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Clustering Analysis on Teachers' Perceptions of Mathematics Pedagogical Content Knowledge

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Abstract: The purpose of this study is to cluster the perceptions of mathematics pedagogical content knowledge (MPCK) for teachers. The subject is 259 primary school teachers in Taiwan. This study constructs dimensions of MPCK according to the review and conclusions of literature. The MPCK assessment includes six dimensions, which are mathematics content knowledge (MCK), students' cognition knowledge (SCK), mathematics instruction knowledge (MIK), mathematics instruction practice (MIP), mathematics assessment knowledge (MAK) and teacher professional responsibility (TPR). The MPCK questionnaire is 4-points Likert scale and its reliability and validity are acceptable. Fuzzy clustering is adopted to cluster the subject based on these six dimensions. Results show that all teachers could be properly classified into six clusters. Each cluster has its own features of mathematics pedagogical content knowledge. There are also significantly differences in the dimensional scores among clusters. Besides, teachers who have more years of in-service tend to have higher dimensional scores on MPCK. These results could provide references for cultivating pre-service teachers and professional promotion for in-service teachers. Based on the findings of this study, some suggestions and recommendations are discussed for future research.

Keywords: Fuzzy Clustering, Mathematics Pedagogical Content Knowledge, Pedagogical Content Knowledge

1. Introduction

Educational studies have indicated that subject matter knowledge is necessary for effective teaching. Many researches have also showed that students' mathematics achievement is attributed to teachers' mathematics instruction knowledge [7, 12]. Therefore, there is widespread agreement that teachers should have special knowledge for teaching mathematics. Following Shulman's original 1986 address discussing pedagogical content knowledge, most researches indicate such knowledge not only exists but also contributes to effective mathematics instruction [18]. Teachers' mathematics pedagogical content knowledge is apparently the most important factor to student learning [5, 14].

In the last decades of the 20th century, the content and the way mathematics is taught are definitely changed from the traditional curriculum. The traditional method of teaching mathematics relies on the assumption that students acquire knowledge and skills by observing a teacher's explanations and practices. Nowadays, learning mathematics is viewed as an active process in which students construct their knowledge by engaging in meaningful and purposeful activities [9]. Most findings from earlier research on the relationships among teachers' mathematical knowledge, their teaching, and student learning indicate connection between teachers' knowledge and student achievement. It shows the role of teachers' mathematics teaching knowledge in their teaching is important. However, some literature indicates the role of teachers' mathematical knowledge in their teaching is not clear [15]. Therefore, clustering on teachers' perceptions of mathematics pedagogical content knowledge to further investigate its features should be prospective. In this study, it aims to investigate primary school teachers' perceptions of mathematics pedagogical content knowledge (MPCK). Questionnaire of MPCK is to evaluate teachers' perceptions. Fuzzy clustering is adopted to cluster the response data of perceptions. The optimal clustering could display the features of MPC for each cluster. The relationship between features of MPCK and experience of in-service could be discussed further. Results could be references to promote the professional development of primary school teachers.

2. Literature Review

2.1. Teaching and Learning Mathematics

It is important to clarify what the teachers need to know and be able to do mathematically in order to be effective in teaching mathematics for understanding [2]. They developed the practice-based theory of for teaching and they clarified [1].

In addition to general pedagogical knowledge and knowledge of the content, teachers need to know things like what topics children find interesting or difficult and the representations most useful for teaching a specific content idea. Pedagogical content knowledge is a unique kind of knowledge that intertwines content with aspects of teaching and learning (p. 4).

An understanding of the different kinds of perspectives that mathematics teachers hold on teaching mathematics can help to highlight some of the key characteristics of instructional practices that teachers' professional development aim to support.

Many definitions of pedagogical content knowledge begin with L. S. Shulman when he stated that pedagogical content knowledge in one's subject area [19]. As L. S. Shulman stated [20].

The key of distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students. (p. 15)

An argument that highlights the role of mathematics teachers in promoting educational improvement can be found in the research on school leadership. School leaders are increasingly considered as mediators of policies to support teachers' professional development of mathematics instruction [6]. Frameworks on teacher knowledge for mathematics teaching create a foundation for educational researchers who investigate the interaction of knowledge occurring in the classroom. The knowledge is multifaceted practice which is a challenge to decompose, evaluate and analyze the type of mathematics pedagogical knowledge.

Since teachers' mathematics knowledge greatly becomes an important issue throughout the past several decades, numerous approaches and methodologies have been undertaken in an attempt to identify the relationships among teachers' mathematics knowledge and student learning [16]. Studies using proxy measures have been mainly sought to demonstrate a relationship between teachers' demographic variables and teachers' mathematical knowledge. Another studies using direct measures to assess teachers' mathematics knowledge consider teachers' knowledge will influence student achievement [12]. Another lines concerning teachers' effectiveness tend to investigate and observe teachers' mathematical knowledge while they are teaching [10]. This study adopts clustering analysis to reveal the features of teachers' mathematical knowledge should be a prospective approach.

2.2. Dimensions of Knowledge for Mathematics Instruction

L. S. Shulman and P. L. Grossman refined the concept of pedagogical content knowledge and developed five subcomponents: knowledge of alternative content frameworks, knowledge of student understanding and misconceptions of a subject, knowledge of curriculum, knowledge of particular content for the purpose of teaching, and knowledge of topic specific pedagogical strategies [21]. In accordance with this point, one is clearly understood what knowledge mathematics teachers' should have.

Several other scholars have also attempted to identify components of teacher mathematics knowledge. To summarize the viewpoints, their common suggested components include four categories of knowledge: knowledge of mathematics, context specific knowledge, pedagogical knowledge, and knowledge of learners' cognition in mathematics [8]. In addition, mathematics teachers' beliefs are partly of their components [23]. Besides, the above four components of teachers' knowledge each influences one another.

P. Mishra and M. Koehler considered the development of technology had influenced the usage in educational environment, they provided the concept of technological pedagogical content knowledge [17]. The complicated relationships are shown in Figure 1.

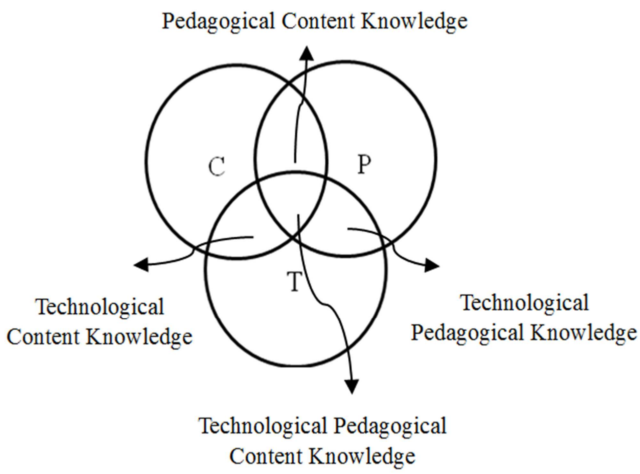

Figure 1. *The structure among content, pedagogy and technology (Mishra & Koehler, 2006, p.1025).*

H. C. Hill, D. L. Ball and S. G. Schilling proposed three types of subject matter knowledge (SMK) and three types of pedagogical content knowledge (PCK) [11]. As to mathematics teachers, the knowledge is non-overlapping categories in the domain of mathematical knowledge for teaching. It is shown in Figure 2.

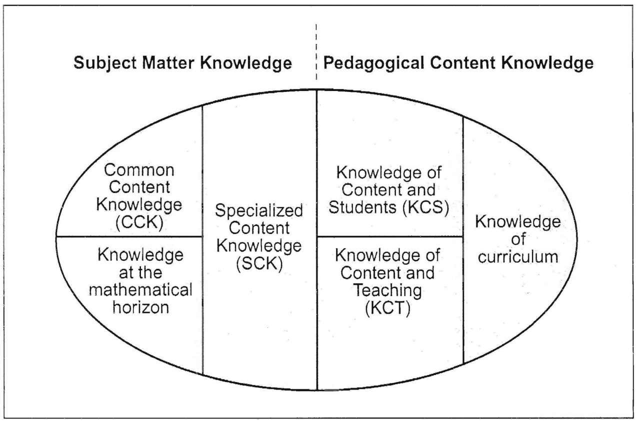

Figure 2*. Domain map for mathematical knowledge for teaching (MKT) from Hill, Ball, and Schilling (2008).*

Teacher knowledge is strongly related to individual experiences and contexts [25]. A major conclusion from literature is that an understanding of teacher knowledge may be useful to improve teacher professional development and to make educational innovations more successful.

Some literature investigated effects of teachers' mathematical knowledge for teaching on student achievement. H. C. Hill, B. Rowan and D. L. Ball used a linear mixed-model methodology to explore whether and how teachers' mathematical knowledge for teaching contributed to gains in students' mathematics achievement [12]. The data included the first and third graders' mathematical achievement gains over a year which was nested within teachers, who in turn were nested within schools. Results showed that teachers' mathematical knowledge was significantly related to student achievement gains in both first and third grades after controlling for key student- and teacher-level covariates. This finding provided suggestions to improve students' mathematics achievement by improving teachers' mathematical knowledge.

2.3. Fuzzy Clustering and Its Application in Education

L. A. Zadeh developed fuzzy theory and it flourished methodologies in many fields. In recent years, fuzzy theory has became one approach of data analysis methodology [4, 27]. Suppose A is a fuzzy set and μ_A is membership function with the membership value $0 \le u_A(x) \le 1$, $u_A(x)$ represents the degree that x belongs to fuzzy set A.

The most widely-used fuzzy clustering algorithm is the fuzzy c-means algorithm (FCM). J. C. Bezdek combined fuzzy theory and clustering technique and he innovated fuzzy clustering greatly since he brought the membership into the objective function [3]. This fuzzy clustering is also called fuzzy c-means which allows partial memberships of data points in the clusters. It is suitable to cluster database so that population of dataset could be classified into some subpopulations. Fuzzy clustering is a useful technique which helps to enrich the semantics of the data by revealing patterns in database. For a data matrix $X = (x_{nm})_{N \times M}$ with N subjects and M variables, the membership matrix $U = (u_{cn})_{C \times N}$ and the group center matrix $V = (v_{cm})_{C \times M}$ are unknown under group number C. The following objective function with optimization problem is applied as follows,

$$J(U,V) = \sum_{n=1}^{N}\sum_{c=1}^{C}(u_{cn})^q d^2(c,n) \quad (1)$$

where $d^2(c,n) = \sum_{m=1}^{M}(x_{nm} - v_{cm})^2$ and u_{cn} , v_{cm} could be acquired by iteration with convergence.

Decision on number of cluster is the issue of clustering validity. There are some indices to help select the optimal number of cluster. Two popular indices, which are partition coefficient $F(U;C)$ and partition entropy $H(U;C)$, are used in this study. The formulas are as follows [3].

$$F(U;C) = \frac{1}{N}\sum_{n=1}^{N}\sum_{c=1}^{C}(u_{cn})^2 \quad (2)$$

$$H(U;C) = \frac{-1}{N}\sum_{n=1}^{N}\sum_{c=1}^{C} ucnln(ucn)\,, \quad \forall ucn \neq 0 \quad (3)$$

Within the range of possible clustering number, the best partition is that corresponding to the highest partition coefficient. As to the partition entropy, the best partition is that corresponding to the lowest partition entropy [26].

Fuzzy clustering has been adopted to identify and classify at-risk students at an early stage of their academic career so that teachers can develop plans to improve their performance [13]. Those at–risk students were classified into weak, average and good clusters. The findings could help educational managers monitor the performance of various groups of students and improve academic achievement. Clustering analysis was also adopted to investigate the knowledge that experienced science teachers have of models and modelling in science in the context of a school curriculum innovation project in which the role and the nature of models and modelling in science are emphasized [24]. In their study, two instruments of questionnaires are used collect response for clustering analysis. Results showed that different clusters had varieties of the teachers' knowledge of models and modelling in science. H. S. Siller, S. Kuntze, S. Lerman and C. Vogl investigated the big idea on the mathematics classroom for teachers to be aware of this big idea related to a variety of curricular contents. They study concentrated on views of Austrian and German pre-service teachers about the significance of modelling as a big idea. Clustering was adopted to analyze the ratings of the big ideas about results concerning the pre-service teachers' perceptions [22]. Its findings indicated the various features of big idea for different clusters. In according to the above the clustering application in education, it is concluded that clustering analysis should be beneficial to probe the mathematics pedagogical content knowledge of teachers.

3. Research Design

3.1. Questionnaire of MPCK

The questionnaire of perceptions of mathematics pedagogical content knowledge (MPCK) is designed by the author according to the related literatures of mathematics teaching knowledge. The questionnaire is four-point Likert scale. The coding and linguistic variables are 1 = strongly disagree 2 = disagree, 3 = agree and 4 = strongly agree. In this study, the questionnaire consists of six dimensions. These dimensions are mathematics content knowledge (MCK), students' cognition knowledge (SCK), mathematics instruction knowledge (MIK), mathematics instruction practice (MIP), mathematics assessment knowledge (MAK) and teacher professional responsibility (TPR). The validity has been confirmed based factor analysis and experts. The Cronbach reliability, which is one of internal consistency indices, with respect to each dimension is between 0.81 and 0.90. It means that the validity and reliability are acceptable.

3.2. Subject and Sample

There are totally 259 primary school teachers in Taiwan participating in the study. Of these teachers, there are 73 male teachers and 186 female teachers. The sample distribution is shown in Table 1. It indicates most teachers are located at 6-15 and 16-25 in-service years.

Table 1. *Sample distribution for years of in-service.*

Years of in-service		Size and percent
under 15 years	under 5 years	33 (12.74 %)
	6-15 years	109 (42.09 %)
16 years and over	16-25 years	94 (36.29 %)
	26 years and over	23 (8.88%)
	Total	259 (100%)

4. Results and Discussions

4.1. Statistics Description and Number of Clusters

Mean and standard deviation of each dimension are depicted in Table 2. It reveals that the lowest mean score is students' cognition knowledge (SCK) but its standard deviation is quite high. On the contrary, the highest mean score is teacher professional responsibility (TPR) but its standard deviation is the lowest. General speaking, the perceptions as to all dimension of MPCK almost locates at "3 = agree".

The average scores of dimensions for each teachers are the raw data for fuzzy clustering. On the process of fuzzy clustering, partition entropy and partition coefficient are used to decide number of C clusters. As shown in Table 3, from C=2 to C=8, the largest partition coefficient (0.889458) and the smallest partition entropy (0.133875) occur when it is C=6. Therefore, it means the subject could be properly classified into six clusters.

Table 2. *Mean and standard deviation of each dimension.*

Dimensions	Mean	Standard deviation
MCK	3.09	.47
SCK	2.90	.52
MIK	3.01	.53
MIP	3.05	.47
MAK	3.04	.46
TPR	3.19	.42

Table 3. *Partition coefficient and partition entropy for number of clusters.*

Number of clusters	Partition coefficient	Partition entropy
2	0.887876	0.177530
3	0.885784	0.193827
4	0.865666	0.233950
5	0.838500	0.289340
6	0.889458	0.133875
7	0.805318	0.368319
8	0.786818	0.41664

Since optimal number of clusters is six which are from cluster I to cluster VI, the sample size and percent are shown in Table 4. Cluster III has the largest sample size (84 and 32.43%) and cluster I has the smallest sample size (20 and 7.72%).

Table 4. *Sample size and percent of each cluster.*

Cluster	Size	Percent (%)
I	20	7.72%
II	50	19.30%
III	84	32.43%
IV	42	16.22%
V	29	11.20%
VI	34	13.13%
Total	259	100%

4.2. Description and Characteristics of Clusters

Based on the results of fuzzy clustering, the cluster centers for each dimension of MPCK are shown in Table 5. Figure 3 displays the line chart of these cluster centers.

According to Table 5, Cluster I have the lowest dimensional scores and the cluster VI has the highest dimensional scores. In Figure 3, the lines across dimensions of MPCK are almost parallel. Hence, it means the dimensional scores almost increase as they are from cluster I to cluster VI. As to the dimensions of mathematics assessment knowledge (MAK) and teacher professional responsibility (TPR), there exist little differences. It reveals that cluster IV has higher MAK and TPR than cluster V. Cluster II also has higher TPR than cluster III.

In comparison with Table 4 and Table 5, one is concluded that cluster III owns the largest sample size among clusters and its scores of cluster center are almost lower than 3. Therefore, quite a few of teachers perceive that their own mathematics pedagogical content knowledge (MPCK) is not very well.

Table 5. *Cluster center for each dimension of MPCK.*

Cluster	Cluster center of dimensions					
	MCK	SCK	MIK	MIP	MAK	TPR
I	2.40165	2.10935	2.15004	2.18792	2.32038	2.64558
II	2.80715	2.30990	2.69484	2.81075	2.76999	3.02980
III	2.89469	2.96046	2.87068	2.89777	2.92389	2.96455
IV	3.35836	3.06129	3.01151	3.18929	3.24765	3.44951
V	3.48844	3.26089	3.60954	3.37494	3.22162	3.32958
VI	3.73809	3.52526	3.79068	3.80906	3.73803	3.85050

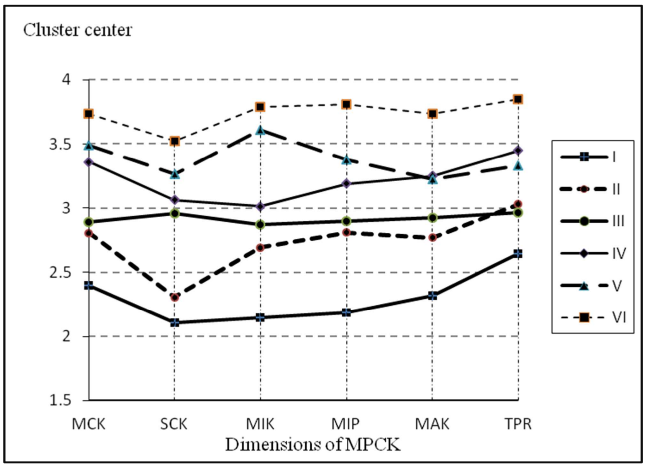

Figure 3. *Line chart of cluster center for each dimension.*

It is prospective to compare the means of all dimensions among clusters. As shown in Table 6, it clearly displays that there are significant difference on means of all dimensions among clusters.

Table 6. *One way ANOVA on dimensions among clusters.*

Dimensions	Source	SS	df	MS	F
	Between Group	36.56	5	7.31	89.38***
MCK	Within Group	20.70	253	.08	
	Total	57.25	258		
	Between Group	47.38	5	9.48	105.16***
SCK	Within Group	22.80	253	.09	
	Total	70.18	258		
	Between Group	52.18	5	10.47	136.57***
MIK	Within Group	19.33	253	.08	
	Total	71.51	258		
	Between Group	42.51	5	8.50	151.96***
MIP	Within Group	14.15	253	.06	
	Total	56.66	258		
	Between Group	33.84	5	6.77	78.56***
MAK	Within Group	21.79	253	.09	
	Total	55.63	258		
	Between Group	30.07	5	6.01	95.61***
TPR	Within Group	15.92	253	.06	
	Total	45.99	258		

*** p<.001

Further investigation on the cross analysis for cluster and years of in-service is shown in Table 7. The Chi-square test is $\chi^2(df=5)=13.387$, $p=0.020$. It indicates there exits significantly difference as to frequency distribution. Teachers who are under 15 years of in-service tend to belong to cluster I, II and III. On the contrary, teachers who are 16 years of in-service and over tend to belong to cluster IV and VI. Therefore, one is concluded that teachers who have more years of in-service will have higher scores on all MPCK dimensions. Experienced teachers would have higher perceptions of MPCK.

Table 7. *Cross analysis of cluster and years of in-service.*

Cluster	Years of in-service	
	under 15 years	16 years and over
I	12	8
II	28	22
III	57	27
IV	17	25
V	15	14
VI	13	21
Total	142	117

5. Conclusions

This study surveys teachers' perceptions of mathematics pedagogical content knowledge and discuss the clustering based on dimensions of MPCK. Results show the subject could be properly classified into six clusters and perception on dimensions of MPCK varies with clusters. It implies there are some latent subpopulations about MPCK for primary school teachers in Taiwan. Moreover, cross analysis shows primary school teachers in Taiwan who have more years of in-service will have higher scores on all MPCK dimensions. It coincides and induces the finding that experiences of mathematics teaching may influence the perception of MPCK [11]. Conclusions of this study may provide some suggestions for cultivation and professional promotion of primary school teacher. Further research could consolidate quality methodologies to investigate the influential factors in mathematics pedagogical content knowledge.

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