

Effects of Temperature Dependent Viscosity on Magnetohydrodynamic Natural Convection Flow Past an Isothermal Sphere

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Abstract: In this study, the effects of temperature dependent viscosity on MHD natural convection flow past an isothermal sphere are determined. The uniformly heated sphere is immersed in a viscous and incompressible fluid where viscosity of the fluid is taken as a non-linear function of temperature. The Partial Differential Equations governing the flow are transformed into non dimensional form and solved using the Direct Numerical Scheme and implemented in MATLAB. The numerical results obtained are presented graphically and in tables and are discussed. In this study, it has been observed that increasing the Magnetic parameter M leads to decrease in velocity, temperature, skin friction and the rate of heat transfer. It has also been noted that increase in the Grashof number G_r leads to increase in velocity and temperature whereas increase in the values of η leads to increase in temperature but there is a decrease in velocity. These results are applicable to engineers in designing electricity plants which have higher life expectancy.

Keywords: Natural Convection Flow, Temperature Dependent Viscosity, Magnetohydrodynamic (MHD), Isothermal Sphere

1. Introduction

Electrically conducting fluids in presence of strong magnetic field is an important phenomenon in our day-to-day lives. The interaction of the current with the magnetic field changes the motion of the fluid and produces an induced magnetic field. An isothermal sphere in this study is made of non-conducting material and is uniformly heated. Molla *et. al* (2005) studied the magnetohydrodynamic natural convection flow from a sphere with uniform heat flux in the presence of heat generation. They observed that for increased values of heat generation parameter, there was an increase in local skin friction, velocity and temperature profiles but there was a decrease in the local rate of heat transfer. Temperature distribution was observed to increase whereas velocity distribution, local rate of heat transfer and local skin friction coefficient were observed to decrease slightly as the Magnetic parameter M increased. The MHD natural convection flow from an isothermal sphere has many applications in the engineering processes. This has been due to its cost and practical advantages which has led to its

usefulness. For example, the fluid flow is considered to be of importance in the geothermal and hydroelectric plants in Kenya.

Temperature is one of the factors that may cause variation in the viscosity of a given fluid. Soares *et al* (2010) investigated the effects of temperature dependent viscosity on forced convection heat transfer from a cylinder in cross flow of power-law fluids and found out that the variation of viscosity with temperature is shown to have an effect on both the local and the surface averaged values of the Nusselt number.

It has been found out that the velocity and temperature of the flow change more or less with the variation of the flow parameters and that for higher values of Prandtl number Pr , both velocity and temperature decreases such that there exists a local maximum value of velocities (Kabir *et. al*, 2013). Alam *et. al* (2007) investigated the effects of viscous dissipation on MHD natural convection flow over a sphere in the presence of heat generation and found out that velocity increases as the values of viscous parameter increases. Also, they found out that velocity distribution increase as the values of heat generation parameter increases.

Shrama *et. al*(2010) studied the effect of temperature dependent electrical conductivity on steady natural convection flow of viscous incompressible low Prandtl ($Pr \ll 1$) electrical conducting fluid along an isothermal vertical plate in the presence of transverse magnetic field and exponentially decaying heat generation. They found out that fluid velocity increases in the presence of heat generation due to increase in the electrical conductivity parameter while it decreases due to increase in the magnetic field intensity. They also deduced that fluid temperature increases in the presence of volumetric rate of heat generation or due to increase in magnetic field intensity while it decreases due to increase in electrical conductivity parameter. They also explained that the skin friction coefficient increases with the increase in electrical conductivity parameter or in the presence of volumetric rate of heat generation while it decreases due to increase in the Prandtl number. Finally, they concluded that the rate of heat transfer increases with the increase in the Prandtl number of electrical conductivity parameter which is not much effective while it decreases due to increase in the magnetic field intensity.

Chaudhary *et. al* (2007) studied combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium and found out that the concentration decreases with an increase in Schmidt number. They found out that in case of cooling of the plate ($Gr > 0$), the velocity decreases with an increase in the phase angle, magnetic parameter, Schmidt number and Prandtl number while it increases with an increase in the value of Grashof number and modified Grashof number, permeability parameter and time. In case of heating of the plate ($Gr < 0$), the velocity increases with an increase in magnetic parameter, Schmidt number and Prandtl number while it decreases with an increase in the value of Grashof number and modified Grashof number, permeability parameter and time. The Skin friction increases with an increase in Schmidt number, Prandtl number and Magnetic parameter while it decreases with an increase in the value of Grashof number, modified Grashof number, permeability parameter and time. They also concluded that Nusselt number increase with an increase in the Prandtl number while temperature decreases with an increase in the value of Prandtl number.

Molla *et. al* (2005) studied Magnetohydrodynamic natural convection flow from an isothermal sphere with temperature dependent heat generation and they found out that when the values of heat generation parameter Q increases, there is an increase in the local skin friction coefficient C_{fx} but there is a decrease in the local rate of heat transfer. Both the velocity and temperature profiles increase significantly when the value of heat generation parameter increases. They also found out that the local rate of heat transfer and the local skin friction coefficient decreases when the value of magnetic parameter increases. Finally, they concluded that increased values of magnetic parameter leads to decrease in the velocity distribution whereas there is an increase in the temperature distribution.

Natural convection flow along an isothermal vertical plate

with temperature dependent viscosity and heat generation has been studied by Molla *et. al* (2014) and they deduced that the effect of viscosity variation parameter and Rayleigh number decreases the skin friction coefficient whereas increasing the local average rate of heat transfer. They also found out that the momentum and thermal boundary layer becomes thinner when the values of viscosity -variation parameter increases. They have also found out that both viscosity and velocity distribution increases with the effect of Rayleigh number. This has also led to significant decrease in temperature distributions whereas the thickness of momentum boundary layer is enhanced. They concluded that the decrease in the local and average rate of heat transfer is more significant.

Miraj *et. al* (2011) studied the effects of viscous dissipation and radiation on MHD free convection flow along a sphere with joule heating and heat generation and they found out that velocity profiles increases with the increasing values of radiation parameter, heat generation parameter, magnetic parameter, joule heating parameter and viscous dissipation parameter whereas the profiles decreases with increasing values of Prandtl number. They also discovered that the skin friction coefficient increases with the increasing values of heat generation parameter whereas decreases for the increasing values of magnetic parameter. Finally, they found out that the rate of heat transfer increases for increasing values of radiation parameter whereas decreases for increasing values of heat generation parameter, joule heating parameter and viscous dissipation parameter.

Molla *et al* (2012) have studied the MHD natural convection flow from an isothermal cylinder under the consideration of temperature dependent viscosity and they found out that increasing the values of magnetic parameter M and viscosity variation parameter leads to decrease in the local skin friction coefficient, Grashof number and the local Nusselt number. It was also seen that the velocity distribution decreases as well as the temperature distribution increases with the increasing values of the magnetic parameter M and viscosity variation parameter.

Hague *et. al* (2014) studied the effects of viscous dissipation on MHD natural convection flow over a sphere with temperature dependent thermal conductivity in presence of heat generation and they found out that the velocity and temperature of the fluid within the boundary layer increases with increasing thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter. They also noted that the skin friction along the surface of the sphere increases with increasing thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter but decreases for the increasing values of M . The heat transfer rate from the surface was found to decrease with the increasing value of magnetic parameter, thermal conductivity variation parameter, heat generation parameter and viscous dissipation parameter.

This study, presents the findings of the studies made on the effects of temperature dependent viscosity on the MHD natural convection flow past an isothermal sphere taking viscosity as a non-linear function of temperature and

analyzing the results using Direct Numerical Scheme to a simple analytical case.

2. Formulation

The configuration of the problem considered in this study is as shown in the diagram below:

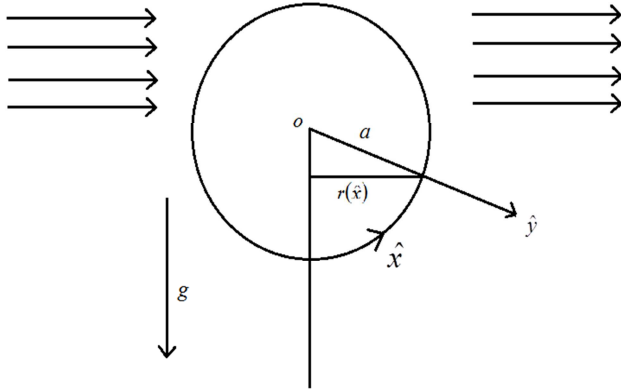


Figure 1. Physical Model of Problem.

The diagram above shows a two-dimensional MHD Laminar free convective flow past a uniformly heated sphere Center o and radius a which is immersed in a viscous and incompressible fluid having a temperature dependent viscosity. In this study, viscosity is considered as a non-linear function of temperature.

2.1. Governing Equations

2.1.1. Equation of Conservation of Mass

This equation states that mass can neither be created nor destroyed under normal circumstances. It is obtained from the fact that the mass of fluid entering and leaving a volume in the flow field have the same mass balance.

$$\frac{\partial \rho}{\partial t} + \bar{\nabla} \cdot (\rho \bar{q}) = 0 \quad (2.1)$$

Where \bar{q} is the velocity in the X, Y direction and

$$\bar{\nabla} = \bar{i} \frac{\partial}{\partial x} + \bar{j} \frac{\partial}{\partial y} \quad (2.2)$$

2.1.2. Equation of Motion

The expression of the equation in tensor form is given as;

$$\frac{\partial \bar{q}}{\partial t} + \bar{q} (\bar{\nabla} \cdot \bar{q}) = -\frac{1}{\rho} \bar{\nabla} p + \nu \nabla^2 \bar{q} + \bar{F} \quad (2.3)$$

Where $\frac{\partial \bar{q}}{\partial t}$ is the temporal acceleration, $\bar{q} (\bar{\nabla} \cdot \bar{q})$ is the convective acceleration, $\bar{\nabla} p$ is the pressure gradient, $\nu \nabla^2 \bar{q}$ is the force due to viscosity and \bar{F} represents the body forces vector in X and Y directions.

2.1.3. Equation of Induced Magnetic Force

The Induced magnetic force for the magnetic fluid is given as:

$$F_m = (\bar{\nabla} x h_m) x B + \mu M_g (\bar{\nabla} h_m) \quad (2.4)$$

Where B is the Magnetic Field Density vector, M_g is magnetization of the magnetic material and $(\bar{\nabla} x h_m)$ represents the Induced free current.

2.1.4. The Energy Equation

The general equation is given as:

$$\rho C_p \left(\bar{u} \frac{\partial T}{\partial x} + \bar{v} \frac{\partial T}{\partial y} \right) = K \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \phi \quad (2.5)$$

Where viscous energy dissipation term ϕ is defined as:

$$\phi = 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \quad (2.6)$$

2.2. Non-dimensional Numbers

Non-dimensional numbers are applicable in this study in order to transform the obtained results obtained in the model into any other dynamically similar case. In this work, the following numbers have been used;

2.2.1. Grashof Number, Gr

This number usually occurs in natural convection problem and is usually defined as:

$$Gr = \frac{g \beta (T_w - T_\infty) a^3}{\nu_\infty^2}$$

2.2.2. Prandtl Number, Pr

This number gives the ratio of viscous force to the thermal and is defined as:

$$Pr = \frac{\mu C_p}{k}$$

2.2.3. Nusselt Number, Nu

This number is important in the heat transfer problems and is defined as;

$$Nu = \frac{a q_w}{k (T_w - T_\infty)}$$

2.2.4. Magnetic Parameter, M

This is defined as;

$$M = \frac{\delta_0 \beta_0^2 a^2}{\mu_\infty Gr^2}$$

2.2.5. Skin Friction Coefficient, C_f

This is defined as;

$$C_f = \frac{2\tau_w}{\rho U_\infty^2}$$

2.2.6. Pressure Coefficient, C_p

This is the ratio of pressure number to the inertia force and it gives the importance of pressure force to the inertia force. It is given as;

$$C_p = \frac{P}{\rho u^2}$$

2.3. Problem Modelling

In this study, the fluid viscosity μ is assumed to vary as an inverse function of temperature T , in the form;

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 + \gamma(T - T_\infty)] \quad (2.7)$$

Where T is the temperature of the fluid and T_∞ is the temperature of the ambient fluid.

The non-dimensional variables used in transforming equations (2.1), (2.3), (2.4) and (2.5) into non-dimensional form are:

$$x = \frac{\hat{x}}{a}, y = \frac{\hat{y}}{a}, u = \frac{\rho a}{\mu} Gr^{\frac{-1}{2}} \hat{u},$$

$$v = \frac{\rho a}{\mu} Gr^{\frac{-1}{2}} \hat{v}, v_\infty = \frac{\mu_\infty}{\rho}, \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\gamma = \frac{1}{\mu_f} \left(\frac{\partial \mu}{\partial T} \right)_f (T_w - T_\infty), r(\hat{x}) = a \sin\left(\frac{\hat{x}}{a}\right) \quad (2.8)$$

Equations (2.8) are used together with the non-dimensional numbers given above.

Equation of conservation of mass reduces to:

$$\frac{\partial(r\hat{u})}{\partial \hat{x}} + \frac{\partial(r\hat{v})}{\partial \hat{y}} = 0 \quad (2.9)$$

Substituting equations (2.8) in Equation (2.1), we obtain;

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \quad (2.10)$$

The expression of the equation of motion reduces to;

$$\hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{u}}{\partial \hat{y}} = \frac{1}{\rho} \frac{\partial}{\partial \hat{y}} \left(\frac{1}{\mu} \frac{\partial \hat{u}}{\partial \hat{y}} \right) + g\beta(T - T_\infty) \sin\left(\frac{\hat{x}}{a}\right) - \frac{\delta_0 B_0^2}{\rho} \hat{u} \quad (2.11)$$

Substituting equations (2.7), (2.8) and the non-dimensional numbers above in (2.11) and simplifying, we obtain;

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\rho a}{\mu^2 Gr^{\frac{1}{2}}} (1 + \gamma\theta) \frac{\partial^2 u}{\partial y^2} + \frac{\rho \gamma}{\mu Gr^{\frac{1}{2}}} \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} + \theta \sin(x) - Mu \quad (2.12)$$

$$\text{Let } \frac{\rho a}{\mu^2 Gr^{\frac{1}{2}}} = \eta \quad (2.13)$$

Thus, the non-dimensionalized equation of motion becomes:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \eta (1 + \gamma\theta) \frac{\partial^2 u}{\partial y^2} + \eta \frac{\mu \gamma}{a} \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} + \theta \sin(x) - Mu \quad (2.14)$$

On simplification of the equation of induced magnetic field, we obtain the Induced Force \vec{F} which is taken to be along the x-axis in our study and is given as:

$$\vec{F} = -\delta_0 B_0^2 \hat{u} \quad (2.15)$$

This result is used in the last term of equation (2.3).

In energy equation (2.5), the flow viscosity is not large; hence the Viscous Energy dissipation $\mu \phi$ is small and is neglected.

$(\nabla^2 \vec{q})$ is considered to be along the y-axis and thus the energy equation becomes;

$$\hat{u} \frac{\partial T}{\partial \hat{x}} + \hat{v} \frac{\partial T}{\partial \hat{y}} = K \frac{\partial^2 T}{\partial \hat{y}^2} \quad (2.16)$$

Substituting equations (2.8) in equation (2.16) and simplifying, we obtain;

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{K Gr^{\frac{-1}{2}}}{\mu C_p} \frac{\partial^2 \theta}{\partial y^2} \quad (2.17)$$

Taking $Pr = \frac{\mu C_p}{K}$, therefore equation (2.17) becomes;

$$\frac{u}{Gr^{\frac{1}{2}}} \frac{\partial \theta}{\partial x} + \frac{v}{Gr^{\frac{1}{2}}} \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (2.18)$$

$$X = x \quad Y = y \quad U = \frac{u}{x} \quad V = v \quad (2.20)$$

Boundary Conditions

$$u = v = 0, \theta = 1, \text{ at } y = 0 \quad (2.19a)$$

$$u \rightarrow 0, \theta \rightarrow 0, \text{ as } y \rightarrow \infty \quad (2.19b)$$

$$r(xa) = a \sin x \text{ thus } r(x) = \sin x \quad (2.21)$$

Thus, substituting equations (2.20) and (2.21) into equation (2.10), we get;

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = X \frac{\partial U}{\partial X} + \left[1 + X \frac{\cos X}{\sin X}\right] U + \frac{\partial V}{\partial Y} = 0 \quad (2.22)$$

2.4. Method of Solution

Direct Numerical Scheme (DNS) is applied in solving equations (2.10), (2.14) and (2.18) subject to the boundary conditions (2.19). The following new set of transformations is introduced;

Substituting transformations (2.20) in equation (2.14), we get;

$$XU \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + U^2 = \eta(1 + \gamma\theta) \frac{\partial^2 U}{\partial Y^2} + \eta \frac{\mu\gamma}{a} \frac{\partial \theta}{\partial Y} \frac{\partial U}{\partial Y} + \theta \sin X - MU \quad (2.23)$$

Substituting transformations (2.20) in equation (2.18), we get;

$$U \rightarrow 0, \theta \rightarrow 0 \text{ as } Y \rightarrow \infty, X > 0 \quad (2.25c)$$

$$\frac{XU}{Gr^{\frac{1}{2}}} \frac{\partial \theta}{\partial X} + \frac{V}{Gr^{\frac{1}{2}}} \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (2.24)$$

Writing equations (2.22), (2.23) and (2.24) in terms of central finite differences, we get the following equations;

$$X_i \left[\frac{U_{i+1}^j - U_{i-1}^j}{2(\Delta X)} \right] + \left[1 + X_i \frac{\cos X_i}{\sin X_i} \right] U_i^j + \frac{V_{i+1}^{j+1} - V_i^{j-1}}{2(\Delta Y)} = 0 \quad (2.26)$$

Boundary Conditions;

$$U = V = 0, \theta = 1 \text{ at } X = 0 \text{ any } Y \quad (2.25a)$$

$$U = V = 0, \theta = 1 \text{ at } Y = 0, X > 0 \quad (2.25b)$$

Making U_i^j the subject of the formulae, we get;

$$U_i^j = -X_i \left[\frac{U_{i+1}^j - U_{i-1}^j}{2(\Delta X)} \right] - \left[\frac{V_{i+1}^{j+1} - V_i^{j-1}}{2(\Delta Y)} \right] \div \left[1 + X_i \frac{\cos X_i}{\sin X_i} \right] \quad (2.27)$$

Writing equation (2.23) in terms of finite differences, we get;

$$\begin{aligned} X_i \left[\frac{U_{i+1}^{j+1} - U_{i-1}^{j-1}}{2} \right] \left[\frac{U_{i+1}^j - U_{i-1}^j}{2(\Delta X)} \right] + \left[\frac{V_{i+1}^{j+1} - V_i^{j-1}}{2} \right] \left[\frac{U_{i+1}^{j+1} - U_{i-1}^{j-1}}{2(\Delta Y)} \right] + \left[\frac{U_{i+1}^j - U_{i-1}^j}{2} \right]^2 = \\ \eta(1 + \gamma\theta_i^j) \left[\frac{U_{i+1}^{j+1} - 2U_i^j + U_{i-1}^{j-1}}{(\Delta Y)^2} \right] + \eta \frac{\mu\gamma}{a} \left[\frac{U_{i+1}^{j+1} - U_{i-1}^{j-1}}{2(\Delta Y)} \right] \left[\frac{\theta_{i+1}^{j+1} - \theta_{i-1}^{j-1}}{2(\Delta Y)} \right] + \theta_i^j \frac{\sin X_i}{\cos X_i} - MU_i^j \end{aligned} \quad (2.28)$$

Consider the term;

$$\eta(1 + \gamma\theta_i^j) \left[\frac{U_{i+1}^{j+1} - 2U_i^j + U_{i-1}^{j-1}}{(\Delta Y)^2} \right];$$

This can be written as;

$$\eta \left[-\frac{2U_i^j}{(\Delta Y)^2} + \frac{U_{i+1}^{j+1} + U_{i-1}^{j-1}}{(\Delta Y)^2} + \gamma\theta_i^j \left[\frac{U_{i+1}^{j+1} + U_{i-1}^{j-1}}{(\Delta Y)^2} \right] - \gamma\theta_i^j 2 \left[\frac{U_{i+1}^j + U_{i-1}^j}{2(\Delta Y)^2} \right] \right] \quad (2.29)$$

Putting this in equation (2.28) and making U_i^j the subject of the formulae, we get;

$$\begin{aligned}
U_i^j = & -X_i \left[\frac{U_i^{j+1} - U_i^{j-1}}{2} \right] \left[\frac{U_{i+1}^j - U_{i-1}^j}{2(\Delta X)} \right] - \left[\frac{V_i^{j+1} - V_i^{j-1}}{2} \right] \left[\frac{U_i^{j+1} - U_i^{j-1}}{2(\Delta Y)} \right] - \left[\frac{U_{i+1}^j + U_{i-1}^j}{2} \right]^2 + \\
& \eta(1 + \gamma \theta_i^j) \left[\frac{U_{i+1}^{j+1} + U_{i-1}^{j-1}}{(\Delta Y)^2} \right] - \eta \gamma \theta_i^j 2 \left[\frac{U_{i+1}^j + U_{i-1}^j}{2(\Delta Y)^2} \right] + \eta \frac{\mu \gamma}{a} \left[\frac{U_i^{j+1} - U_i^{j-1}}{2(\Delta Y)} \right] \left[\frac{\theta_i^{j+1} - \theta_i^{j-1}}{2(\Delta Y)} \right] + \theta_i^j \frac{\sin X_i}{X_i} + \\
& \left[\frac{2}{(\Delta Y)^2} \eta + M \right]
\end{aligned} \quad (2.30)$$

Writing equation (2.24) in terms finite differences, we get;

$$X_i U_i^j \frac{1}{Gr^{\frac{1}{2}}} \left[\frac{\theta_{i+1}^j - \theta_{i-1}^j}{2(\Delta X)} \right] + V_i^j \frac{1}{Gr^{\frac{1}{2}}} \left[\frac{\theta_i^{j+1} - \theta_i^{j-1}}{2(\Delta Y)} \right] = \frac{1}{Pr} \left[\frac{\theta_i^{j+1} - 2\theta_i^j + \theta_i^{j-1}}{(\Delta Y)^2} \right] \quad (2.31)$$

From equation (2.31), consider;

$$\frac{1}{Pr} \left[\frac{\theta_i^{j+1} - 2\theta_i^j + \theta_i^{j-1}}{(\Delta Y)^2} \right] = \frac{-2\theta_i^j}{Pr(\Delta Y)^2} + \frac{1}{Pr} \left[\frac{\theta_i^{j+1} + \theta_i^{j-1}}{(\Delta Y)^2} \right] \quad (2.32)$$

Putting equation (2.32) in (2.31) and making θ_i^j the subject of the formulae, we get;

$$\theta_i^j = -X_i U_i^j \frac{1}{Gr^{\frac{1}{2}}} \left[\frac{\theta_{i+1}^j - \theta_{i-1}^j}{2(\Delta X)} \right] - V_i^j \frac{1}{Gr^{\frac{1}{2}}} \left[\frac{\theta_i^{j+1} - \theta_i^{j-1}}{2(\Delta Y)} \right] + \frac{1}{Pr} \left[\frac{\theta_i^{j+1} + \theta_i^{j-1}}{(\Delta Y)^2} \right] + \frac{2}{Pr(\Delta Y)^2} \quad (2.33)$$

The physical quantities to be obtained are the shearing stress (the rate of skin friction) and the rate of heat transfer. The equations to be used in obtaining the graphs are;

$$\frac{C_f Gr^{\frac{1}{4}}}{2(1 + \gamma)} = X \left(\frac{\partial U}{\partial Y} \right)_{Y=0} \quad (2.34)$$

$$Nu Gr^{\frac{-1}{4}} = - \left(\frac{\partial \theta}{\partial Y} \right)_{Y=0} \quad (2.35)$$

Writing the equation in terms of finite differences, we have;

$$\frac{C_f Gr^{\frac{1}{4}}}{2(1 + \gamma)} = X_i \left(\frac{U_i^{j+1} - U_i^{j-1}}{2\Delta Y} \right)_{Y=0} \quad (2.36)$$

$$Nu Gr^{\frac{-1}{4}} = - \left(\frac{\theta_i^{j+1} - \theta_i^{j-1}}{2\Delta Y} \right)_{Y=0} \quad (2.37)$$

Equations (2.27), (2.30), (2.33), (2.36) and (2.37) are the final equations which are solved using MATLAB in order to obtain the results shown below.

3. Results and Discussion

In this study, we have investigated effects of temperature dependent viscosity on MHD natural convection flow past an isothermal sphere. Here, we have considered the viscosity of the fluid as a non-linear function of temperature. The numerical solutions start from the lower stagnation point

$x = 0$, round the sphere to the upper stagnation point where $x = \pi$. The MHD Parameter values are taken as $M (= 0.0, 1.0, 5.0, 10.0)$ and the variable eta $\eta (= 0.015, 0.02, 0.025, 0.03)$ and Grashof number is given as $Gr (= 1.0, 2.0, 4.0, 6.0)$.

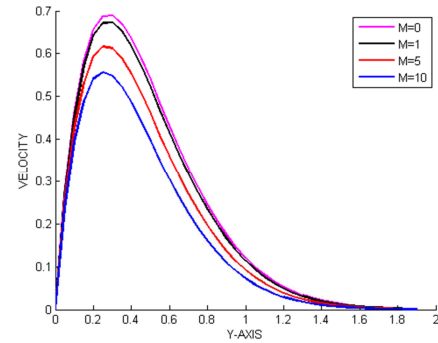


Figure 2. Velocity varying Magnetic Parameter M .

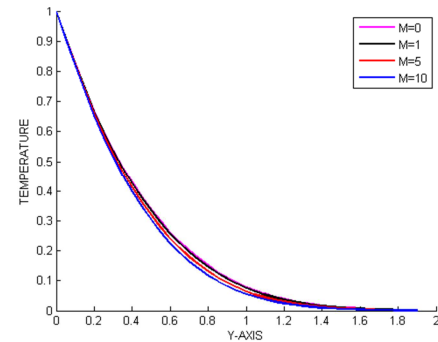


Figure 3. Temperature varying Magnetic Parameter M .

From figure 2 above, velocity varying MHD parameter M is presented as shown. Setting $Pr = 0.73, \eta = 0.015, Gr = 4$. It can be seen that, as the magnetic parameter M increases the velocity profile decrease. From figure 3, as the magnetic parameter M increases, the temperature profile decreases slightly. This is because the interaction of the magnetic field and the moving electric charge carried by the fluid induces a force which tends to oppose the fluid motion. Velocity increases and then decreases slowly and finally approaches to zero near the surface of the sphere. This shows that there is a maximum local velocity within the boundary layer.

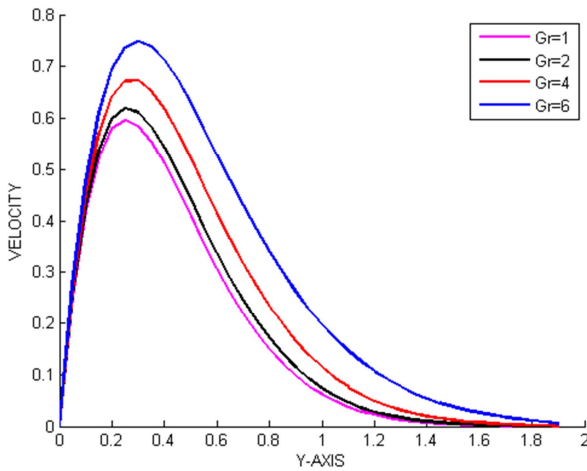


Figure 4. Velocity varying Grashof number Gr .

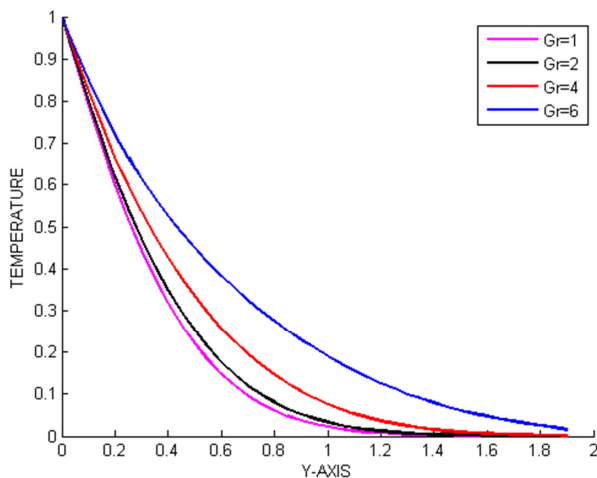


Figure 5. Temperature varying Grashof number Gr .

Figure 4 and figure 5 shows the Velocity and Temperature profiles respectively. $Gr (= 1.0, 2.0, 4.0, 6.0)$, $M = 1$, $Pr = 0.73$, $\eta = 0.015$.

It can be seen that increase in Grashof number Gr leads to increase in the velocity (figure 4) as well increase in Gr leads to increase in temperature as in figure 5. This is because as temperature of fluid increases, viscosity of the fluid decreases and therefore, the drag force in the fluid is reduced and thus the velocity of the fluid increases.

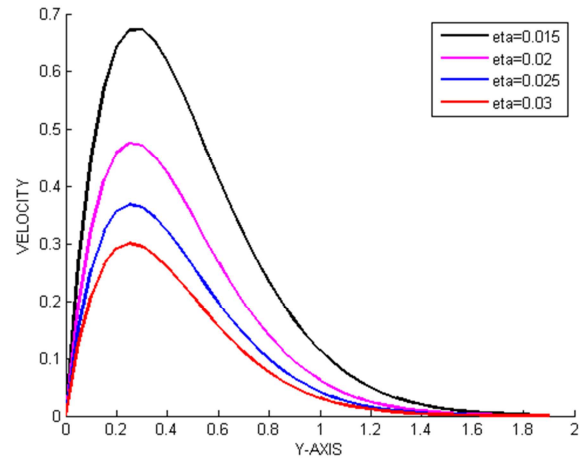


Figure 6. Velocity varying η .

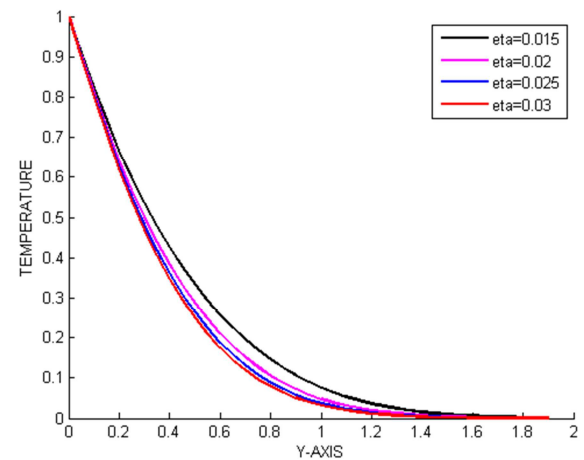


Figure 7. Temperature varying η .

Figure 6 and figure 7 shows the velocity and temperature distribution against the variable Y for different values of η ($\eta = 0.015, 0.02, 0.025, 0.03$) while $Pr = 0.73$, $M = 1$, $Gr = 4$. It can be observed that the velocity decreases and temperature distribution increases with the increasing values of the η . It can be deduced that at each value of η , the velocity profile has a local maximum value within the boundary layer.

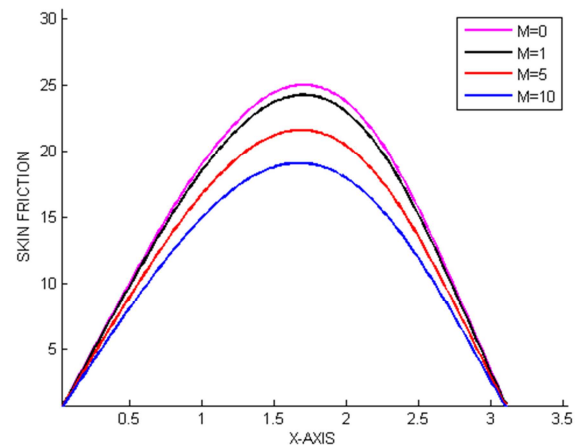


Figure 8. Skin friction varying Magnetic parameter M .

Figure 8 represents skin friction varying Magnetic parameter M whereas figure 9 represents Rate of Heat Transfer varying the Magnetic Parameter M . $Pr = 0.73, \eta = 0.015, Gr = 4$. In this case the values are taken at the stagnation point of the sphere, $x = \pi$. It can be observed that increasing the value M leads to decrease in skin friction and rate of Heat transfer. This is due to the fact increasing the value of M leads to increase in Lorentz force which opposes the motion hence decreasing the velocity and temperature gradients thus causing a decrease in Skin Friction as well as the Rate of heat transfer. In figure 9, it can be observed that there is an inverse behavior of the curves. This can be explained from the fact that viscosity is taken to vary inversely proportional to temperature thus a varying change of the curves.

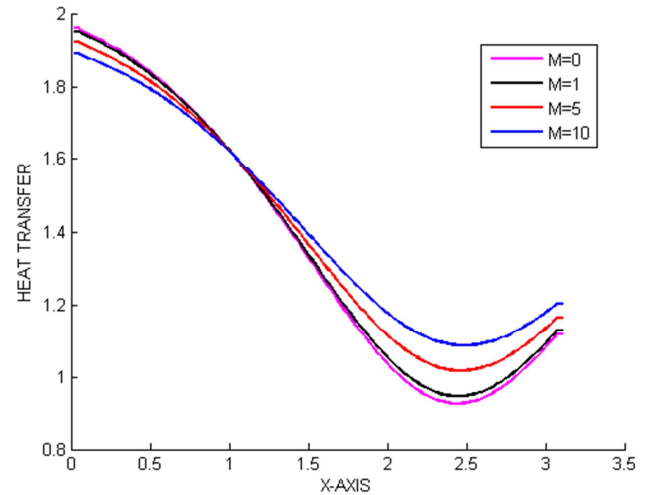


Figure 9. Heat Transfer varying Magnetic Parameter M .

Table 1. The results of skin friction and Rate of heat Transfer (Nusselt number) varying Magnetic parameter while $Pr=0.073$, $\eta=0.015$, $Gr=4$.

M=0			M=1			M=5			M=10		
X^0	Skin friction	Nusselt number	X^0	Skin friction	Nusselt number	X^0	Skin friction	Nusselt number	X^0	Skin friction	Nusselt number
0.02	0.4015	1.9581	0.02	0.3914	1.9498	0.02	0.3558	1.9202	0.02	0.3198	1.8890
0.40	8.0984	1.8704	0.40	7.9050	1.8642	0.40	7.2220	1.8417	0.40	6.5319	1.8181
0.79	15.4196	1.7279	0.79	15.0242	1.7250	0.79	13.6387	1.7143	0.79	12.2564	1.7029
1.17	21.3508	1.5315	1.17	20.7584	1.5336	1.17	18.7002	1.5409	1.17	16.6756	1.5481
1.55	24.7245	1.2949	1.55	23.9855	1.3039	1.55	21.4388	1.3359	1.55	18.9665	1.3679
1.94	24.2875	1.0665	1.94	23.5241	1.0830	1.94	20.9063	1.1409	1.94	18.3863	1.1987
2.32	19.3930	0.9343	2.32	18.7799	0.9546	2.32	16.6764	1.0262	2.32	14.6496	1.0972
2.71	10.9739	0.9644	2.71	10.6430	0.9816	2.71	9.4990	1.0428	2.71	8.3833	1.1045
3.09	1.2630	1.1182	3.09	1.2298	1.1281	3.09	1.1125	1.1642	3.09	0.9942	1.2023

4. Conclusions

In this study, the effects of viscosity which is a non-linear function of temperature on MHD natural convection flow past an isothermal sphere has been determined. Numerical results of the study have been obtained using the Direct Numerical Scheme. From this study, the following conclusions can be made;

- Increase in Magnetic parameter M leads to decrease in velocity profile as well as the temperature profile.
- Increase in the Grashof number Gr leads to increase in velocity and temperature.
- Increase in the values of η leads to increase in temperature and decrease in velocity.
- Increase in the value of M leads to a decrease in skin friction and the rate of heat transfer

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Nomenclature

Roman Symbols

a	Radius of the sphere, m
C_p	Specific heat at constant pressure, J/deg kg
C_f	Skin friction coefficient
f	Dimensionless stream function
g	Acceleration due to gravity, g
Gr	Grashof number
K	Thermal conductivity of the fluid, W/mK
M	MHD parameter
Nu	Nusselt number
Pr	Prandtl number
q_w	Heat flux at the surface, W/m ²
T	Temperature of the fluid, K
T_∞	Temperature of the ambient fluid, K
T_w	Temperature at the surface, K
u, v	Dimensionless velocity component

x, y	Axis direction
\vec{q}	Fluid velocity in the x, y direction, m^3/s
\vec{F}	Body forces vector in x and y directions, N
h_m	Magnetic field intensity, T
C_{fx}	Local skin Friction coefficient
M_g	Magnetization of the magnetic Material
Q	Heat Generation Parameter
$r(\hat{x})$	Radial distance from the symmetrical axis to the surface of the sphere, m
o	The center of the sphere
Greek Symbols	Quantity
β	Volumetric coefficient of thermal expansion, $(C^\circ)^{-1}$
τ_w	Shearing stress, N/m^2
ρ_∞	Density of the ambient fluid, Kg/m^3
ρ	Density of the fluid, Kg/m^3
μ	Viscosity of the fluid, Ns/m^2
θ	Dimensionless temperature function
β_0	Strength of magnetic field, A/m
δ_0	Electric conduction, S/m
$\vec{\nabla}$	Gradient operator $i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$

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