American Journal of Applied Mathematics
2015; 3(5): 229-232
Published online September 25, 2015 (http://www.sciencepublishinggroup.com/j/ajam)
doi: 10.11648/j.ajam.20150305.14
ISSN: 2330-0043 (Print); ISSN: 2330-006X (Online)

Investigation of Order among Some Known T-norms

Shohel Babu[1], Fatema Tuj Johora[1], Abdul Alim[2]

[1]Mathematics, IUBAT-International University of Business Agriculture and Technology, Dhaka, Bangladesh
[2]Mathematics, BGMEA University of Fashion and Technology, Dhaka, Bangladesh

Email address:
shohel.babu@iubat.edu (S. Babu), fatema.johorea@iubat.edu (F. T. Johora), abdulalim@buft.edu.bd (A. Alim)

To cite this article:
Shohel Babu, Fatema Tuj Johora, Abdul Alim. Investigation of Order among Some Known T-norms. *American Journal of Applied Mathematics*. Vol. 3, No. 5, 2015, pp. 229-232. doi: 10.11648/j.ajam.20150305.14

Abstract: In this paper, order among some known T-norms is investigated. Firstly, the T-norm which is the strongest or greatest and the T-norm which is the weakest is observed. Comparing two T-norms we establish the relation which is strong or weak. In addition, for parametric T-norms after changing the interval of their parameter a relation has established which is strong or weak. Finally, compared has done among three or more T-norms.

Keywords: T-norm Algebraic Product T_P, T-norm Min T_M, T-norm Drastic Product T_D, T-norm Franks Product T_F, T-norm Einstein Product T_E, T-norm Hamacher Product T_H, T-norm Dubois & Prade Product T_{DP}

1. Introduction

T-norms or triangular norms are generalization of the classical triangular inequality according to K. Menger in 1942. In 1960, B. Schweizer and A. Sklar after revision of this work redefined the concept of triangular norm as an associative and commutative binary operation. They play a fundamental role in probabilistic metric spaces, probabilistic norms and scalar products, multiple-valued logic, fuzzy sets theory.

Definition:

A T-norm is a function$T$: [0, 1] × [0, 1] → [0, 1] which satisfies the following properties:

$[T_1]$:Monotonicity: $T(a,b) \leq T(c,d) if a \leq c and b \leq d$
$[T_2]$:Commutativity: $T(a,b) = T(b,a)$
$[T_3]$:Associativity: $T\big(a,T(b,c)\big) = T(T(a,b),c)$
$[T_4]$:Boundary condition: $T(a,1) = a$

2. Ordering of T-norms

2.1. Proposition: Among all T-norm "Min" is the Strongest

Proof*:* Suppose T is any T-norm.
$\forall x, y \epsilon$I;
$T(x,y) \leq T(x,1)$[by boundary condition]
=x [by boundary condition]
So $T(x,y) \leq x$.
Again $T(x,y) \leq T(y,x)$ [by commutativity]
$\leq T(y,1)$ [by monotonicity]
=y.
So $T(x,y) \leq y$.
Consequently,$T(x,y)$is a lower bound of $\{x,y\}$.
Again min (x,y) is the greatest lower bound(glb) of $\{x,y\}$.
But I is in chain, so $T_M(x,y) \geq T(x,y)$.
Hence "min" is the strongest or greatest t-norm.

2.2. Proposition: Among all T-norm "Drastic Product" is the Weakest

Proof*:* Suppose T is any T-norm.
$\forall x, y \epsilon$I; there are two cases:
Case1*:* Without loss of generality suppose x=1, then
$T_D(x,y) = T_D(1,y) = y$
and $T(x,y) = T(1,y) = y$ [by boundary condition].
Consequently T_D=1.
Case2*:* When $x \neq 1$ and $y \neq 1$, then
$T_D(x,y) = 0$.
But in I; clearly, 0≤ $T(x,y)$
Therefore $T_D(x,y) \leq T(x,y)$.
Hence among all T-norm T_D is the weakest.

2.3. Proposition: Let $k\epsilon]0,1[$, Frank's Product

$$T_F(x,y)_k = \log_k\left(1 + \frac{(k^x-1)(k^y-1)}{k-1}\right)$$

And Algebraic Product $T_P(x,y) = xy\ , \forall (x,y) \epsilon I \times I$.
Then $T_F(x,y)_k \geq T_P(x,y)\ , \forall (x,y) \epsilon I \times I$.
Proof: Let $x\epsilon]0,1[$ and $R_x: [0,1] \to \mathbb{R}: R_x(v) = xv^{x-1}, \forall v \epsilon [0,1]$.

Then $R_x(v) \geq 0$, and $\frac{\delta}{\delta x}(R_x) = x(x-1)v^{x-2} \leq 0$, which means that $R_x(v)$ is monotonically decreasing.

If we have $U_x(v) = v^x$, then by mean value theorem we get

$$\frac{U_x(1) - U_x(v)}{1 - v} = \frac{\delta}{\delta x} U_x(\xi_0)$$
$$= R_x(\xi_0), \text{ because } \frac{\delta}{\delta x} U_x(v) = xv^{x-1}$$
$$\leq \mathrm{R_x(v)}, \xi_0 \epsilon]\mathrm{v}, 1[.$$

i.e.

$$\frac{1 - \mathrm{v^x}}{1 - \mathrm{v}} \leq \mathrm{xv^{x-1}}, \forall \mathrm{x}, \mathrm{v}\epsilon]0,1[.$$

Now let $H_x: [0,1] \to \mathbb{R}$,

$$\mathrm{H_x(v)} = \frac{1 - \mathrm{v^x}}{1 - \mathrm{v}}, \mathrm{v}\epsilon]0,1[.$$

Here

$$\frac{\delta}{\delta \mathrm{x}} \mathrm{H_x(v)} = \frac{-\mathrm{xv^{x-1}}(1 - \mathrm{v}) + 1 - \mathrm{v^x}}{(1 - \mathrm{v})^2} \leq 0, \forall \mathrm{x}, \mathrm{v}\epsilon]0,1[.$$

Which means that $H_x(v)$ is monotonically decreasing and, Furthermore,

$$\mathrm{H_x(k)} \leq \mathrm{H_x(k^y)}, \forall \mathrm{k}, \mathrm{y}\epsilon]0,1[,$$

i.e

$$\frac{1 - k^x}{1 - k} \geq \frac{1 - k^{xy}}{1 - k^y}$$
$$\Rightarrow \frac{k^x - 1}{k - 1} \geq \frac{k^{xy} - 1}{k^y - 1}$$
$$\Rightarrow \frac{(k^x - 1)(k^y - 1)}{k - 1} \leq k^{sy} - 1 \; [\because k^y - 1 \leq 0]$$
$$\Rightarrow 1 + \frac{(k^x - 1)(k^y - 1)}{k - 1} \leq k^{sy}$$
$$\Rightarrow 1 + \frac{(k^x - 1)(k^y - 1)}{k - 1} \leq k^{sy}$$
$$\Rightarrow \log_k \left(1 + \frac{(k^x - 1)(k^y - 1)}{k - 1}\right) \geq \log_k(k^{sy}) = xy.$$

This means that $T_F(x,y)_k \geq T_P(x,y), \forall k, x, y\epsilon]0,1[$.

Again, we clam that

$$T_F(0,x)_k = T_F(x,0)_k = T_P(0,x) = T_P(x,0) = 0.$$

And

$$T_F(x,1)_k = T_F(1,x)_k = T_P(x,1) = T_P(1,x)$$
$$= x, \forall k, x, \epsilon]0,1[.$$

Hence $T_F(x,y)_k \geq T_A(x,y), \forall (x,y)\epsilon I \times I$.

2.4. Proposition: Let $k\epsilon]1,\infty[$, Frank's Product $(x,y)_k = \log_k\left(1 + \frac{(k^x-1)(k^y-1)}{k-1}\right)$

and Algebraic Product $T_P(x,y) = xy, \forall (x,y)\epsilon I \times I$.

Then $T_F(x,y)_k \leq T_P(x,y), \forall (x,y)\epsilon I \times I$.

Proof: Let $x\epsilon]0,1[$ and $R_x: [1,\infty] \to \mathbb{R}: R_x(v) = xv^{x-1}, \forall v\epsilon[0,1]$.

Then $R_x(v) \geq 0$, and $\frac{\delta}{\delta x}(R_x) = x(x-1)v^{x-2} \leq 0$, which means that $R_x(v)$ is monotonically decreasing.

Now we have ,

$U_x: [1,\infty[\to \mathbb{R}, U_x(v) = v^x, \forall x\epsilon]0,1[$ and

$$\frac{U_x(v) - U_x(1)}{v - 1} = \frac{\delta}{\delta x} U_x(\xi_0)$$
$$= R_x(\xi_0), \forall \xi_0 \epsilon]0,1[\forall$$
$$\geq R_x(v)$$

because $R_x(v)$ is monotonically decreasing and $\xi_0 \leq v$.

i.e.

$$\frac{v^x - 1}{v - 1} = x{\xi_0}^{x-1}, \forall x\epsilon]0,1[, v\epsilon]1,\infty[\qquad (1)$$

Now let $H_x(v): [1,\infty] \to \mathbb{R}$,

$$\mathrm{H_x(v)} = \frac{\mathrm{v^x} - 1}{\mathrm{v} - 1}, \forall \mathrm{v}\epsilon]1,\infty[, \mathrm{x}\epsilon]0,1[$$

We see that

$$\frac{\delta}{\delta \mathrm{x}} \mathrm{H_x(v)} = \frac{\mathrm{xv^{x-1}}(\mathrm{v} - 1) - \mathrm{v^x} + 1}{(\mathrm{v} - 1)^2} \leq 0$$

According to (1) and $H_x(v)$ is monotonically decreasing.

Then,

$$\mathrm{H_x(k)} \leq \mathrm{H_x(k^y)}, \forall \mathrm{y}\epsilon]0,1[, \mathrm{k}\epsilon]1,\infty[.$$

i.e.

$$\frac{k^x - 1}{k - 1} \geq \frac{k^{xy} - 1}{k^y - 1} \qquad (2)$$

From (2) we get

$$1 + \frac{(\mathrm{k^x} - 1)(\mathrm{k^y} - 1)}{\mathrm{k} - 1} \leq \mathrm{k^{sy}}$$
$$\Rightarrow \log_\mathrm{k}\left(1 + \frac{(\mathrm{k^x} - 1)(\mathrm{k^y} - 1)}{\mathrm{k} - 1}\right) \leq \log_\mathrm{k}(\mathrm{k^{xy}})$$
$$\Rightarrow \log_\mathrm{k}\left(1 + \frac{(\mathrm{k^x} - 1)(\mathrm{k^y} - 1)}{\mathrm{k} - 1}\right) \leq \mathrm{xy}.$$

This means that $T_F(x,y)_k \leq T_P(x,y), \forall (x,y)\epsilon I \times I$, $\forall k\epsilon]1,\infty[$.

Moreover$T_F(0,x)_k = T_F(x,0)_k = T_P(0,x)$
$= T_P(x,0) = 0$

And

$$\mathrm{T_F(x,1)_k} = \mathrm{T_F(1,x)_k} = \mathrm{T_P(x,1)} = \mathrm{T_P(1,x)}$$
$$= \mathrm{x}, \forall \mathrm{x}\epsilon]0,1[, \forall \mathrm{k}, \epsilon]1,\infty[.$$

Hence $T_F(x,y)_k \leq T_P(x,y), \forall (x,y)\epsilon I \times I, \forall k\epsilon]1,\infty[$.

2.5. Proposition: Let $k\epsilon]1,\infty[$, Frank's Product

$$T_F(x,y)_k = \log_k\left(1 + \frac{(k^x - 1)(k^y - 1)}{k - 1}\right)$$

And Boundary Product

$$T_B(x,y) = (0 \wedge x + y - 1), \forall (x,y)\epsilon I \times I.$$

Then $T_B(x,y) \leq T_F(x,y)_k, \forall (x,y)\epsilon I \times I$.

Proof: We will distinguish two cases:

Case1: If $x + y \geq 1$ then $T_B(x,y) = x + y - 1$

Now for $T_B(x,y) \leq T_F(x,y)_k$ we get

$$x + y - 1 \leq \log_k\left(1 + \frac{(k^x - 1)(k^y - 1)}{k - 1}\right)$$
$$\Leftrightarrow k^{x+y-1} \leq 1 + \frac{(k^x - 1)(k^y - 1)}{k - 1}$$
$$\Leftrightarrow k^{x+y-1} - 1 \leq \frac{(k^x - 1)(k^y - 1)}{k - 1}$$
$$\Leftrightarrow (k^{x+y-1} - 1)(k - 1) \leq (k^x - 1)(k^y - 1)$$
$$\Leftrightarrow k^{x+y} - k - k^{x+y-1} + 1 \leq k^{x+y} - k^x - k^y + 1$$
$$\Leftrightarrow k^x + k^y \leq k + k^{x+y-1}$$
$$\Leftrightarrow k \cdot k^{x-1} + k \cdot k^{y-1} \leq k + k \cdot k^{(x-1)(y-1)}$$
$$\Leftrightarrow k^{x-1} + k^{y-1} \leq 1 + k^{(x-1)(y-1)}$$
$$\Leftrightarrow 1 - k^{x-1} - k^{y-1} + k^{(x-1)(y-1)} \geq 0$$
$$\Leftrightarrow 1(1 - k^{x-1}) - k^{y-1}(1 - k^{x-1}) \geq 0$$
$$\Leftrightarrow (1 - k^{x-1})(1 - k^{y-1}) \geq 0$$

Which is true, because $k > 1, x - 1 \leq 0, y - 1 \leq 0$ and

$k^{x-1} \le k^0 = 1$
$k^{y-1} \le k^0 = 1.$

Case2: If $x + y < 1$ then
$T_B(x, y) = 0$
For$T_B(x, y) \le T_F(x, y)_k \Longrightarrow 0 \le T_F(x, y)_k$
This is obvious.
Hence $T_B(x, y) \le T_F(x, y)_k, \forall(x, y)\epsilon I \times I, k\epsilon]1, \infty[$.

2.6. Proposition: Let $k\epsilon[0,1]$, Duboi's & Prade Product

$$T_{DP}(x, y)_k = \frac{xy}{x \vee y \vee k}$$

and Algebrai Product $T_P(x, y) = xy, \forall(x, y)\epsilon I \times I$.
Then $T_P(x, y) \le T_{DP}(x, y), \forall(x, y)\epsilon I \times I, \forall(x, y)\epsilon I \times I$.

Proof: We will distinguish three cases, according to the maximum value of $x, yandk$.

Caes1: If $x \vee y \vee k = x$, then
$T_{DP}(x, y)_k = \frac{xy}{x}$
$=y$
$\ge xy, [\because x, y\epsilon I]$
$= T_P(x, y).$
$\therefore T_P(x, y) \le T_{DP}(x, y)_k.$

Case2: If $x \vee y \vee k = y$, then
$T_{DP}(x, y)_k = \frac{xy}{y}$
$= x$
$\ge xy, [\because x, y\epsilon I]$
$= T_P(x, y).$
$\therefore T_P(x, y) \le T_{DP}(x, y)_{k.}$

Case3: If $x \vee y \vee k = k$, then
$T_{DP}(x, y)_k = \frac{xy}{k}$
$= \frac{1}{k}(xy)$
$= \frac{1}{k}T_P(x, y)$
$\ge T_a(x, y).$
$\therefore T_P(x, y) \le T_{DP}(x, y)_k, \forall(x, y)\epsilon I \times I, \forall k\epsilon[0,1].$
Hence we can say from three cases $T_P(x, y) \le T_{DP}(x, y)_k$.

2.7. Proposition: Let Einstein Product T_E: $I \times I \to I$

$$T_E(x, y) = \frac{xy}{1 + (1 - x)(1 - y)}$$

Boundery Produuct$T_B(x, y) = 0 \vee x + y - 1$,
Algebraicc Prodducct$T_P(x, y) = xy, \forall(x, y)\epsilon I \times I$.
Then $T_B(x, y) \le T_E(x, y) \le T_P(x, y), \forall(x, y)\epsilon I \times I$.

Proof: Now we will split the proof into two cases:

Case1: If $T_E(x, y) \le T_P(x, y)$, then

$$\frac{xy}{1 + (1 - x)(1 - y)} \le xy$$
$$\Leftrightarrow \frac{xy}{1 + (1 - x)(1 - y)} \le 1$$
$$\Leftrightarrow 1 + (1 - x)(1 - y) \ge 1$$
$$\Leftrightarrow (1 - x)(1 - y) \ge 0$$

Which is true, because $\forall(x, y)\epsilon I \times I$.

Case2: If $T_B(x, y) \le T_E(x, y)$,then

$$0 \vee (x + y - 1) \le \frac{xy}{1 + (1 - x)(1 - y)}.$$

(i)If $x + y \le 1$, then $0 \vee (x + y - 1) = 0$.
In this case we get,

$$0 \le \frac{xy}{1 + (1 - x)(1 - y)}$$

Since $x, y\epsilon[0,1], soxy \ge 0 \ and \frac{xy}{1+(1-x)(1-y)} \ge 0$
$\therefore T_B(x, y) \le T_E(x, y)$

(i)Otherwise $x + y \ge 1, then \ 0 \vee (x + y - 1) = x + y - 1.$

$(x + y - 1) \le \frac{xy}{1 + (1 - x)(1 - y)}$
$\Leftrightarrow (x + y - 1)(1 - x)(1 - y) \le xy$
$\Leftrightarrow xy - (x + y - 1)(1 - x)(1 - y) \ge 0$
$\Leftrightarrow xy - (x + y - 1)(2 - x - y + xy) \ge 0$
$\Leftrightarrow xy - (2x - x^2 - xy + x^2y + 2y - xy - y^2 + xy^2 - 2 + x + y - xy) \ge 0$
$\Leftrightarrow xy - (3x - x^2 + x^2y - 3xy + 3y - y^2 + xy^2 - 2) \ge 0$
$\Leftrightarrow xy - 3x + x^2 - x^2y + 3xy - 3y + y^2 - xy^2 + 2) \ge 0$
$\Leftrightarrow x^2(1 - y) - 3x + 4xy - xy^2 + 2 - 3y + y^2 \ge 0$
$\Leftrightarrow x^2(1 - y) - x(3 - 4y + y^2) + (2 - y)(1 - y) \ge 0$
$\Leftrightarrow x^2(1 - y) - x(3 - y)(1 - y) + (1 - y0(2 - y) \ge 0$
$\Leftrightarrow (1 - y)(x^2 - x(3 - y) + 2 - y) \ge 0$
$\Leftrightarrow (1 - y)(x^2 - 3x + xy + 2 - y) \ge 0$
$\Leftrightarrow (1 - y)(2 - 2x - x + x^2 - y + xy) \ge 0$
$\Leftrightarrow (1 - y)(1 - x)(2 - x - y) \ge 0.$

This is true $\forall x, y\epsilon[0,1]$.
From above we may conclude that

$$T_B(x, y) \le T_E(x, y) \le T_P(x, y), \forall(x, y)\epsilon I \times I.$$

2.8. Proposition:Let Hamacher Product T_H: $I \times I \to I$

$$T_H(x, y)_k = \frac{xy}{k + (1 - k)(x + y - xy)}, k\epsilon[1,2].$$

Then $T_B(x, y) \le T_H(x, y)_k \le T_P(x, y), \forall(x, y)\epsilon I \times I$.

Proof: As above we will distinguish two cases:

Case1*:* In this case, it will prove that $T_H(x, y)_k \le T_P(x, y)$, then

$$\frac{xy}{k + (1 - k)(x + y - xy)} \le xy$$
$$\Leftrightarrow \frac{1}{k + (1 - k)(x + y - xy)} \le 1$$
$$\Leftrightarrow k + (1 - k)(x + y - xy) \ge 1$$
$$\Leftrightarrow -1 + k + (1 - k)(x + y - xy) \ge 0$$
$$\Leftrightarrow (1 - k)(x + y - 1 - xy) \ge 0$$
$$\Leftrightarrow (1 - k)(1 - x)(1 - y) \ge 0.$$

This is obvious, since $x, y\epsilon[0,1]$and$k\epsilon[1,2]$.

Case2*:* Now we prove the relation between $T_H(x, y)_k andT_B(x, y)$.
We may see that if $x + y \le 1$, then

$$T_B(x, y) = 0 \le T_P(x, y).$$

Otherwise, if $x + y \ge 1$, and consider $T_B(x, y) \le T_H(x, y)_k$, then

$x + y - 1 \le \frac{xy}{k + (1 - k)(x + y - xy)}$
$\Leftrightarrow (x + y - 1)(k + (1 - k)(x + y - xy)) \le xy$
$\Leftrightarrow (x + y - 1)(k + x + y - xy - kx - ky + kxy) - xy \le 0$

$$\Leftrightarrow kx + x^2 + xy - x^2y - kx^2 - kxy + kx^2y + ky + xy + y^2 - xy^2 - kxy - ky^2 + kxy^2 - k - x - y + xy + kx + ky - kxy - xy \leq 0$$
$$\Leftrightarrow -k + kx - x + x^2 + kx - kx^2 - y + xy + 2ky - 2kx + xy - x^2 - kxy + kx^2y + y^2 - xy^2 - ky^2 + kxy^2 \leq 0$$
$$\Leftrightarrow -k(1-x) - x(1-x) + kx(1-x) - y(1-x) + 2ky(1-x) + xy(1-x) - kxy(1-x) + y^2(1-x) - ky^2(1-x) \leq 0$$
$$\Leftrightarrow (1-x)(-k - x + kx - y + 2ky + xy - kxy + y^2 - ky^2 \leq 0$$
$$\Leftrightarrow (1-x)(-k + ky + ky - ky^2 - x + xy + kx - kxy - y + y^2 \leq 0$$
$$\Leftrightarrow (1-x)(-k(1-y) + ky(1-y) - x(1-y) + kx(1-y) - y(1-y) \leq 0$$
$$\Leftrightarrow (1-x)(1-y)(-k + ky - x + kx - y) \leq 0$$
$$\Leftrightarrow (1-x)(1-y)(k(x+y-1) - x - y) \leq 0$$
$$\Leftrightarrow (1-x)(1-y)(x+y-1)(k-1) - 1 \leq 0.$$

This is true because

$$1 - x \leq 0, 1 - y \leq 0 \; and (x+y-1)(k-1) - 1 \leq 0$$
$$\Leftrightarrow (x+y-1)(k-1) \leq 1 \Leftrightarrow x + y - 1 \leq \frac{1}{k-1}$$

Since $x + y - 1 \leq 1 \leq \frac{1}{k-1}, \forall k \epsilon [1,2]$.

Hence we may conclude that

$$T_D(x,y) \leq T_B(x,y) \leq T_H(x,y)_k \leq T_P(x,y) \leq T_M(x,y), \forall (x,y) \epsilon I \times I.$$

3. Conclusion

Form above discussions, it conclude that T-norm Min is the strongest and T-norm Drastic product is the weakest T-norms. Also, T-norm Algebraic product is stronger than T-norm Hamacher product and Boundery Product. Similarly, T-norm Dubois & Prade product T_{DP} is stronger than T-norm Algebraic product. Since, T-norms are using for taking suitable decision from multi-valued logic. So, the ordering of T-norms, that's means, this paper will help to take correct decision using sufficient T-norms according to their order.

References

[1] Didier Dobois, Prade Henri, FUZZY SET AND SYSTEM, THEORY AND APPLICATIONS, Academic press INC, New York.

[2] Peter Vicenik, A NOTE TO CONSTRUCTION OF T-NORMS BASED ON PSEUDO-INVERSE OF MONOTONE FUNCTIONS; Department of Mathematics, Slovak Technical University, Radlinskeho 11, 813 68 Bratislava, Slovak Republic Received June 1998.

[3] George J Klir, Yuan Bo, FUZZY SETS AND FUZZY LOGIC, Theory and Applications, Prentice-Hall Inc. N.J. U.S.A. 1995.

[4] Lowen, FUZZYSET THEORY, Department of Mathematics and Computer Science, University of Antwerp; Belgium, Basic Concepts, Techniques and Bibliography, Kluwer Academic Publishers Dordeecht/Boston/London.

[5] Peter Vicenik, A NOTE ON GENERATORS OF T-NORMS; Department of Mathematics, Slovak Technical University, Radlinskeho 11, 813 68 Bratislava, Slovak Republic.

[6] Matteo Bianchi, The logic of the strongest and the weakest t-norms, Fuzzy Sets Syst. 276 (2015) 31–42, http://dx.doi.org/10.1016/j.fss.2015.01. 13.

[7] Mirko Navara(2007), "*Triangular norms and conforms*" Scholarpedia

[8] Wladyslaw Homenda, TRIANGULAR NORMS, UNI-AND NULLNORMS,BALANCED NORMS,THE CASES OF THE HIERACHY OF ITERATIVE OPERATORS, Faculty of Mathematics and Information Science, Warsaw University of Technology, Warsaw, Poland.

[9] Mirta N.K., FUZZY SET THEORY RELATIONAL STRUCTURE USING T-NORMS AND MATHLAB, Department of Mathematics, University of Dhaka.

[10] Klement, Erich Perer; Mesiar, Radko; and Pap, Endre(2000),Triangular Norms. Dordrecht: Kluwer. ISBN 0-7923-6416-3.