mathbf{U}+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right] \quad (3)$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{V}}{\partial \tau}+\frac{1}{\varepsilon^2}\left[U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right]=-\frac{\partial \mathbf{P}}{\partial \mathrm{Y}}-\frac{1}{\mathrm{ReDa}}\mathbf{V}+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right]+Ri(\theta+NC) \quad (4)$$

$$\frac{\partial \boldsymbol{\theta}}{\partial \tau}+\left(U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial X}\right)=\frac{1}{\mathrm{Re\,Pr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (5)$$

$$\frac{\partial \mathbf{C}}{\partial \tau}+\left(U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}\right)=\frac{1}{\mathrm{Re}\,Sc}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right) \quad (6)$$

The initial and boundary conditions are expressed in dimensionless form as:

$$\left.\begin{array}{c}
\text{At } \tau = 0, U = V = P = C = 0 \text{ and } \theta = 0.5 - Y \\
\text{At the inlet of the channel } (x = 0, 0 \le Y \le 1): U = 6Y(1-Y), V = 0 \text{ and } \theta = 1 - Y \\
\text{At } Y = 0 \text{ and } 0 \le X \le 5: U = V = 0, \theta = 1, C = 0 \\
\text{At } Y = 1 \text{ and } 0 \le X \le 5: U = V = 0, \theta = 0, C = 1
\end{array}\right] \quad (7)$$

The major difficulty in the numerical study related to the mixed convection in the channel is that the physical domain is unlimited, whereas the numerical domain is limited. For this reason, the convective boundary condition (CBC) is imposed at the outflow. It's given in dimensionless form by:

$$\frac{\partial\varphi}{\partial\tau}+\frac{\partial\varphi}{\partial X}=0 \tag{8}$$

Where φ can be one of the dimensionless parameters: U, V, Θ or C.

3.2. Entropy Generation Formulation

According to the local thermodynamic equilibrium and using the equation of Woods (1975), the dimensionless local entropy generation (S_l) in the porous medium is given by:

$$S_l=(\nabla\boldsymbol{\theta})^2+\frac{Br^*}{Da}\left(\boldsymbol{U}^2+\boldsymbol{V}^2\right)+Br^*\left(2\left(\frac{\partial\boldsymbol{U}}{\partial X}\right)^2+2\left(\frac{\partial\boldsymbol{V}}{\partial Y}\right)^2+\left(\frac{\partial\boldsymbol{U}}{\partial Y}+\frac{\partial\boldsymbol{V}}{\partial X}\right)^2\right)+\varphi_1\left(\left(\frac{\partial\boldsymbol{C}}{\partial X}\right)^2+\left(\frac{\partial\boldsymbol{C}}{\partial Y}\right)^2\right)+\varphi_2\left(\left(\frac{\partial\theta}{\partial X}\right)\left(\frac{\partial\boldsymbol{C}}{\partial X}\right)+\left(\frac{\partial\theta}{\partial y}\right)\left(\frac{\partial\boldsymbol{C}}{\partial y}\right)\right) \tag{9}$$

The third first terms in the right hand side of Eq. (9) represents respectively the heat transfer irreversibility, Darcy-Brinkman viscous fluid irreversibility, clear viscous fluid irreversibility. The fourth and fifth terms are linked to the mass transfer irreversibility. The dimensionless total entropy generation (S) is obtained by integrating the dimensionless local irreversibility over the entire volume of the channel:

$$\mathrm{S}=\int_{\Omega}\mathrm{S_l d\Omega} \tag{10}$$

3.3. Numerical Scheme and Accuracy Tests

The present study is based on a modified version of the Control Volume Finite Element Method (CVFEM) of Patankar. [8] and Saabas and Baliga. [9], adapted to the standard-staggered grids in which pressure and velocity components are stored at different nodal points. SIMPLER algorithm was applied to resolve the pressure-velocity coupling in conjunction with an alternating direction implicit scheme, for performing the time evolution[8]. From the known velocity and temperature fields, at any given time τ, by solving Eqs. (2) - (6), the local entropy generation S_l is therefore evaluated at any nodal point of the porous channel by using Eq. (9). More details related to the numerical code used in this study are available in Abbassi *et al.* [10, 11] for further details about CVFEM method, see Parakash. [12] and Hookey. [13].

In this study, imposed global and local convergence criteria are used, and should verify the following conditions

$$\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)\le 10^{-5}\ ,\ \max\left|\frac{\chi^{t+\Delta t}-\chi^{t}}{\chi^{t+\Delta t}}\right|\le 10^{-5} \tag{11}$$

χ is the dependent variable $\chi=(U,V\theta,C)$. The continuity equation should verify the first convergence criterion at each time step of the calculation, and the dependent variable χ should verify the second criterion at each point of the space channel, and at each time step. The use of a time step $\Delta\tau=10^{-4}$ for all Darcy number is found to be sufficient to achieve the imposed convergence criteria.

The space-averaged Nusselt number at the bottom wall is used for the grid independence analysis. Grid refinement tests have been performed for the case where: Re=10, Pr=0.7, ε=0.85 using three uniform grids of sizes: $F_1=70\times20$, $F_2=101\times26$ and $F_3=132\times31$ nodal points. An imposed relative error should satisfy the following criterion:

$$Error=\left|\frac{\langle Nu\rangle_{(i+31)}-\langle Nu\rangle_{i}}{\langle Nu\rangle_{(i+31)}}\right|\le 2\% \tag{12}$$

(i+31) represents the number of the nodal point through the X-axis.

Results reveals that, when passing from grid $F_1$to grid $F_2$the relative error is close to 4, 24%. Whereas when we pass from F_2 to F_3, the error (Er) is about 1, 74%. Thus, the grid F_2 is sufficient enough to carry out the calculations related to the present problem.

To validate the numerical simulation, results concerning maximum horizontal velocity component in laminar flow through a horizontal porous channel, U_{max} have been compared to those published by Shohel and Fraser. [14] and Abdulhassan *et al.* [15] related to the laminar flow in a porous channel. A good agreement (Table 1) is seen between the present results and the results of the previous work.

Another accuracy test has been performed by comparing values of the space average Nusselt number given by the present numerical study with those obtained by Shohel and Fraser. 14 as indicated in Table 2. A good agreement is also shown between the two works.

Table 1. *Variation of maximum horizontal velocity component versus Darcy number.*

Da	10^{-3}	10^{-2}	10^{-1}	1
Present study	1.06	1.23	1.44	1.53
Shohel et al.	1.06	1.11	1.33	1.48
Abdulhassan et al.	1.09	1.30	1.55	1.59

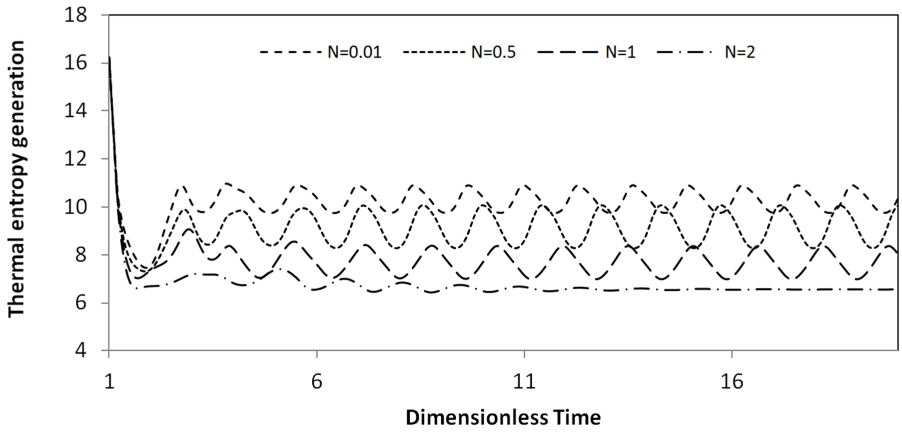
Table 2. *Variation of Average Nusselt number versus Darcy number.*

Da	10^{-2}	10^{-1}	1	10
Present study	100.324	10.785	1.4759	0.5293
Shohel et al.	99.936	11.098	1.5849	0.5828

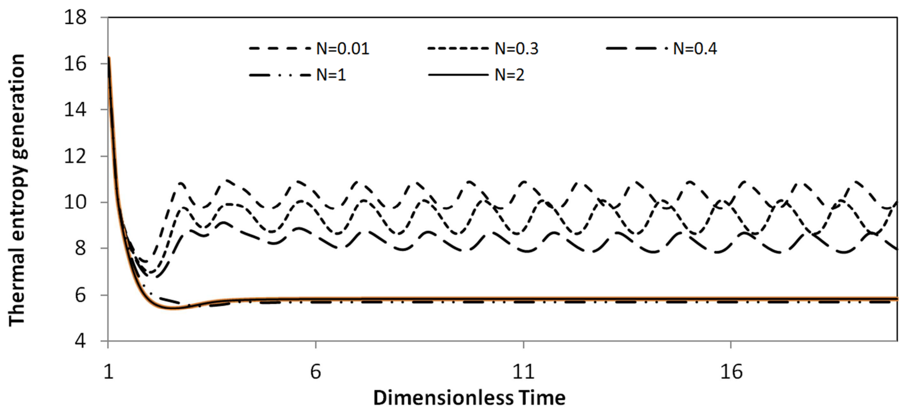
4. Results and Discussion

The medium porosity is fixed at 0.85. The Rayleigh, Reynolds and Prandtl numbers are fixed to 10^4, 10 and 0.7 respectively. The Darcy and Brinkman numbers are fixed at 5.10^{-2} and 10^{-4} respectively.

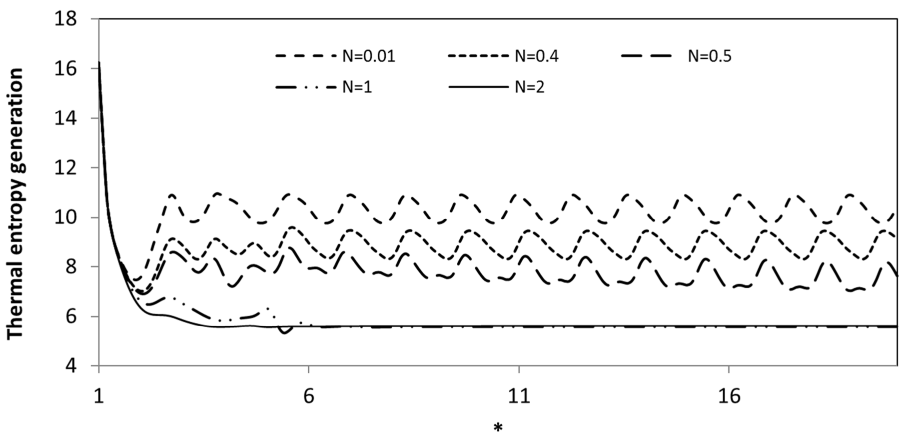
This investigation is focalised on the effect of both the buoyancy ratio and the Schmidt number on entropy generation fluctuation in mixed convection on a saturated porous channel under the Darcy-Brinkman formulation. For this reason the buoyancy ratio and the Schmidt number are considered varying from10^{-2} to 2 and from10^{-1} to 10 respectively. The irreversibility ratios are chosen small and equal to 10^{-4} in the goal to eliminate the intrinsic effect of mass transfer irreversibility on total entropy generation. Then, the contribution of the mass transfer irreversibility to the total entropy generation is extrinsic through the Navier Stokes and energy equations. Additionally, the clear and Darcy viscous fluid irreversibilities are unimportant because of the insignificant value of Darcy and Brinkman numbers. Finally, the total irreversibility is only reduced to the thermal contribution.



(a)



(b)



(c)

Figure 2. *Thermal entropy generation variation in transient mixed convection: a) Sc=0.1; b) Sc=1.5, c) Sc=6.*

Figure. 2 illustrates the transient entropy generation in double diffusive mixed convection for different Schmidt numbers and buoyancy ratios. In general case, Figure. 2 shows that the total irreversibility takes important value at the very beginning of mixed convection due to the important initial values of the thermal gradients, and then it decreases to reach the steady state of mixed convection with a behavior that depends on both the Schmidt number and the buoyancy ratio values. As seen from this figure, the system approaches the steady state with three behaviors. The first corresponds to a periodic one, the second is pseudo-periodic and the third is practically asymptotic. Remark that, these observations are similar to the results found by Tayari *et al*. [16], related to the study of the influence of Darcy number on the entropy generation fluctuation in mixed convective heat transfer in a porous channel.

Let's start with the case of relative small buoyancy ratio (N = 0.01), which corresponds to a weak effect of solutal buoyancy force induced by small mass gradients. In this case, which is not far from the case of simple heat transfer in porous channel, the entropy generation oscillates with a periodic behavior. This situation which persists for all selected Schmidt numbers proves the existence of thermo-convective cells in the porous channel. [16].

From a point of view of thermodynamics for irreversible processes (TIP), this configuration maintained by the energy dissipation is known as dissipative structure. [17]. The case of periodic fluctuations of entropy generation corresponds to a rotation of the system around the steady state that is in this case far from the equilibrium one. The theorem of minimum entropy generation of Prigogine[17] is therefore unproven and the system evolves in the nonlinear branch of the TIP, for which relations between thermodynamic forces and fluxes loss their linearity. This periodic behavior remains until the buoyancy ratio reaches a critical value (Nc) which depends on the Schmidt number. Beyond this critical buoyancy ratio and according to the Schmidt number value, the entropy generation exhibits a pseudo-periodic or an asymptotic approach towards the steady state. From the TIP view point, the pseudo-periodic regime of the irreversibility shows that the system develops a spiral approach towards the steady state, for which the entropy generation takes a constant value. The short time of fluctuations of the irreversibility (pseudo-periodic oscillations) may be the result of the birth of thermo-convective cells in the double diffusive porous channel, which rapidly vanish as time proceeds, under the increasingly effect of mass transfer. This pseudo-periodic behavior, characterized by an irreversibility constant value at steady state, implies that the system is in the frontier between the nonlinear and the linear domains of TIP. The asymptotic profile of the total entropy generation, at critical buoyancy ratio and for given Schmidt number, implies that the system progress directly towards the steady state, which is in this case a new equilibrium one. The Prigogine's theorem of minimum entropy generation is verified and therefore the system evolves, in this case, in the linear branch of TIP.

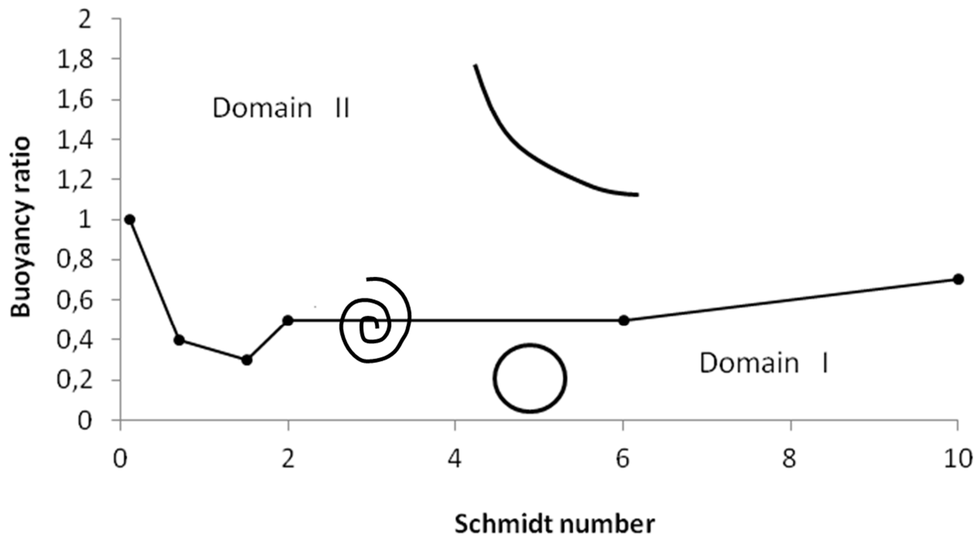


Figure 3. *Frontier between linear and non-linear domain of TIP with different approaches towards the steady state (Re=10, Ra=10^4, Pr=0.7).*

The frontier between the non linear and the linear domains of TIP, which depends on both the buoyancy ratio and the Schmidt number is plotted in Figure 3. The domain (I) corresponds to the non linear branch of TIP, which is characterized by a periodic fluctuation of the entropy generation and consequently by a rotation of the system around the steady state in the phase space. Whereas the domain (II) is related to the linear domain of TIP which illustrates the pseudo-periodic and the asymptotic behaviors and which corresponds, in the phase space, to a spiral approach near the frontier and a direct approach towards the steady state respectively.

5. Conclusion

A numerical model was employed to analyse the entropy generation fluctuations in transient state of mixed convective heat and mass transfer in a Darcy-Brinkman porous channel.

The following conclusions are drawn: It was observed that the total entropy generation takes important value at the very beginning of mixed convection and then it decreases to reach the steady state, with a behavior that depends on both the Schmidt number and the buoyancy ratio. At fixed Schmidt number, the entropy generation fluctuations are periodic until the buoyancy ratio reaches a critical value. This case corresponds to a rotation of the system around the steady state and the system evolves in the nonlinear branch of TIP. It was found that beyond the critical buoyancy ratio the entropy generation behavior can be pseudo-periodic or asymptotic depending on the Schmidt number. For the pseudo-periodic approach, the system develops a spiral approach towards the steady state and for the asymptotic approach the system progress directly towards the steady state and evolves in the linear branch of TIP.

Nomenclature:

Br:	Brinkman number	$Br = \frac{\mu U_0^2}{\lambda_e \Delta T}$ (T)
Br^*:	modified Brikmann number	$Br^* = \frac{Br}{\Omega}$
g	gravitational acceleration	
Da:	Darcy number	
H:	Height of the channel	
L:	length of the channel	
P:	dimensionless pressure	
Pe	Peclet number	
Pr:	Prandtl number	
Ra:	Rayleigh number	$Ra = g\beta_\theta(\theta_h - \theta_c)H^3 / \nu\alpha$
Re:	Reynolds number	$Re = U_0 H / \nu$
Ri:	Richardson number	$Ri = Ra / Pe\ Re$
U, V:	dimensionless velocity components	
X, Y:	dimensionless Cartesian coordinates	
	Greek letters	
φ	irreversibility distribution ratio	
α	thermal diffusivity	
β	thermal expansion coefficient	
ν	kinematic viscosity	
ε :	Porosity of the media $0 \prec \varepsilon \prec 1$	
Θ:	dimensionless temperature	$\theta = \frac{T - T_c}{T_h - T_c}$
Ω:	dimensionless temperature ratio $\Omega = \frac{\Delta T}{T_0}$	(ΔT/T)
	Subscripts	
H:	high	
l:	Low, local	

References

[1] Nakayama A, Hasebe K, and Sugiyama Y, Nonlinear, and Soft Matter Physics, 77 (2008).

[2] Ostrach S and KamotaniY, Journal of Heat Transfer, 2 (1975) 220.

[3] Evans G and Paolucci S, International Journal for Numerical Methods in Fluids, 11 (1990) 1001.

[4] Hasnaoui M, Bilgen E, Vasseur R, and Robillard L, Numerical Heat Transfer Applications, 3 (1991) 297·

[5] Nield D A and Bejan A, Convection in Porous Media, 2nd ed, Springer, New York, (1999).

[6] Nield DA and Bejan A, Convection in Porous Media, 3nd ed, Springer-Verleg, New York, (2006).

[7] Vafai K, Handbook of Porous Media, Second edition, New YORK, (2005).

[8] Patankar SV, In Computational Methods in Mechanics and Thermal Sciences, Hemisphere/Mac Graw-Hill, New York, NY, USA (1980).

[9] Saabas H J, and Baliga B R, Part I: FNHT Pt B-Fund, 26 (1994).

[10] Abbassi H, Turki S and Ben Nasrallah S, HTPA, 39 (2001) 307.

[11] Abbassi H, Turki S, And Ben Nasrallah S, IJTS, 40 (2001) 649.

[12] Prakash C, Numerical Heat Transfer, 3 (1986) 253.

[13] Hooky NA, Ph D thesis. Mc Gill University, Montreal, QC, Canada.

[14] Shohel M R and Fraser A, International Journal of Thermal Sciences, 1 (2005) 21–32.

[15] Abdulhassan A, Karamallah A, Mohammad WS, Khalil W H, Eng. & Tech. Journal, 9 (2011).

[16] Tayari A, Hidouri N, Magherbi M, BenBrahim A, Journal of Heat Transfer, 2 (2015).

[17] Prigogine I, Editions Odile Jacob, Paris 1996.