American Journal of Applied Mathematics
2015; 3(4): 185-188
Published online July 15, 2015 (http://www.sciencepublishinggroup.com/j/ajam)
doi: 10.11648/j.ajam.20150304.14
ISSN: 2330-0043 (Print); ISSN: 2330-006X (Online)

A Study on (Q,L)-Fuzzy Normal Subsemiring of a Semiring

S. Sampathu[1], S. Anita Shanthi[2], A. Praveen Prakash[3]

[1]Department of Mathematics, Sri Muthukumaran College of Education, Chikkarayapuram, Chennai, Tamil Nadu, India
[2]Department of Mathematics, Annamalai University, Tamil Nadu, India
[3]Department of Mathematics, Hindustan University, Padur, Tamil Nadu, India

Email address:
sampathugokul@yahoo.in (S. Sampathu), shanthi.Anita@yahoo.com (S. A. Shanthi), apraveenprakash@gmail.com (A. P. Prakash)

To cite this article:
S. Sampathu, S. Anita Shanthi, A. Praveen Prakash. A Study on (Q,L)-Fuzzy Normal Subsemiring of a Semiring. *American Journal of Applied Mathematics*. Vol. 3, No. 4, 2015, pp. 185-188. doi: 10.11648/j.ajam.20150304.14

Abstract: In this paper, we introduce the concept of (Q,L)-fuzzy normal subsemirings of a semiring and establish some results on these. We also made an attempt to study the properties of (Q,L)-fuzzy normal subsemirings of semiring under homomorphism and anti-homomorphism , and study the main theorem for this. We shall also give new results on this subject.

Keywords: (Q,L)-Fuzzy Subset, (Q,L)-Fuzzy Subsemiring, (Q,L)-Fuzzy Normal Subsemiring, Product Of (Q,L)-Fuzzy Subsets, Strongest (Q, L)-Fuzzy Relation, Pseudo (Q, L)-Fuzzy Coset

1. Introduction

There are many concepts of universal algebras generalizing an associative ring (R ;+; .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R;+) and (R; .) are semigroups satisfying a.(b+c)=a.b+a.c and (b+c).a=b.a+c.a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a=a=a+0 and a.0=0=0.a for all a in R. After the introduction of fuzzy sets by L.A.Zadeh[15], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearrings and ideals was introduced by S.Abou Zaid[10]. A.Solairaju and R.Nagarajan [12] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. In this paper, we introduce the concept of (Q,L)-fuzzy normal subsemiring of a semiring and established some results.

2. Preliminaries

2.1. Definition 1

Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function A: X×Q → L.

2.2. Definition 2

Let (R, +, ·) be a semiring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-fuzzy subsemiring (QLFSSR) of R if the following conditions are satisfied:

(i) A(x+y, q) ≥ A(x, q) ∧ A(y, q),

(ii) A(xy, q) ≥ A(x, q) ∧ A(y, q), for all x and y in R and q in Q.

2.3. Definition 3

Let R be a semiring and Q be a non-empty set. An (Q, L)-fuzzy subsemiring A of R is said to be an (Q, L)-fuzzy normal subsemiring (QLFNSSR) of R if it satisfies the following conditions:

(i) A(x+y,q) = A(y+x,q),

(ii) A(xy,q) = A(yx,q), for all x and y in R and q in Q.

2.4. Definition 4

Let A and B be any two (Q,L)-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as A×B={<((x, y), q),A×B((x,y),q)>/ for all x in R and y in H and q in Q}, where A×B((x,y),q)=A(x, q)∧ B(y, q).

2.5. Definition 5

Let (R,+,·) and (R^{I},+,·) be any two semirings and Q be a non empty set. Let f:R→R^{I} be any function and A be a (Q,L)-fuzzy subsemiring in R, V be a (Q,L)-fuzzy subsemiring in f(R)=R^{I}, defined by V(y,q) = $\sup_{x\in f^{-1}(y)}$ A(x,q), for all x in R and y in R^{l} and q in Q. Then A is called a pre-image of V under f and is denoted by $f^{-1}(V)$.

2.6. Definition 6

Let A be a (Q,L)-fuzzy subset in a set S, the strongest (Q, L)-fuzzy relation on S, that is a (Q,L)-fuzzy relation V with respect to A given by $V((x,y),q) = A(x,q)\wedge A(y,q)$, for all x and y in S and q in Q.

2.7. Definition 7

A (Q,L)-fuzzy subset A of a set X is said to be normalized if there exists an element x in X such that A(x,q)=1.

2.8. Definition 8

Let A be an (Q, L)-fuzzy subsemiring of a semiring $(R, +, \cdot)$ and a in R. Then the pseudo (Q, L)-fuzzy coset $(aA)^{p}$ is defined by $((aA)^{p})(x,q) = p(a)A(x,q)$, for every x in R and for some p in P and q in Q.

2.9. Definition 9

Let A be a (Q,L)-fuzzy subset of X. For α in L, a Q-level subset of A is the set $A_{\alpha} = \{ x\in X : A(x,q) \geq \alpha\}$.

3. Properties of (Q,L)-Fuzzy Normal Subsemiring of a Semiring

3.1. Theorem 1

Let (R,+,.) be a semiring and Q be a non-empty set. If A and B are two (Q,L)-fuzzy normal subsemirings of R, then their intersection A∩B is an (Q,L)-fuzzy normal subsemiring of R.

Proof: Let x and y∈R. Let A={⟨(x,q),A(x,q)⟩/ x in R and q in Q} and B={⟨(x,q), B(x,q)⟩/ x in R and q in Q} be (Q,L)-fuzzy normal subsemirings of a semiring R. Let C=A∩B and C={⟨(x,q),C(x,q)⟩/x in R and q in Q}. Then, Clearly C is an (Q,L)-fuzzy subsemiring of a semiring R, since A and B are two (Q,L)-fuzzy subsemirings of a semiring R. And (i) $C(x+y,q)=A(x+y,q)\wedge B(x+y,q)=A(y+x,q)\wedge B(y+x,q)=C(y+x,q)$, for all x and y in R and q in Q. Therefore,C(x+y,q) =(y+x,q), for all x and y in R and q in Q. (ii) $C(xy,q) = A(xy,q)\wedge B(xy,q)=A(yx,q)\wedge B(yx,q)=C(yx,q)$, for all x and y in R and q in Q. Therefore, C(xy,q)=C(yx,q), for all x and y in R and q in Q. Hence A∩B is an (Q,L)-fuzzy normal subsemiring of a semiring R.

3.2. Theorem 2

Let R be a semiring and Q be a non-empty set. The intersection of a family of (Q,L)-fuzzy normal subsemirings of R is an (Q,L)-fuzzy normal subsemiring of R.

Proof: Let $\{A_i\}_{i\in I}$ be a family of (Q,L)-fuzzy normal subsemirings of a semiring R and let $A=\bigcap_{i\in I} A_i$. Then for x and y in R and q in Q. Clearly the intersection of a family of (Q,L)-fuzzy subsemirings of a semiring R is an (Q,L)-fuzzy subsemiring of a semiring R.(i) $A(x+y,q) = \inf_{x\in f^{-1}(y)} A_i(x+y,q)= \inf_{x\in f^{-1}(y)} A_i(y+x,q)=A(y+x,q)$. Therefore, A(x+y,q)=A(y+x,q), for all x and y in R and q in Q. (ii) $A(xy,q)= \inf_{x\in f^{-1}(y)} A_i(xy,q)= \inf_{x\in f^{-1}(y)} A_i(yx,q)=A(yx,q)$. Therefore, A(xy,q)=A(yx,q), for all x and y in R and q in Q.Hence the intersection of a family of (Q,L)-fuzzy normal subsemirings of a semiring R is an (Q,L)-fuzzy normal subsemiring of a semiring R.

3.3. Theorem 3

Let A and B be (Q,L)-fuzzy subsemiring of the semirings G and H, respectively. If A and B are (Q,L)-fuzzy normal subsemirings, then A×B is an (Q,L)- fuzzy normal subsemiring of G×H.

Proof: Let A and B be (Q,L)-fuzzy normal subsemirings of the semirings G and H respectively. Clearly A×B is an (Q,L)-fuzzy subsemiring of G×H. Let x_1 and x_2 be in G, y_1 and y_2 be in H and q in Q. Then (x_1,y_1) and (x_2,y_2) are in G×H.
Now,$A\times B[(x_1,y_1)+(x_2,y_2),q]=A\times B((x_1+x_2,y_1+y_2),q)$
$=A(x_1+x_2,q)\wedge B(y_1+y_2,q)=A(x_2+x_1,q)\wedge B(y_2+y_1,q)$
$=A\times B((x_2+x_1,y_2+y_1),q)=A\times B[(x_2,y_2)+(x_1,y_1),q]$.
Therefore,$A\times B[(x_1,y_1)+(x_2,y_2),q]=A\times B[(x_2,y_2)+(x_1,y_1),q]$.
And,$A\times B[(x_1,y_1)(x_2,y_2),q]=A\times B((x_1x_2,y_1y_2),q)$
$=A(x_1x_2,q)\wedge B(y_1y_2,q)=A(x_2x_1,q),B(y_2y_1,q)=$
$A\times B((x_2x_1,y_2y_1),q)=A\times B[(x_2,y_2)(x_1,y_1),q]$.
Therefore, $A\times B[(x_1,y_1)(x_2,y_2),q]=A\times B[(x_2,y_2)(x_1,y_1),q]$.
Hence A×B is an (Q,L)-fuzzy normal subsemiring of G×H.

3.4. Theorem 4

Let A be a fuzzy subset in a semiring R and V be the strongest (Q,L)-fuzzy relation on R. Then A is an (Q,L)-fuzzy normal subsemiring of R if and only if V is an (Q,L)-fuzzy normal subsemiring of R×R.

Proof: Suppose that A is a (Q,L)-fuzzy normal subsemiring of R. Then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in R×R and q in Q. Clearly V is a (Q,L)-fuzzy subsemiring of R×R. We have,
$V(x+y,q)=V[(x_1,x_2)+(y_1,y_2),q]=V((x_1+y_1,x_2+y_2),q)$
$=A((x_1+y_1),q)\wedge A((x_2+y_2),q)=A((y_1+x_1),q)\wedge A((y_2+x_2),q)$
$=V((y_1+x_1,y_2+x_2),q)=V[(y_1,y_2)+(x_1,x_2),q]=V(y+x,q)$
Therefore,V(x+y,q)=V(y+x,q), for all x and y in R×R and q in Q. We have, $V(xy,q)=V[(x_1,x_2)(y_1,y_2),q]=V((x_1y_1,x_2y_2),q)$
$=A((x_1y_1),q)\wedge A((x_2y_2),q)=A((y_1x_1),q)\wedge A((y_2x_2),q)$
$=V((y_1x_1,y_2x_2),q)=V[(y_1,y_2)(x_1,x_2),q]=V(yx,q)$
Therefore, V(xy,q)=V(yx,q), for all x and y in R×R and q in Q. This proves that V is a (Q,L)-fuzzy normal subsemiring of R×R. Conversely, assume that V is a (Q,L)-fuzzy normal subsemiring of R×R, then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in R×R,we have
$A(x_1+y_1,q)\wedge A(x_2+y_2,q)=V((x_1+y_1,x_2+y_2),q)$
$=V[(x_1,x_2)+(y_1,y_2),q]=V(x+y,q)=V(y+x,q)$
$=V[(y_1,y_2)+(x_1,x_2),q]=V((y_1+x_1,y_2+x_2),q)$
$=A(y_1+x_1,q)\wedge A(y_2+x_2,q)$. We get, $A((x_1+y_1),q)=A((y_1+ x_1),q)$, for all x_1 and y_1 in R and q in Q. And $A(x_1y_1,q)\wedge A(x_2y_2,q)=V((x_1y_1,x_2y_2),q)=V[(x_1,x_2)(y_1,y_2),q]$
$=V(xy,q)=V(yx,q)=V[(y_1,y_2)(x_1,x_2),q]=V((y_1x_1,y_2x_2),q)$
$=A(y_1x_1,q)\wedge A(y_2x_2,q)$. We get, $A((x_1y_1),q)=A((y_1x_1),q)$, for all x_1 and y_1 in R and q in Q. Hence A is a (Q, L)-fuzzy normal

subsemiring of R.

3.5. Theorem 5

Let (R,+, .) and (R^I,+, .) be any two semirings and Q be a non-empty set. The homomorphic image of an (Q,L)-fuzzy normal subsemiring of R is an (Q,L)-fuzzy normal subsemiring of R^I.

Proof: Let (R,+,.) and (R^I,+,.) be any two semirings Q be a non-empty set and f :R→R ^{I}be a homomorphism. Then, f(x+y)=f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R. Let V=f(A), where A is an (Q,L)-fuzzy normal subsemiring of a semiring R. We have to prove that V is an (Q,L)-fuzzy normal subsemiring of a semiring R^I. Now, for f(x), f(y) in R^I, clearly V is an (Q,L)-fuzzy subsemiring of a semiring R^I, since A is an (Q,L)-fuzzy subsemiring of a semiring R. Now, V(f(x)+f(y),q)=V(f(x+y),q)≥A(x+y,q)=A(y+x,q)≤V(f(y+x),q) =V(f(y)+f(x),q), which implies that V(f(x)+f(y),q)= V(f(y)+(f(x),q), for all f(x) and f(y) in R^I. Again, V(f(x)f(y),q)=V(f(xy),q)≥A(xy,q)=A(yx,q)≤V(f(yx),q) =V(f(y)f(x),q), which implies that V(f(x)f(y),q)=V(f(y)f(x),q), for all f(x) and f(y) in R^I. Hence V is an (Q,L)-fuzzy normal subsemiring of a semiring R^I.

3.6. Theorem 6

Let (R,+,.) and (R^I,+,.) be any two semirings and Q be a non-empty set. The homomorphic preimage of an (Q,L)-fuzzy normal subsemiring of R^I is an (Q,L)-fuzzy normal subsemiring of R.

Proof: Let (R, +, .) and (R^I, +, .) be any two semirings and Q be a non-empty set and f : R → R^I be a homomorphism. Then, f(x+y)=f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R. Let V=f(A), where V is an (Q,L)-fuzzy normal subsemiring of a semiring R^I. We have to prove that A is an (Q,L)-fuzzy normal subsemirring of a semiring R. Let x and y in R. Then, clearly A is an (Q,L)-fuzzy subsemiring of a semiring R, since V is an (Q,L)-fuzzy subsemiring of a semiring R^I. Now, A(x+y,q)=V(f(x+y),q)=V(f(x)+f(y),q)=V(f(y)+f(x),q)= V(f(y+x),q)=A(y+x,q), which implies that A(x+y,q)=A(y+x,q), for all x and y in R and q in Q. Again, A(xy,q)=V(f(xy),q)=V(f(x)f(y),q)=V(f(y)f(x),q)=V(f(yx),q)= A(yx,q), which implies that A(xy,q)= A(yx,q), for all x and y in R and q in Q. Hence A is an (Q,L)-fuzzy normal subsemiring of a semiring R.

3.7. Theorem 7

Let (R, +, .) and (R^I, +, .) be any two semirings and Q be a non-empty set. The anti-homomorphic image of an (Q, L)-fuzzy normal subsemiring of R is an (Q, L)-fuzzy normal subsemiring of R^I.

Proof: Let (R,+,.) and (R^I,+,.) be any two semirings and Q be a non-empty set and f:R→R^I be an anti-homomorphism. Then, f(x+y)=f(y)+f(x) and f(xy)=f(y) f(x), for all x and y in R. Let V=f(A), where A is an (Q,L)-fuzzy normal subsemiring of a semiring R. We have to prove that V is an (Q,L)-fuzzy normal subsemiring of a semiring R^I. Now, for f(x) and f(y) in R^I, clearly V is an (Q,L)-fuzzy subsemiring of a semiring R^I, since A is an (Q,L)-fuzzy subsemiring of a semiring R. Now, V(f(x)+f(y),q)=V(f(y+x),q)≥A(y+x,q)=A(x+y,q)≤V(f(x+y),q) =V(f(y)+f(x),q), which implies that V(f(x)+f(y),q)=V(f(y)+f(x),q), for all f(x) and f(y) in R^I. Again,V(f(x)f(y),q)=V(f(yx),q)≥A(yx,q)=A(xy,q)≤V(f(xy),q) =V(f(y)f(x),q), which implies that V(f(x)f(y),q)=V(f(y)f(x),q), for all f(x) and f(y) in R^I. Hence V is an (Q,L)-fuzzy normal subsemiring of a semiring R^I.

3.8. Theorem 8

Let (R,+,.) and (R^I,+,.) be any two semirings and Q be a non-empty set. The anti-homomorphic preimage of an (Q,L)-fuzzy normal subsemiring of R^I is an (Q,L)-fuzzy normal subsemiring of R.

Proof: Let (R, +, .) and (R^I, +, .) be any two semirings and Q be a non-empty set and f :R→ R^I be an anti-homomorphism. Then, f(x+y)=f(y)+f(x) and f(xy)=f(y)f(x), for all x and y in R.Let V=f(A), where V is an (Q,L)-fuzzy normal subsemiring of a semiring R^I. We have to prove that A is an (Q,L)-fuzzy normal subsemiring of a semiring R. Let x and y in R, then clearly A is an (Q,L)-fuzzy subsemiring of a semiring R, since V is an (Q,L)-fuzzy subsemiring of a semiring R^I. Now, A(x+ y,q)=V(f(x+y),q)=V(f(y)+f(x),q)=V(f(x)+f(y),q)=V(f(y+x),q) =A(y+x,q), which implies that A(x+y,q)= A(y+x,q), for all x and y in R and q in Q. Again, A(xy,q)=V(f(xy),q)=V(f(y)f(x),q)=V(f(x)f(y),q)=V(f(yx),q)= A(yx,q), which implies that A(xy,q)=A(yx,q), for all x and y in R and q in Q. Hence A is an (Q,L)-fuzzy normal subsemiring of a semiring R.

3.9. Theorem 9

Let A be an (Q, L)-fuzzy normal subsemiring of a semiring (R, +, .), then the pseudo (Q, L)-fuzzy coset $(aA)^p$ is an (Q, L)-fuzzy normal subsemiring of a semiring R, for a in Rand q in Q.

Proof: Let A be an (Q, L)-fuzzy normal subsemiring of a semiring R. For every x and y in Rand q in Q, we have, $((aA)^p)(x+y)=p(a)A(x+y)\geq p(a)\{(A(x)\wedge A(y)\}=\{p(a)A(x)\wedge p(a)A(y)\}=\{((aA)^p)(x)\wedge((aA)^p)(y)\}$.
Therefore, $((aA)^p)(x+y)=\{((aA)^p)(x)\wedge((aA)^p)(y)\}$.
Now, $((aA)^p)(xy)=p(a)A(xy)\geq p(a)\{A(x)\wedge A(y)\} = \{p(a)A(x)\wedge p(a)A(y)\}=\{((aA)^p)(x)\wedge((aA)^p)(y)\}$.
Therefore, $((aA)^p)(xy)=\{((aA)^p)(x)\wedge((aA)^p)(y)\}$. Hence $(aA)^p$ is an (Q, L)-fuzzy normal subsemiring of a semiring R.

3.10. Theorem 10

Let A and B be (Q,L)-fuzzy subsets of the sets R and H respectively , and let α in L. Then $(A\times B)_\alpha = A_\alpha \times B_\alpha$.

Proof: Let α in L. Let (x,y) be in $(A\times B)_\alpha$ if and only if $A\times B((x,y),q) \geq \alpha$

if and only if $\{A(x,q)\wedge B(x,q)\} \geq \alpha$

if and only if $A(x,q)\geq\alpha$ and $B(x,q) \geq \alpha$

if and only if $x\in A_\alpha$ and $y\in B_\alpha$

if and only if $(x,y) \in A_\alpha \times B_\alpha$.

Therefore, $(A\times B)_\alpha = A_\alpha \times B_\alpha$.

3.11. Theorem 11

Let A be a (Q,L)-fuzzy normal subsemiring of a semiring R. If A(x,q) < A(y,q), for some x and y in R and q in Q, then A(x+y,q)=A(x,q)= A(y+x,q), for some x and y in R and q in Q.

Proof: It is trivial.

3.12. Theorem 12

Let A be a (Q,L)-fuzzy normal subsemiring of a semiring R. If A(x,q) > A(y,q), for some x and y in R and q in Q, then A(x+y,q)=A(y,q)= A(y+x,q), for some x and y in R and q in Q.
Proof: It is trivial.

3.13. Theorem 13

Let A be a (Q,L)-fuzzy normal subsemiring of a semiring R such that Im A ={α}, where α in L. If A=B $\bigcup$ C, where B and C are (Q,L)-fuzzy normal subsemiring of a semiring R, then either B $\subseteq$ C or C $\subseteq$ B.
Proof: It is trivial.

4. In the Following Theorem is the Composition Operation of Functions

4.1. Theorem 1

Let A be an (Q, L)-fuzzy normal subsemiring of a semiring H and f is an isomorphism from a semiring R onto H. Then A◦f is an (Q,L)-fuzzy normal subsemiring of the semiring R.
Proof: Let x and y in R and A be an (Q,L)-fuzzy normal subsemiring of a semiring H. Then clearly A◦f is an (Q,L)-fuzzy subsemiring of a semiring R. Now, (A◦f)(x+y, q) =A(f(x+y),q)=A(f(x)+f(y),q)=A(f(y)+f(x),q)=A(f(y+x),q) =(A◦f)(y+x,q),which implies that (A◦f)(x+y,q)=(A◦f)(y+x,q) , for all x and y in R and q in Q. And, (A◦f)(xy,q)= A(f(xy),q)=A(f(x)f(y),q)=A(f(y)f(x),q)=A(f(yx),q)= (A◦f)(yx,q), which implies that (A◦f)(xy,q)=(A◦f)(yx,q) , for all x and y in R and q in Q. Hence A◦f is an (Q,L)-fuzzy normal subsemiring of a semiring R.

4.2. Theorem 2

Let A be an (Q,L)-fuzzy normal subsemiring of a semiring H and f is an anti-isomorphism from a semiring R onto H. Then A◦f is an (Q,L)-fuzzy normal subsemiring of the semiring R.
Proof: Let x and y in R and A be an (Q,L)-fuzzy normal subsemiring of a semiring H. Then clearly A◦f is an (Q,L)-fuzzy subsemiring of a semiring R. Now, (A◦f)(x+y,q) = A(f(x+y),q)=A(f(y)+f(x),q)=A(f(x)+f(y),q)=A(f(y+x),q)= (A◦f)(y+x,q),which implies that (A◦f)(x+y,q)=(A◦f)(y+x,q) , for all x and y in R and q in Q. And, (A◦f)(xy,q) = A(f(xy),q)=A(f(y)f(x),q)=A(f(x)f(y),q)=A(f(yx),q)= (A◦f)(yx,q), which implies that (A◦f)(xy,q)=(A◦f)(yx,q) , for all x and y in R and q in Q. Hence A◦f is an (Q,L)-fuzzy normal subsemiring of a semiring R.

Acknowledgements

The authors would like to be thankful to the anonymous reviewers for their valuable suggestions.

References

[1] Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications, 35, 512-517 (1971).

[2] Anthony. J. M. and Sherwood. H, Fuzzy groups Redefined, Journal of mathematical analysis and applications, 69,124 -130 (1979).

[3] Asok Kumer Ray, On product of fuzzy subgroups, fuzzy sets and sysrems, 105, 181-183 (1999).

[4] Biswas. R, Fuzzy subgroups and Anti-fuzzy subgroups, Fuzzy sets and systems, 35,121-124 (1990).

[5] Mustafa Akgul, Some properties of fuzzy groups, Journal of mathematical analysis and applications, 133, 93-100 (1988).

[6] Mohamed Asaad, Groups and fuzzy subgroups, fuzzy sets and systems (1991), North-Holland.

[7] Palaniappan. N & Arjunan. K, Operation on fuzzy and anti fuzzy ideals, Antartica J. Math., 4(1) (2007), 59-64.

[8] Prabir Bhattacharya, Fuzzy Subgroups: Some Characterizations, Journal of Mathematical Analysis and Applications, 128, 241-252 (1987).

[9] Rajesh Kumar, Fuzzy Algebra, Volume 1, University of Delhi Publication Division, July -1993.

[10] Salah Abou-Zaid, On generalized characteristic fuzzy subgroups of a finite group, fuzzy sets and systems, 235-241 (1991).

[11] Sivaramakrishna das. P, Fuzzy groups and level subgroups, Journal of Mathematical Analysis and Applications, 84, 264-269 (1981).

[12] Solairaju. A and Nagarajan. R, A New Structure and Construction of Q-Fuzzy Groups, Advances in fuzzy mathematics, Volume 4, Number 1 (2009), 23-29.

[13] Tang J, Zhang X (2001). Product Operations in the Category of L –fuzzy groups. J. Fuzzy Math., 9:1-10.

[14] Vasantha kandasamy. W. B, Smarandache fuzzy algebra, American research press, Rehoboth -2003.

[15] Zadeh. L. A., Fuzzy sets , Information and control ,Vol.8, 338-353 (1965).