

# Temperature Fluctuations in Dusty Fluid Homogeneous Turbulence at Four Point Correlations

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**Abstract:** In this study the temperature fluctuations in dusty fluid homogeneous turbulence prior to the final period is considered by using the four point correlation equations for fluctuating quantities in the flow field. The correlation equations are converted into spectral form by their Fourier-transform. The set of equations are made to determinate by neglecting the quintuple correlations in comparison to the fourth- order correlation terms. Finally by integration of the energy spectrum over all wave numbers and have obtained the energy equation of decaying of temperature fluctuations in dusty fluid homogeneous turbulence for four point correlations. The obtained results have been shown by graphically at different Prandtl no. and at the different state of temperature. It is also determined the values of the constant appear at the energy equation by using the values of the parameters which exist in it for different fluids. It has also been shown the effects of the parameters by graphically.

**Keywords:** Four-Point Correlation, Decay, Temperature Fluctuations, Final Period, Dust Particles, Prandtl No

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## 1. Introduction

Very recently the motion in dusty fluid in homogeneous turbulence problem is developed rapidly. The motion of dusty fluid is observed in the movement of dust-laden air, in a gas cooling system, in the tidal rivers and in other fluids. The behavior of dust particles in a turbulent flow depend on the concentration of the particles, the size of the particles and quantities of the particles with respect to scale of the turbulent fluid.

In the past, some researchers had done their research considering dust particles in turbulence and MHD<sup>1</sup> turbulent flow. Corrsin [1] considered on the spectrum of isotropic temperature fluctuations in isotropic turbulence. Deissler [2, 3] developed a theory on decay of homogeneous turbulence for times before the final period for three and four point correlation. In the next, Loeffler and Deissler [4] extended their theory for the case of decaying of temperature fluctuations in homogeneous turbulence. Saffman [5] derived an equation that describes the fluid containing small dust particles which is applicable laminar flow as well as turbulent flow. Kishore and Sinha [6] studied the statistical

theory of decay process of homogeneous hydromagnetic turbulence, Kishor and Golsefied [7] have done their research on the effect of Coriolis force on acceleration covariance in MHD turbulent flow of a dusty incompressible fluid. Kishor and Sarker [8] also studied the rate of change of vorticity covariance in MHD turbulent flow of dusty incompressible fluid. Recently, Azad and Sarker [9, 10, 12] calculated a results on decaying of MHD turbulence before the final period for the case of multi-point and multi-time in presence of dust particle taking Coriolis force. Azad et al [11, 13] studied the decay of temperature fluctuations in dusty fluid MHD turbulence in a rotating system before the final period. Sarker et al [14] have been done their work on first order reactant in MHD turbulence before the final Period of decay for the case of multi-point and multi-time in presence of dust particles. Azad et al [15, 16] had done their research on the decay law of temperature fluctuations in dusty fluid MHD turbulence before the final period. They also had done their work on joint distribution function of velocity, temperature and concentration in convective dusty fluid turbulent flow. Azad and Mumtahirah [17, 18] considered the decay of temperature fluctuations in dusty fluid homogeneous turbulence prior to final period with Coriolis force. Bkar et al [19] studied on 4-point correlations of dusty fluid MHD

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<sup>1</sup> MHD= Magneto-hydro Dynamic.

turbulent flow in a 1<sup>st</sup> order chemical reaction. Azad et al [20, 21, 22, 23] had done their work on the effect of chemical reaction on statistical theory of dusty fluid MHD turbulent flow for certain variables at three-point correlation. Alam et al [24] Study the Effect of Chemical Reaction and Magnetic Field on Free Convection Boundary Layer Flow of Heat and Mass Transfer with Variable Prandtl Number. Most recently Mamun et al [25] also studied the effect of 1st order chemical reaction in convective dusty fluids turbulent flow for three-point Joint Distribution Functions.

The above researchers have considered two and three point correlation equations and solved these equations after

neglecting the fourth and higher order correlation terms. But in this work we have solved for four point correlations equations after neglecting the quintuple and higher order correlation terms for dusty fluid turbulent flow.

The main purpose of the present study is to find a possible solution for the dynamics of decaying the temperature fluctuation in homogeneous dusty fluid turbulence for four point correlation. Through this study, we have obtained energy equation of decaying of temperature fluctuations in dusty fluid homogeneous turbulence at times before the final period for four-point correlation systems which comes out in the following form

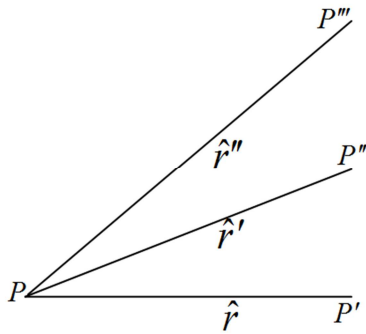
$$\langle T^2 \rangle = W(t-t_0)^{-3/2} + X(t-t_0)^{-5} + Y(t-t_1)^{-7/2} \exp[l(t-t_1)] + Z(t-t_1)^{-7} \exp[l(t-t_1)],$$

Where  $\langle T^2 \rangle$  denotes the total energy and  $t$  is the time,  $W, X,$

$Y$  and  $Z$  are arbitrary constants determined by initial conditions. Through this study we have tried to show the effect of varying Prandtl no. by graphically. In this study, it observed that the energy more decays than the clean fluid.

## 2. Four Point Correlation and Spectral Equations

In order to find the four point correlations and spectral equations for single time and take the momentum equation of turbulence at the point  $P$  and the energy equation of Temperature fluctuation for four point correlations in presence of dust particle at  $P', P''$  and  $P'''$  with position vectors  $\hat{r}, \hat{r}'$  and  $\hat{r}''$



Vector configuration for four point correlation equations.

$$\begin{aligned} & \frac{\partial(u_j T_i' T_j'' T_k''')}{\partial t} + \frac{\partial(u_j u_i' T_j'' T_k''')}{\partial x_i} + \frac{\partial(u_i u_j' T_j'' T_k''')}{\partial x_i'} + \frac{\partial(u_i u_j'' T_j'' T_k''')}{\partial x_i''} + \frac{\partial(u_i u_j''' T_j'' T_k''')}{\partial x_i'''} \\ &= -\frac{1}{\rho} \frac{\partial(P T_i' T_j'' T_k''')}{\partial x_j} + \frac{\nu}{P_r} \frac{\partial^2(u_j T_i' T_j'' T_k''')}{\partial x_j \partial x_i} + \frac{\nu}{p_r} \frac{\partial^2(u_i T_j' T_j'' T_k''')}{\partial x_i' \partial x_i'} + \frac{\nu}{p_r} \frac{\partial^2(u_i T_j'' T_j'' T_k''')}{\partial x_i'' \partial x_i''} + \\ & \frac{\nu}{p_r} \frac{\partial^2(u_i T_j''' T_j'' T_k''')}{\partial x_i''' \partial x_i'''} + f(u_i - v_i) T_i' T_j'' T_k''' \end{aligned} \quad (5)$$

Now we will use the transformations

$$\frac{\partial}{\partial x_i'''} = \frac{\partial}{\partial r_i'''} , \frac{\partial}{\partial x_i''} = \frac{\partial}{\partial r_i''} , \frac{\partial}{\partial x_i'} = \frac{\partial}{\partial r_i'} , \frac{\partial}{\partial x_i} = \frac{\partial}{\partial r_i} , \frac{\partial}{\partial x_i} = -\left(\frac{\partial}{\partial r_i} + \frac{\partial}{\partial r_i'} + \frac{\partial}{\partial r_i''}\right) \text{ into equations (5) then,}$$

$$\frac{\partial u_j}{\partial t} + \frac{\partial(u_j u_i)}{\partial x_i} = -\frac{1}{\rho} + \frac{\nu}{P_r} \frac{\partial^2 u_j}{\partial x_i \partial x_i} + f(u_i - v_i) \quad (1)$$

$$\frac{\partial T'}{\partial t} + u_i' \frac{\partial T'}{\partial x_i} = \frac{\nu}{p_r} \frac{\partial^2 T'}{\partial x_i' \partial x_i'} \quad (2)$$

$$\frac{\partial T''}{\partial t} + u_i'' \frac{\partial T''}{\partial x_i} = \frac{\nu}{p_r} \frac{\partial^2 T''}{\partial x_i'' \partial x_i''} \quad (3)$$

$$\frac{\partial T'''}{\partial t} + u_i''' \frac{\partial T'''}{\partial x_i} = \frac{\nu}{p_r} \frac{\partial^2 T'''}{\partial x_i''' \partial x_i'''} \quad (4)$$

Where,  $T, T', T'', T'''$  denote temperature at points  $P, P', P''$  and  $P'''$ ,  $u_i$  = Instantaneous velocity,

$\rho$  = Fluid density,  $C_p$  = Heat capacity at constant pressure,  $k$  = Thermal conductivity,

$x_i$  = Space co-ordinate,  $t$  = Time,  $f = \frac{KN}{\rho}$  the

dimensions of frequency,  $K$  = Stock's drug resistance,  $N$  = Constant number density of dust particles,  $v_i$  = velocity component of dust particles and the repeated subscripts are assumed from 1 to 3.

Multiplying equation (1) by  $T_i' T_j'' T_k'''$  (2) by  $u_i T_j'' T_k'''$  (3) by  $u_i T_i' T_j'' T_k'''$  (4) by  $u_i T_i' T_j''$  and adding these equations, after taking time averages, result is

$$\begin{aligned}
& \frac{\partial(\overline{u_j T_i' T_j'' T_k''''})}{\partial t} - \frac{\partial(\overline{u_j u_i T_i' T_j'' T_k''''})}{\partial r_i} - \frac{\partial(\overline{u_j u_i T_i' T_j'' T_k''''})}{\partial r_i'} - \frac{\partial(\overline{u_j u_i T_i' T_j'' T_k''''})}{\partial r_i''} + \frac{\partial(\overline{u_i u_i' T_i' T_j'' T_k''''})}{\partial r_i} \\
& + \frac{\partial(\overline{u_i u_i'' T_i' T_j'' T_k''''})}{\partial r_i'} + \frac{\partial(\overline{u_i u_i''' T_i' T_j'' T_k''''})}{\partial r_i''} = \frac{1}{\rho} \left[ \frac{\partial(\overline{P T_i' T_j'' T_k''''})}{\partial r_j} + \frac{\partial(\overline{P T_i' T_j'' T_k''''})}{\partial r_j'} + \frac{\partial(\overline{P T_i' T_j'' T_k''''})}{\partial r_j''} \right] \\
& + \frac{\nu}{P_r} [(1+P_r) \frac{\partial^2(\overline{u_i T_i' T_j'' T_k''''})}{\partial r_i \partial r_i} + (1+P_r) \frac{\partial^2(\overline{u_i T_i' T_j'' T_k''''})}{\partial r_i \partial r_i'} + (1+P_r) \frac{\partial^2(\overline{u_i T_i' T_j'' T_k''''})}{\partial r_i \partial r_i''} \\
& + 2P_r \frac{\partial^2(\overline{u_i T_i' T_j'' T_k''''})}{\partial r_i \partial r_i'} + 2P_r \frac{\partial^2(\overline{u_i T_i' T_j'' T_k''''})}{\partial r_i \partial r_i''} + 2P_r \frac{\partial^2(\overline{u_i T_i' T_j'' T_k''''})}{\partial r_i \partial r_i''} + f(u_i - v_i) T_i' T_j'' T_m'''
\end{aligned} \quad (6)$$

Converting the equation (6) into spectral form, now we can define the following nine dimensional Fourier transforms

$$\langle u_i T_i'(\hat{r}) T_j''(\hat{r}') T_m'''(\hat{r}'') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_i \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}' + \hat{k}'' \cdot \hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \quad (7)$$

$$\langle u_j u_i T_i'(\hat{r}) T_j''(\hat{r}') T_m'''(\hat{r}'') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_j \phi_i'(\hat{k}) \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}' + \hat{k}'' \cdot \hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \quad (8)$$

$$\langle u_i u_i T_i'(\hat{r}) T_j''(\hat{r}') T_m'''(\hat{r}'') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_i \phi_i'(\hat{k}) \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}' + \hat{k}'' \cdot \hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \quad (9)$$

$$\langle u_j u_i' T_i'(\hat{r}) T_j''(\hat{r}') T_m'''(\hat{r}'') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_j \phi_i'(\hat{k}) \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}' + \hat{k}'' \cdot \hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \quad (10)$$

$$\langle u_j u_i'' T_i'(\hat{r}) T_j''(\hat{r}') T_m'''(\hat{r}'') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_j \phi_i''(\hat{k}) \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}' + \hat{k}'' \cdot \hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \quad (11)$$

$$\langle u_j u_i''' T_i'(\hat{r}) T_j''(\hat{r}') T_m'''(\hat{r}'') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_j \phi_i'''(\hat{k}) \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}' + \hat{k}'' \cdot \hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \quad (12)$$

$$\langle P T_i'(\hat{r}) T_j''(\hat{r}') T_m'''(\hat{r}'') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \delta \gamma_i'(\hat{k}) \gamma_j''(\hat{k}') \gamma_m'''(\hat{k}'') \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}' + \hat{k}'' \cdot \hat{r}'')] d\hat{k} d\hat{k}' d\hat{k}'' \quad (13)$$

Interchange of points  $p'$  and  $p''$ ,  $p'$  and  $p'''$  the subscripts i and j; i and m results in the relations  $\overline{(u_j u_i' T_i' T_j'' T_m''')} = \overline{(u_j u_i' T_i' T_j'' T_m''')} , \overline{(u_j u_i' T_i' T_j'' T_m''')} = \overline{(u_j u_i' T_i' T_j'' T_m''')}$

By using equations (7) to (13), one can write equation (6) in the form

$$\begin{aligned}
& \frac{\partial}{\partial t} (\overline{\phi_i \gamma_i' \gamma_j'' \gamma_m'''}) + \frac{\nu}{P_r} [(1+P_r) K^2 + (1+P_r) K'^2 + (1+P_r) K''^2 + 2P_r K K' + 2P_r K K'' + 2P_r K K'''] (\overline{\phi_i \gamma_i' \gamma_j'' \gamma_m'''}) = \\
& i(K_k + K'_k + K'') (\overline{\phi_i \phi_k \gamma_i' \gamma_j'' \gamma_m'''}) - i(K_k + K'_k + K'') (\overline{\gamma_i \gamma_k \gamma_i' \gamma_j'' \gamma_m'''}) - i(K_k + K'_k + K'') (\overline{\phi_i \phi_k \gamma_i' \gamma_j'' \gamma_m'''}) \\
& + i(K_k + K'_k + K'') (\overline{\gamma_i \gamma_k \gamma_i' \gamma_j'' \gamma_m'''}) + i \frac{1}{\rho} (K_k + K'_k + K'') (\overline{P \gamma_i' \gamma_j'' \gamma_m'''})
\end{aligned} \quad (14)$$

The tensor equation (14) can be converted to the scalar equation by contraction of the indices i and j;

$$\begin{aligned}
& \frac{\partial}{\partial t} (\overline{\phi_i \gamma_i' \gamma_i'' \gamma_m'''}) + \frac{\nu}{P_r} [(1+P_r) K^2 + (1+P_r) K'^2 + (1+P_r) K''^2 + 2P_r K K' + 2P_r K K'' + \\
& 2P_r K K'''] (\overline{\phi_i \gamma_i' \gamma_i'' \gamma_m'''}) - f(u_i - v_i) \overline{\phi_i \gamma_i' \gamma_i'' \gamma_m'''} = i(K_k + K'_k + K'') (\overline{\phi_i \phi_k \gamma_i' \gamma_i'' \gamma_m'''}) - i(K_k + K'_k + K'') (\overline{\gamma_i \gamma_k \gamma_i' \gamma_i'' \gamma_m'''}) - \\
& i(K_k + K'_k + K'') (\overline{\phi_i \phi_k \gamma_i' \gamma_i'' \gamma_m'''}) + i(K_k + K'_k + K'') (\overline{\gamma_i \gamma_k \gamma_i' \gamma_i'' \gamma_m'''}) + i \frac{1}{\rho} (K_k + K'_k + K'') (\overline{P \gamma_i' \gamma_i'' \gamma_m'''})
\end{aligned} \quad (15)$$

If we take the derivative with respect to  $x_i$  of the momentum equation (1) at p, we have

$$\frac{\partial^2 u_j u_i}{\partial x_i \partial x_i} = -\frac{1}{\rho} \frac{\partial^2 P}{\partial x_i \partial x_j} \quad (16)$$

Multiplying equation (16) by  $T_i' T_j'' T_m'''$ ; and then taking time averages

$$\frac{\partial^2 \overline{(u_j u_i T_i' T_j'' T_m''')}}{\partial x_i \partial x_i} = -\frac{1}{\rho} \frac{\partial^2 \overline{(P T_i' T_j'' T_m''')}}{\partial x_i \partial x_j} \quad (17)$$

Equation (17) can be written in terms of the independent variables  $\hat{r}$ ,  $\hat{r}'$  and  $\hat{r}''$  we have,

$$\left[ \frac{\partial^2}{\partial r_i \partial r_i} + \frac{\partial^2}{\partial r_i \partial r_i'} + \frac{\partial^2}{\partial r_i \partial r_i''} + \frac{\partial^2}{\partial r_i \partial r_i'} + \frac{\partial^2}{\partial r_i \partial r_i''} + \frac{\partial^2}{\partial r_i \partial r_i''} \right] \overline{(u_j u_i T_i' T_j'' T_m''')} =$$

$$-\frac{1}{\rho} \left[ \frac{\partial^2}{\partial r_i \partial r_k} + \frac{\partial^2}{\partial r_i \partial r_k'} + \frac{\partial^2}{\partial r_i \partial r_k''} + \frac{\partial^2}{\partial r_i \partial r_k''} + \frac{\partial^2}{\partial r_i \partial r_k''} + \frac{\partial^2}{\partial r_i \partial r_k''} + \frac{\partial^2}{\partial r_i \partial r_k''} + \frac{\partial^2}{\partial r_i \partial r_k''} \right] \overline{(P T_i' T_j'' T_m''')} \quad (18)$$

and

$$-\frac{1}{\rho} \overline{(P \gamma_i' \gamma_j'' \gamma_k''')} = \frac{K_i K_k + K_i K_k' + K_i' K_k + K_i' K_k' + K_i'' K_k + K_i'' K_k' + K_i'' K_k' + K_i'' K_k''}{K_i K_i + K_i' K_i' + K_i'' K_i'' + 2 K_i K_i' + 2 K_i' K_i' + 2 K_i'' K_i''} \overline{\phi_i \phi_k \gamma_i' \gamma_j'' \gamma_k'''} \quad (19)$$

Equation (19) can be used to eliminate  $\left( -\frac{1}{\rho} P \gamma_i' \gamma_j'' \gamma_k''' \right)$  from equation (16) and (17). Taking contraction of the indices i and j in equation (19). Equations (16) and (19) are the spectral equation corresponding to the four –point correlation equation. The spectral equations corresponding to the three-point correlation equations by contraction of the indices i and m are

$$\frac{\partial}{\partial t} \overline{(\phi_i \beta_i' \beta_i'')} + \frac{\nu}{P_r} [(1 + P_r)(K^2 + K'^2 + 2 P_r K K') \overline{(\phi_i \beta_i' \beta_i'')} - i(K_k + K_k') \overline{(\phi_i \phi_k \beta_i' \beta_i'')} - i(K_k + K_k') \overline{(\beta_i \phi_k \beta_i' \beta_i'')}]$$

$$- i(K_k + K_k') \overline{(\phi_i \phi_k \beta_i' \beta_i'')} + i(K_k + K_k') \overline{(\phi_i' \phi_k \beta_i' \beta_i'')} + i(K_k + K_k') \overline{(\gamma \beta_i' \beta_i'')} + f(u_i - v_i) \overline{\phi_i \gamma_i' \gamma_i''} \quad (20)$$

And

$$-\frac{1}{\rho} \overline{(\gamma \beta_i' \beta_i'')} = \frac{K_i K_k + K_i K_k' + K_i' K_k + K_i' K_k'}{K_i^2 + K_i'^2 + 2 K_i K_i'} \overline{(\phi_i \phi_k \beta_i' \beta_i'')} \quad (21)$$

the six dimensional spectral tensors are defined by

$$\langle u_i T_j' T_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_i \beta_i'(\hat{k}) \beta_j''(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}')] d\hat{k} d\hat{k}' \quad (22)$$

$$\langle u_j u_i T_i' T_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_j \phi_i \beta_i'(\hat{k}) \beta_j''(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}')] d\hat{k} d\hat{k}' \quad (23)$$

$$\langle u_i u_i T_j' T_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_i \phi_i \beta_i'(\hat{k}) \beta_j''(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}')] d\hat{k} d\hat{k}' \quad (24)$$

$$\langle u_j u_i' T_i' T_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_j \phi_i \beta_i'(\hat{k}) \beta_j''(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}')] d\hat{k} d\hat{k}' \quad (25)$$

$$\langle u_j u_i'' T_i' T_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \phi_j \phi_i'' \beta_i'(\hat{k}) \beta_j''(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}')] d\hat{k} d\hat{k}' \quad (26)$$

$$\langle P T_i' T_j''(\hat{r}') \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \gamma \beta_i'(\hat{k}) \beta_j''(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}')] d\hat{k} d\hat{k}' \quad (27)$$

Interchanging the points  $P'$  and  $P''$  shows that

$$\overline{\langle u_j u_i'' T'' T' \rangle} = \overline{\langle u_j u_i' T' T'' \rangle} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \overline{\beta_j \beta_i' \theta' \theta''} \exp[i(\hat{k} \cdot \hat{r} + \hat{k}' \cdot \hat{r}')] d\hat{k} d\hat{k}' \quad (28)$$

The spectral equations corresponding to the two-point correlation equations by contraction of the indices  $i$  and  $j$  are

$$\frac{\partial}{\partial t} (\overline{\phi_i \beta_i'}) + \frac{2\nu}{P_r} K^2 (\overline{\phi_i \beta_i'}) = 2iK_k [(\overline{\phi_i \phi_k' \beta_i'}) - (\overline{\phi_i' \phi_k \beta_i'})] \quad (29)$$

A relation between  $\phi_i \phi_k' \beta_i'(\hat{k})$  and  $\phi_i' \phi_k \beta_i(\hat{k})$  can be obtained by letting  $\hat{r}'' = 0$  in equation (7) and comparing the result with equation (23)

$$\langle u_i' T_i'(\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \phi_i' \beta_i'(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r})] d\hat{k} \quad (30)$$

$$\langle u_i u_i' T_i'(\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \phi_i \phi_k' \beta_i'(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r})] d\hat{k} \quad (31)$$

$$\langle u_i' u_k T_i'(\hat{r}) \rangle = \int_{-\infty}^{\infty} \langle \phi_i' \phi_k \beta_i(\hat{k}) \rangle \exp[i(\hat{k} \cdot \hat{r})] d\hat{k} \quad (32)$$

### 3. Solution

To obtain the equation for the times before the final period of decay, the four point correlations are considered and the quintuple correlation terms decays faster than the lower order correlation terms. If this assumption is made then we can neglect all the terms on the right side of the equation (15).

So we get

$$\frac{\partial}{\partial t} (\overline{\phi_i \gamma_i' \gamma_j'' \gamma_m''}) + \frac{\nu}{P_r} [(1+P_r)(K^2 + (1+P_r)K'^2 + (1+P_r)K''^2 + 2P_r K K' + 2P_r K K'' + 2P_r K K'')(\overline{\phi_i \gamma_i' \gamma_j'' \gamma_m''}) - f(u_i - v_i) \overline{\phi_i \gamma_i' \gamma_j'' \gamma_m''}] = 0$$

Integrating this equation between  $t_1$  and  $t$  with inner multiplication by  $k_k$  we have,

$$\langle \phi_i \gamma_i' \gamma_j'' \gamma_m'' \rangle = \langle \phi_i \gamma_i' \gamma_j'' \gamma_m'' \rangle_1 \exp\left\{ \frac{\nu}{P_r} [(1+P_r)(K^2 + (1+P_r)K'^2 + (1+P_r)K''^2 + 2P_r K K' + 2P_r K K'' + 2P_r K K'') + f(u_i - v_i)](t - t_1) \right\} \quad (33)$$

Where  $\langle \phi_i \gamma_i' \gamma_j'' \gamma_m'' \rangle_1$  is the value of  $\langle \phi_i \gamma_i' \gamma_j'' \gamma_m'' \rangle$  at  $t = t_1$  for small values of  $k$ ,

Substituting of equation (21), (28), (33) in equation (20), we get,

$$\begin{aligned} \frac{\partial}{\partial t} (k_k \overline{\phi_i \beta_i' \beta_i''}) + \frac{\nu}{P_r} [(1+P_r)(K^2 + K'^2 + 2P_r K K') (k_k \overline{\phi_i \beta_i' \beta_i''}) = \\ [a]_1 \int_{-\infty}^{\infty} \exp\left[-\frac{\nu}{P_r}(t - t_1)[(1+P_r)(k^2 + k'^2 + k''^2) + 2p_r(kk' + k'k'' + k''k)]\right] \exp[fs(t - t_1)] dk'' + \\ [b]_1 \int_{-\infty}^{\infty} \exp\left[-\frac{\nu}{P_r}(t - t_1)[(1+P_r)(k^2 + k'^2 + k''^2) + 2p_r kk' - 2p_r k''k']\right] \exp[fs(t - t_1)] dk'' + \\ [c]_1 \int_{-\infty}^{\infty} \exp\left[-\frac{\nu}{P_r}(t - t_1)[(1+P_r)(k^2 + k'^2 + k''^2) + 2p_r kk' - 2p_r k''k']\right] \exp[fs(t - t_1)] dk'' \end{aligned} \quad (34)$$

At  $t_1$ , the functions  $\gamma$  have been assumed independent that assumption is not, made for other times. This is one of several assumptions made concerning the initial conditions, although continuity equation satisfied the conditions. The complete specification of initial condition is difficult; the assumptions for the initial conditions made here. Substituting  $dk'' = dk_1'' dk_2'' dk_3''$  and integrating equation (34) with respect to  $k_1''$ ,  $k_2''$  and  $k_3''$ ,

We get,

$$\begin{aligned} \frac{\partial}{\partial t} (k_k \overline{\phi_i \beta_i' \beta_i''}) + \frac{\nu}{P_r} [(1 + P_r)(K^2 + K'^2) + 2P_r K K'] (k_k \overline{\phi_i \beta_i' \beta_i''}) &= \left( \frac{\pi p_r}{\nu(t-t_1)(1+p_r)} \right)^{\frac{1}{2}} \\ [a_1] \exp \left[ -\frac{\nu(t-t_1)[(1+P_r)] \left\{ \frac{(1+2P_r)(k^2+k'^2)}{(1+P_r)^2} + \frac{2P_r k k'}{(1+P_r)^2} \right\}}{P_r} \right] &+ \left( \frac{\pi p_r}{\nu(t-t_1)(1+p_r)} \right)^{\frac{1}{2}} \\ [b_1] \exp \left[ -\frac{\nu(t-t_1)[(1+P_r)] \left\{ \frac{(1+2P_r)k^2}{(1+P_r)^2} + \frac{2P_r k k'}{(1+P_r)} + k'^2 \right\}}{P_r} \right] &+ \left( \frac{\pi p_r}{\nu(t-t_1)(1+p_r)} \right)^{\frac{1}{2}} \\ [c_1] \exp \left[ -\frac{\nu(t-t_1)[(1+P_r)] \left\{ k^2 + \frac{(1+2P_r)(k'^2)}{(1+P_r)^2} + \frac{2P_r k k'}{(1+P_r)} \right\}}{P_r} \right] &\exp [fs(t-t_1)] \end{aligned} \quad (35)$$

Integrating equation (35) with respect to time, and in order to simplify the calculations, we will assume that  $[a]_1 = 0$ ; Substituting of equation (32) in equation (29) and setting  $T = 2\pi k^2 \phi_i \phi_i'$ , result in

$$\frac{\partial T}{\partial t} + \frac{2\nu k^2}{P_r} T = W \quad (36)$$

Where,

$$\begin{aligned} W &= k^2 \int_{-\infty}^{\infty} 2\pi i \left[ \langle k_k \phi_i \beta_i' \beta_i''(\hat{k}, \hat{k}') \rangle - \langle k_k \phi_i \beta_i' \beta_i''(-\hat{k}, -\hat{k}') \rangle \right]_0 \exp \left[ -\frac{\nu}{P_r} (t-t_1) \{ (1+P_r)(k^2+k'^2) + 2p_r k k' \} \right] dk' \\ &+ k^2 \int_{-\infty}^{\infty} \frac{2p_r \pi^{\frac{5}{2}}}{\nu} i \left[ b(\hat{k} \hat{k}') + b(\hat{k} - \hat{k}') \right] \exp [fs(t-t_1)] + \left[ k \exp \{ (-\omega^2)(1+p_r)(k^2+k'^2) + 2p_r k k' \} \int_0^{\omega k'/2} \{ \exp(x^2) dx \} dk' \right. \\ &\left. + (\exp k) \times \exp \{ (-\omega^2)((1+p_r)(k^2+k'^2) + 2p_r k k') \} \int_0^{\omega k'/2} \{ \exp(x^2) dx \} dk' \right] \end{aligned} \quad (37)$$

T is the temperature fluctuations spectrum function, which represent contributions from various wave numbers to the energy and W is the energy transfer function, which is responsible for the transfer of energy between wave numbers. In order to make further calculations, an assumption must be made for the forms of the bracketed quantities with the subscripts 0 and 1 in equation (37) which depends on the initial conditions.

$$(2\pi)^2 \left[ \langle k_k \phi_i \beta_i' \beta_i''(\hat{k}, \hat{k}') \rangle - \langle k_k \phi_i \beta_i' \beta_i''(-\hat{k}, -\hat{k}') \rangle \right]_0 - \xi_0 (k^4 k'^6 - k^6 k'^4) \quad (38)$$

where,  $\xi_0$  is a constant depending on the initial conditions. For the other bracketed quantities in equation (37),

$$\frac{4p_r \pi^{\frac{7}{2}}}{\nu} i \left[ b(\hat{k} \hat{k}') + b(\hat{k} + \hat{k}') \right] = \frac{4p_r \pi^{\frac{7}{2}}}{\nu} i \left[ c(\hat{k} \hat{k}') + c(\hat{k} + \hat{k}') \right] = -2\xi_1 (k^6 k'^8 - k^8 k'^6) \quad (39)$$

Remembering that  $d\hat{k}' = -2\pi \hat{k}'^2 d(\cos \theta)$  and  $kk' = kk' \cos \theta$ ,  $\theta$  is the angle between  $\hat{k}$  and  $\hat{k}'$  and carrying out the integration with respect to  $\theta$ , we get,

$$\begin{aligned} W &= - \int_0^{\infty} \left[ \frac{\xi_0 (k^2 k'^4 - k^4 k'^2) k k'}{\nu(t-t_0)} \{ \exp \left[ -\frac{\nu}{P_r} (t-t_0) \{ (1+P_r)(k^2+k'^2) - 2p_r k k' \} \right] - \right. \\ &\left. \exp \left[ -\frac{\nu}{P_r} (t-t_0) \{ (1+P_r)(k^2+k'^2) + 2p_r k k' \} \right] \} + \frac{\xi_1 (k^4 k'^6 - k^6 k'^4) k k'}{\nu(t-t_0)} + \right. \end{aligned}$$

$$\begin{aligned}
& +k. \exp[(-\omega^2)((1+p_r)(k^2+k'^2)-2p_r k k')] - k. \exp[(-\omega^2)((1+p_r)(k^2+k'^2)+2p_r k k')] \int_0^{\frac{\omega k}{2}} \exp(x^2) dx + \\
& \{k' \exp[-\omega^2((1+p_r)(k^2+k'^2)-2p_r k k')] - k' \exp[-\omega^2((1+p_r)(k^2+k'^2)+2p_r k k')]\} \int_0^{\frac{\omega k}{2}} \exp(x^2) dx dk' \quad (40)
\end{aligned}$$

$$\text{Where, } \omega = \left( \frac{\nu(t-t_1)(1+P_r)}{P_r} \right)^{\frac{1}{2}}$$

By integrating equation (40) with respect to  $k'$  We have,

$$W = W_\beta + W_\gamma \exp[fs(t-t_1)] \quad (41)$$

Where,

$$\begin{aligned}
W_\beta = & \frac{\pi^{\frac{1}{2}} \xi_0 p_r}{\nu^{\frac{1}{2}}(t-t_0)^{\frac{5}{2}}(1+p_r)^{\frac{7}{2}}} \exp\left\{ \frac{\nu(t-t_0)(1+p_r)k^2}{2p_r(1+p_r)} \right\} \left[ \frac{5p_r k^4}{4\nu^2(t-t_0)^2(1+p_r)} + \right. \\
& \left. \left\{ \frac{5p_r^2}{(1+p_r)^2\nu(t-t_0)} - \frac{3}{2\nu(t-t_0)} \right\} k^6 + \frac{p_r}{(1+2p_r)} \left\{ \frac{1+p_r^2}{(1+p_r)^2} \right\} k^8 \right] \quad (42)
\end{aligned}$$

And

$$\begin{aligned}
W_\gamma = & \frac{\pi^{\frac{1}{2}} \xi_1 p_r^5}{8\nu^2(t-t_1)^{\frac{3}{2}}(1+p_r)^5} \exp\left\{ \frac{-\nu(t-t_1)(1+2p_r-p_r^2)}{p_r(1+p_r)} \right\} k^2 \left[ \frac{90p_r k^6}{\nu^4(t-t_1)^4(1+p_r)} + \right. \\
& 3 \left\{ \frac{4p_r}{(1+p_r)\nu^2(t-t_1)^2} + \frac{2p_r^2}{(1+p_r)^2\nu^3(t-t_1)^3} - \frac{1}{\nu^3(t-t_1)^3} \right\} k^8 + \\
& \left\{ \frac{64p_r^2}{(1+p_r)^2\nu(t-t_1)} + \frac{10p_r^3}{(1+p_r)^3\nu^2(t-t_1)^2} - \frac{40}{\nu(t-t_1)} \right\} k^{10} + 8 \left\{ \frac{p_r^2}{(1+p_r)^2} + \frac{p_r}{(1+p_r)} \right\} k^{12} \Big] + \\
& \left\{ \frac{64p_r^2(1+p_r)^2}{(1+2p_r)^2\nu(t-t_1)} + \frac{10p_r^3(1+p_r)^3}{(1+2p_r)^3\nu^2(t-t_1)^2} - \frac{40}{\nu(t-t_1)} \right\} k^{10} + \left\{ \frac{8p_r^3(1+p_r)^3}{(1+2p_r)^2} - \frac{p_r(1+p_r)}{(1+2p_r)} \right\} k^{12} \Big] + \\
& \frac{\pi^{\frac{1}{2}} \xi_1 p_r^{\frac{9}{2}}}{8\nu^{\frac{3}{2}}(t-t_1)^2(1+2p_r)^{\frac{9}{2}}} \exp\left\{ \frac{-\nu(t-t_1)(1+2p_r)}{p_r} \right\} k^2 \left[ \frac{90p_r k^7}{\nu^4(t-t_1)^4(1+p_r)^2} + \right. \\
& \left\{ \frac{120p_r}{\nu^2(t-t_1)^2} + \frac{60p_r^2}{(1+p_r)^2\nu^3(t-t_1)^3} - \frac{1}{\nu^3(t-t_1)^3} \right\} k^9 + \\
& \left\{ \frac{64p_r^2}{\nu(t-t_1)} + \frac{60p_r^3}{\nu^2(t-t_1)^2(1+p_r)^2} - \frac{40(1+p_r)^2}{\nu(t-t_1)} \right\} k^{11} + \{p_r^2 - p_r(1+p_r)^2\} \int_0^{\frac{\omega_1}{2}} \exp(y^2) dy
\end{aligned}$$

$$\text{Where, } \omega_1 = \left( \frac{\nu(t-t_1)(1+P_r)}{P_r} \right)^{\frac{1}{2}} k$$

The integral expression in equation (41), The quantity  $W_\beta$  is the energy transfer function arising from consideration of Temperature fluctuation field at three point correlation equation;  $W_\gamma$  arises from consideration of the four –point correlation equation. Integration of equation (41) over all wave numbers shows that

$$\int_0^\infty W \cdot d\vec{k} = 0 \quad (43)$$

The expression for  $W$  satisfies the conditions of continuity and homogeneity, physically, it was to be expected, Since  $W$  is a measure of transfer of energy and the numbers must be zero. From (36), we get,

$$\begin{aligned} T &= \exp\left[-\frac{2\nu k^2(t-t_0)}{p_r}\right] \int W \exp\left[-\frac{2\nu k^2(t-t_0)}{p_r}\right] dt + J(k) \exp\left[-\frac{2\nu k^2(t-t_0)}{p_r}\right] \\ &= J(k) \exp\left[-\frac{2\nu k^2(t-t_0)}{p_r}\right] + \exp\left[-\frac{2\nu k^2(t-t_0)}{p_r}\right] \int (W_\beta + W_\gamma) \exp\left[-\frac{2\nu k^2(t-t_0)}{p_r}\right] dt \end{aligned} \quad (44)$$

Where,  $J(k) = \frac{N_0 k^2}{\pi}$  is a constant of integration and can be obtained as by Corrsin S

Where,

$$W = W_\beta + W_\gamma \quad (45)$$

After integration of equation (43) becomes

$$T = J(k) \exp\left[-\frac{2\nu k^2(t-t_0)}{p_r}\right] + T_\beta + T_\gamma \quad (46)$$

Where,

$$\begin{aligned} T_\beta &= \frac{\pi^{\frac{1}{2}} \xi_0 p_r^{\frac{5}{2}}}{8\nu^{\frac{3}{2}}(1+p_r)^{\frac{7}{2}}} \exp\left\{-\frac{\nu(t-t_0)(1+2p_r)k^2}{p_r(1+p_r)}\right\} + \left[\frac{3p_r k^4}{2\nu^2(t-t_0)^{\frac{5}{2}}} + \right. \\ &\quad \left. \left\{\frac{5p_r^2 - 6p_r}{3\nu(1+p_r)(t-t_0)^{\frac{3}{2}}}\right\} k^6 - \left\{\frac{3p_r^2 - 2p_r + 3}{3(1+p_r)(t-t_0)^{\frac{1}{2}}}\right\} k^8 + \left\{\frac{8\nu^{\frac{1}{2}}(3p_r^2 - 2p_r + 3)}{3(1+p_r)^{\frac{5}{2}} p_r^{\frac{1}{2}}}\right\} k^9 F(\omega)] \end{aligned} \quad (47)$$

Where,

$$F(\omega) = \exp(-\omega^2) \int_0^\omega \exp(x^2) dx, \quad \omega = \left(\frac{\nu(t-t_0)}{p_r(1+p_r)}\right)^{\frac{1}{2}} k$$

And,

$$\begin{aligned} T_\gamma &= \frac{\pi^{\frac{1}{2}} \xi_1 p_r^5}{8\nu^2(1+p_r)^5} \exp\left\{\frac{-\nu(t-t_1)(1+2p_r-p_r^2)}{p_r(1+p_r)}\right\} k^2 \left[\frac{18p_r k^6}{\nu^4(t-t_1)^5(1+p_r)} + \right. \\ &\quad \left. \left\{\frac{15-6p_r+21p_r^2}{4(1+p_r)^2 \nu^3(t-t_1)^4} + \frac{4p_r}{(1+p_r)\nu^2(t-t_1)^3} - \frac{1}{\nu^3(t-t_1)^3}\right\} k^8 + \right. \\ &\quad \left. \left\{\frac{15-6p_r+36p_r^2-6p_r^4}{12(1+p_r)^3 \nu^2(t-t_1)^3} + \frac{14p_r^2-404p_r-18}{(1+p_r)^2 \nu(t-t_1)}\right\} k^{10} \cdot \exp(-\omega_1) \int Ei(\omega_1) dt + \right. \\ &\quad \left. \frac{(-\pi^{\frac{1}{2}} \xi_1 p_r^{\frac{9}{2}})}{128 \nu^2(1+p_r)^{\frac{11}{2}}} \exp\left\{\frac{-\nu(t-t_1)(1+2p_r)}{p_r}\right\} k^2 \left[\frac{180p_r(1+2p_r)}{\nu^4(t-t_1)^4(1+2p_r)^6}\right] k^6 + \right. \\ &\quad \left. \left\{\frac{1710-1036p_r-1820p_r^2}{4(1+p_r)^2 \nu^3(t-t_1)^4} + \frac{140p_r(1+p_r)}{3(1+2p_r)\nu^2(t-t_1)^3} - \frac{1}{\nu^3(t-t_1)^3}\right\} k^8 + \right. \end{aligned} \quad (48)$$

Where,

$$Ei(\omega_1) = \frac{\exp\left\{\frac{-(1+2p_r^2)tk^2}{p_r(1+p_r)}\right\}}{t-t_1} \quad \text{and} \quad \omega_1 = \frac{-(1+2p_r^2)tk^2}{p_r(1+p_r)},$$

From equation (45),

$$T = T_1 + T_2 \quad (49)$$



Where,  $T_1 = J(k) \exp[-\frac{2\nu k^2(t-t_0)}{p_r}] + T_\beta$  and  $T_2 = T_\gamma$

In equation (48)  $T_1$  and  $T_2$  are temperature fluctuation field spectrum arising from consideration of the three and four –point correlation equations respectively. Equation (48) can be integrated over of all wave numbers to give the total Temperature fluctuation turbulent energy. That is

$$\left\langle \frac{T.T'}{2} \right\rangle = \int_0^\infty T dk = \int_0^\infty (T_1 + T_2) dk \quad (50)$$

$$\text{Now, } \int_0^\infty T_1 dk = \frac{N_0 p_r^{\frac{3}{2}} \nu^{-\frac{3}{2}} (t-t_0)^{-\frac{5}{2}}}{32\sqrt{2\pi}} + \xi_0 B_0 \nu^{-5} (t-t_0)^{-7} \quad (51)$$

And

$$\int_0^\infty T_2 dk = \xi_1 [R \nu^{-\frac{13}{2}} (t-t_1)^{-\frac{11}{2}} + S \nu^{-\frac{15}{2}} (t-t_1)^{-\frac{13}{2}}] \exp[fs(t-t_1)]. \quad (52)$$

Where,  $R=C_2+C_4+C_6+\dots$  And  $S=C_1+C_3+C_5+\dots$

$$B_0 = \frac{\pi p_r^6}{(1+p_r)(1+2p_r)^2} \left[ \frac{8}{13} + \frac{3p_r(1+5p_r)}{2(1+2p_r)} + \dots \right]$$

$$C_1 = \frac{\pi p_r^{\frac{7}{2}}}{(1+p_r)^{\frac{5}{2}}(1+2p_r-P_m^2)^{\frac{7}{2}}} \left[ \frac{13.7}{32} + \frac{13.5(13-4p_r+15p_r^2)}{2(1+2p_r)} + \dots \right]$$

$$C_2 = \frac{\pi p_r^{\frac{19}{2}}}{(1+p_r)^{\frac{3}{2}}(1+2p_r-P_r^2)^{\frac{5}{2}}} \left[ \frac{11.7}{32} + \frac{13.7.5(15p_r-14+20p_r^2)}{2^7(1+2p_r-P_r^2)} + \dots \right]$$

$$C_3 = \frac{\pi p_r^{\frac{17}{2}}(1+p_r)^{\frac{1}{2}}}{(1+p_r)^2(1+2p_r-P_r^2)^{\frac{5}{2}}} \left[ \frac{13.7}{32} + \frac{13.5(15+28p_r-14P_r^2+20p_r^4)}{2^9(1+p_r)^2(1+2p_r-P_r^2)} + \dots \right]$$

$$C_4 = \frac{\pi p_r^{\frac{21}{2}}}{(1+p_r)^{\frac{3}{2}}(1+2p_r-P_m^2)^{\frac{7}{2}}} \left[ \frac{11.7.3}{16} + \frac{13.7.5(-20p_r-17P_r^2+20p_r^3)}{2^7(1+2p_r-P_m^2)} + \dots \right]$$

$$C_5 = \frac{\pi p_r^{\frac{17}{2}}(1+p_r)^{\frac{1}{2}}}{(1+p_r)^{\frac{17}{2}}(1+2p_r)^{\frac{7}{2}}} \left[ \frac{13.7.5.3}{32} - \frac{13.5.3(15+28p_r-14P_r^3+20p_r^5)}{2^{11}(1+p_r)^2} + \dots \right]$$

$$C_6 = \frac{\pi p_r^{\frac{21}{2}}}{(1+p_r)^{\frac{11}{2}}(1+2p_r)^{\frac{9}{2}}} \left[ \frac{13.11.7.3}{16} - \frac{11.9.7.5(20+20p_r-17P_r^2+20p_r^3)}{2^7(1+2p_r-P_m^2)} + \dots \right]$$

$$\frac{\langle T.T' \rangle}{2} = \frac{N_0 p_r^{\frac{3}{2}} \nu^{-\frac{3}{2}} (t-t_0)^{-\frac{3}{2}}}{6\sqrt{2\pi}} + \xi_0 X \nu^{-5} (t-t_0)^{-5} \exp[fs(t-t_1)] + \xi_1 [R \nu^{-\frac{13}{2}} (t-t_1)^{-\frac{11}{2}} + S \nu^{-\frac{15}{2}} (t-t_1)^{-\frac{13}{2}}] \exp[fs(t-t_1)] \quad (53)$$

Also, we can write equation (50) of the following form,

$$\langle T^2 \rangle = W(t-t_0)^{-3/2} + X(t-t_0)^{-5} \exp[l(t-t_1)] + Y(t-t_1)^{-7/2} \exp[l(t-t_1)] + Z(t-t_1)^{-7} \exp[l(t-t_1)], \quad (54)$$

This is the decay of energy of Temperature fluctuation in dusty fluid homogeneous turbulence for four point correlations.

Where,  $\langle T^2 \rangle = \langle TT \rangle$ ,  $W = \frac{N_0 P_r^{3/2} \nu^{3/2}}{4\sqrt{2\pi}}$ ,  $X = 2\xi_0 \nu^{-6}$ ,  $Y = 2\xi_1 R \nu^{-9/2}$  and  $Z = 2\xi_1 S \nu^{-11/2}$ ,

$l = fs$  the dust particle parameters

If  $R=0$  and  $S=0$  that is  $Y=0$  and  $Z=0$  in equation (53), it will be in the form

$$\langle T^2 \rangle = A(t-t_0)^{-3/2} + B(t-t_0)^{-5} \exp[fs(t-t_1)], \text{ Taking } W=A, X=B \quad (55)$$

Which is completely same with the result of Azad and Sarker (2009) for the case of three -point correlation for dusty fluid. In equation (54), the third and fourth term on the right hand side comes due to four point correlations for dusty

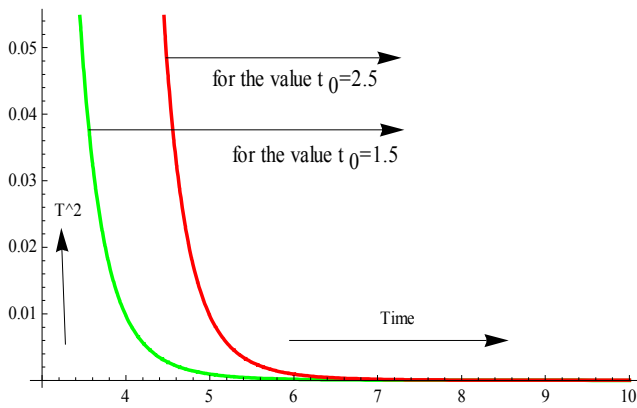
fluid.

## 4. Result and Discussions

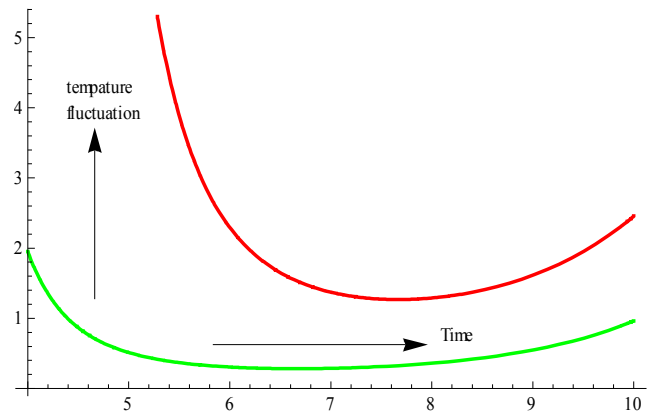
The evaluation of analytical results reported in this paper is performed and representative set of results is reported graphically. These results are obtained to illustrate the influence of various parameters on the temperature fluctuations and dust particle. For numerical validation of the analytical results, we have taken the results obtained in equations (54) and (56). The constants  $W, X, Y, Z$  is calculated in terms of Prandtl no.  $P_r$ , constants  $N_0, \xi_0, \xi_1$ , kinematic viscosity  $\nu$ , thermal conductivity  $k$ . In the present study we adopted the following default parametric values for some fluid in the table has discussed step by step

**Table 1.** The value of the constants and parameter used in equation (54) and (55).

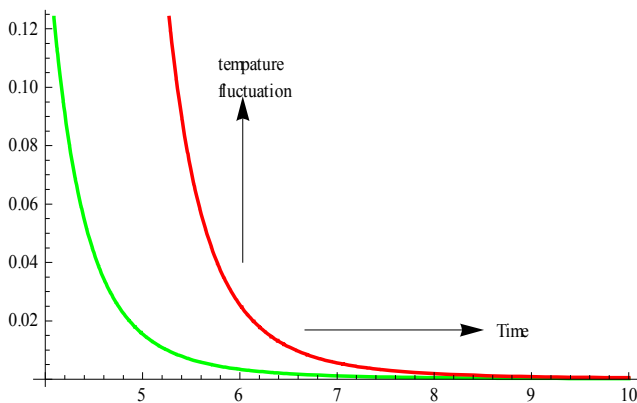
Fluid	$P_r$	$N_0$	$\xi_0$	$\xi_1$	$W$	$X$	$Y$	$Z$
Mercury	0.015	.1	.01	.02	.00058	$4.18 \times 10^{-7}$	$3.69 \times 10^{-13}$	5.87
MixGas	0.2	.1	.01	.02	$1.15 \times 10^{-6}$	$5.75 \times 10^{-18}$	$3.78 \times 10^{-16}$	$9.95 \times 10^{-13}$
Hyd.Gas	0.4	.1	.01	.02	$4.86 \times 10^{-7}$	$9.4 \times 10^{-20}$	$1.9 \times 10^{-16}$	$2.3 \times 10^{-15}$
HelGas	0.7	.1	.01	.02	$4.6 \times 10^{-6}$	$4.8 \times 10^{-16}$	$7.4 \times 10^{-13}$	$9.4 \times 10^{-23}$



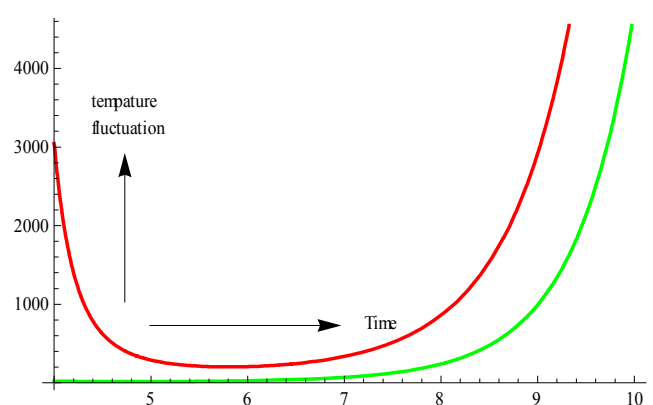
**Figure 1.** When  $P_r=.01$ ,  $l=0$  (for clean fluid).



**Figure 3.** For  $l=1.5$ .



**Figure 2.** For  $l=0.5$ .



**Figure 4.** For  $l=2.5$ .

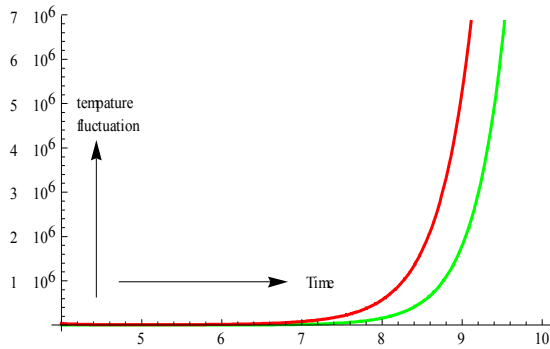
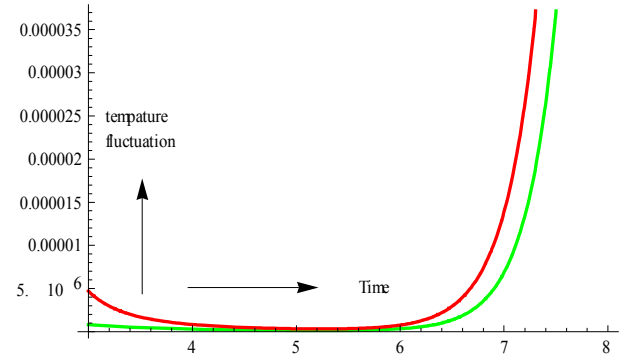
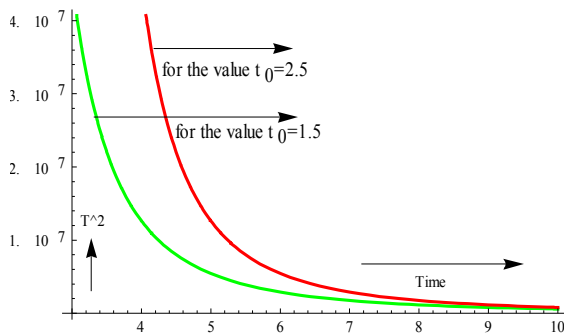
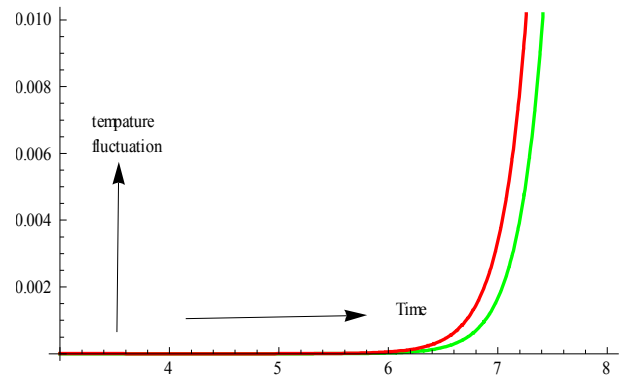
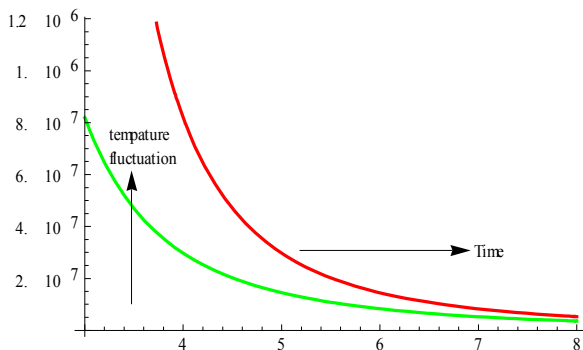
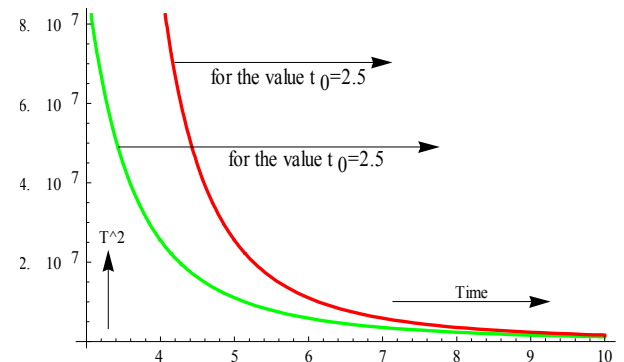
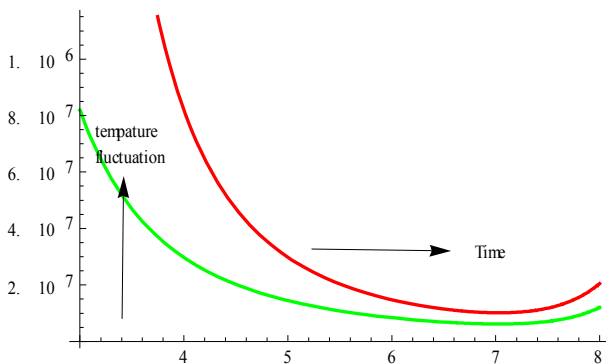
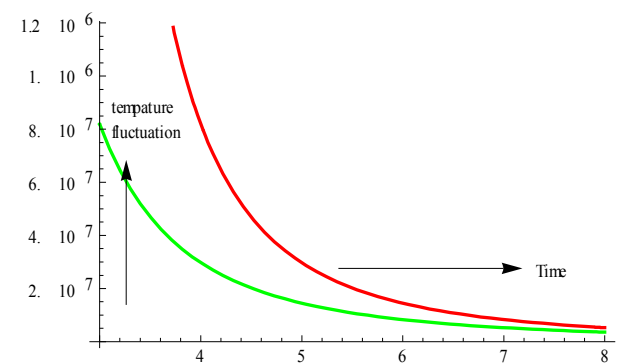
Figure 5. For  $l=3.5$ .Figure 9. For  $l=2.5$ .

Figure 1 and Figure 2, Figure 3, Figure 4, Figure 5, represents the energy decay of temperature fluctuation for four-point correlations of equation (54). When the Prandtl no. is small as of mercury  $P_r=0.15$ . It is observed from the above figures the energy decays more rapidly as dust particle increases from 0.5 to 3.5 that of clean fluid.

Figure 6. Energy decay for mix. of gas  $p_r=.2$  for clean fluid.Figure 10. For  $l=3.5$ .

The above figures represent the energy decay of temperature fluctuation for four-point correlations of equation (54). With fixed Prandtl no. is as of mix. of gas  $P_r=.2$ , if we increases the quantity of  $l$  the energy decays rapidly more and more.

Figure 7. For  $l=0.5$ .Figure 11.  $P_r=.4$ ,  $l=0$ Figure 8. For  $l=1.5$ .Figure 12. For  $l=0.5$ .

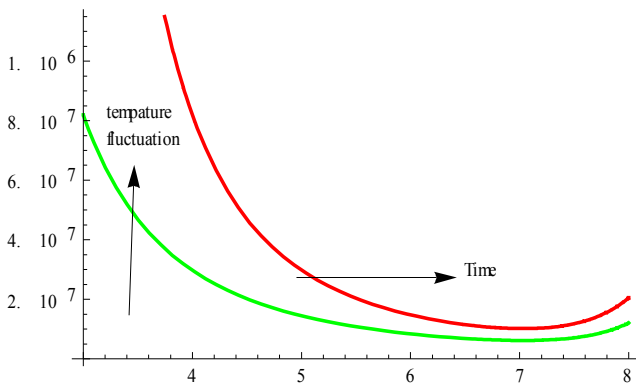
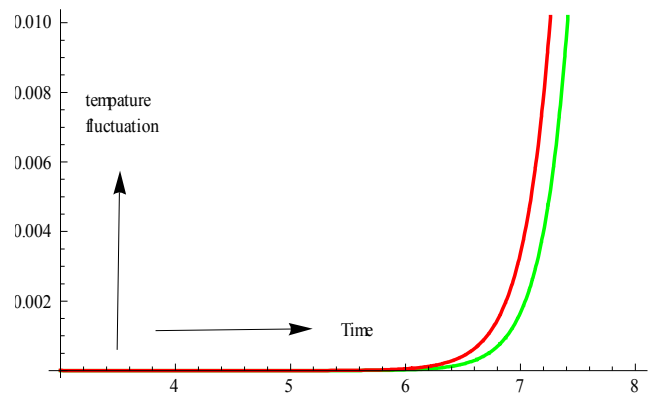
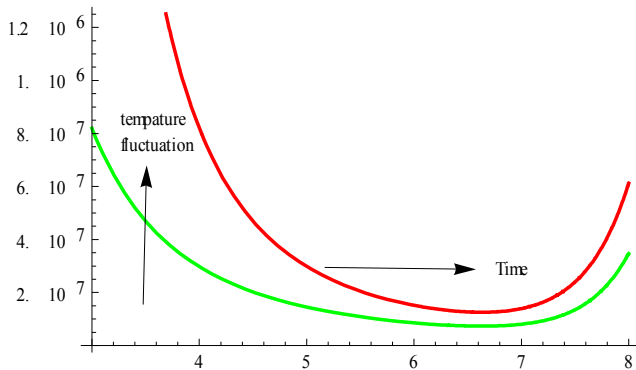
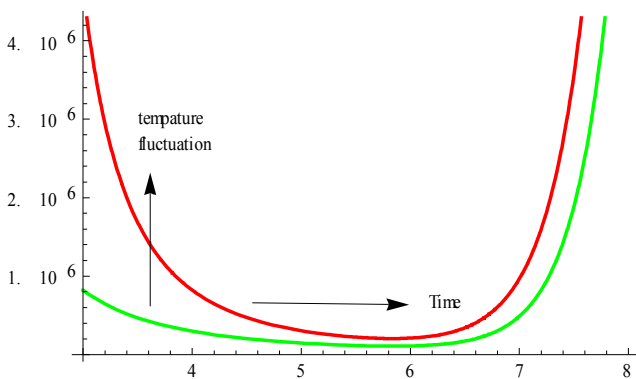
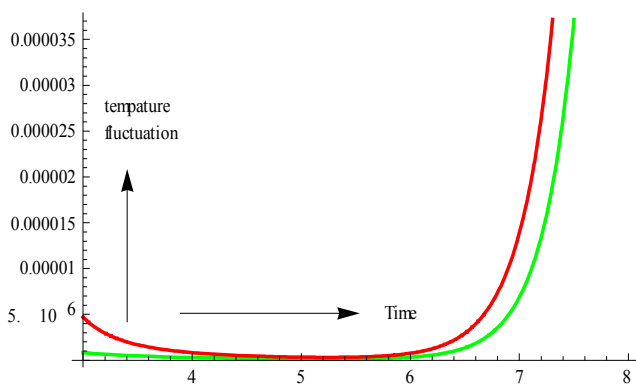
Figure 13. For  $l=1.5$ .Figure 17. For  $l=7$ .Figure 14. For  $l=2$ .Figure 15. For  $l=3$ .Figure 16. For  $l=4$ .

Figure 11 represents the energy decay of temperature fluctuation for four -point correlations of equation (54). When the Prandtl no. is as of Hyd gas  $Pr=4$  without dust particles and Figure 12- Figure 17 represents the energy decay of temperature fluctuation for four -point correlations of equation (54). When the Prandtl no. is as of Hyd gas  $Pr=4$  with dust particles for  $l=0.5, 1.5, 2, 3, 4$  and  $7$  respectively

## 5. Conclusions

Through this study we conclude that the temperature energy in homogeneous dusty fluid turbulent flow for four-point correlations decays rapidly more and more by exponential manner than that of clean fluid. It is observed that the energy decreases more rapidly as Prandtl number increases which shows in Figure 1, Figure 6 and Figure 11 in the case of clean fluid. From the resulting equations and its figures we have tried to show the effect of dust particle in the Figure 2- Figure 5; Figure 7- Figure 10; Figure 12- Figure 17 that the energy decays more faster and abruptly change than that of clean fluids the quantity of dust particle gradually increases.

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