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Stellar Aberration from Earth and from a Satellite

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Abstract: In his endeavor to find a concrete evidence in favor of the Copernican picture of the solar system, the English astronomer James Bradley made a series of astronomical observations during the period (1725-1728) aiming to detect a stellar parallax. His findings, which manifested indeed an annual apparent cyclic motion of a star, were however at conflict with what is expected in a parallax. To his surprise, the result of every measurement obtained corresponded to what he expected to get in a measurement done three months earlier. Bradley realized that he was witnessing a new physical effect, and he presented an explanation that conceived light as a corpuscular stream travelling at finite velocity. Despite that Bradley's explanation of the stellar aberration effect was inadequate, the equation which he derived to quantify the aberration angle, predicted a better estimation of light velocity, and the aberration phenomenon itself was a concrete support of heliocentrism. Stellar aberration as well as some other optical experiments, whose explanations posed challenges to the existing physical theories in the late nineteenth century paved the way for the emergence of the special theory of relativity. In the current work we employ the theory of universal space and time to show that a given direction in a frame of reference is tilted when observed in a moving frame by an angle that depends on the direction itself and the velocity of the moving frame. The latter fact is utilized to explain stellar aberration, determine the deviation of a star's vision direction from its true one, and deduce its apparent position at any instant as a function of its latitude and time. The novel concept of aberration correction vector is employed to derive the apparent elliptic path of an observed celestial object at any time. The concept of graded inertial frames is introduced and utilized to deal with aberration when observed from a satellite in a similar way to its treatment when observed from Earth. The transformation matrix between a geocentric frame and a satellite's non-rotating frame is derived and used to transform temporary Earthly vision directions to the satellite's frame. Furthermore, the transformed vectors are adopted as transient fixed directions relative to which the vision directions of a star from the satellite are specified throughout one revolution. Satellites connective matrices are constructed to make geometric information regarding the celestial sphere in one frame immediately usable by observers on Earth and in all other satellites.

Keywords: Stellar Aberration, Aberration in a Satellite, Graded Inertial Frames, Stars' Apparent Elliptic Trajectories, Aberration Correction Vector, Satellites Connective Matrices

1. Introduction

Stellar aberration [1], as observed from Earth, is the phenomena of apparent periodic motion of a star about its "true" position with a period of one year. The observed star appears displaced towards the Earth's orbital velocity, and the successive displacements associated with the Earth's revolution around the Sun add up to make the apparent annual periodic trajectory. The angle δ between the directions of the true and apparent positions of the observed star attains a maximum $\delta_{max} = 20.4955''$ each half a year starting from Earth's nearest (or furthest) position to the observed star. The latter maximum, also called aberration constant, is quite close to the ratio of the Earth's orbital velocity to light's speed.

Stellar aberration, was discovered by Bradley in 1727 [2-6], who also employed the corpuscular model of light to explain this effect [4-6]. Aberration can also be accounted for in terms of light's waves traveling through the ether, provided the ether remains completely undisturbed by the earth's motion [7-9]. Bradley's explanation of stellar aberration however, proved incorrect since it couldn't account for the negative results of Airy Experiment [4].

In the current work we employ the theory of universal space and time (TUST) [10-13] to explain stellar aberration. The goals and main results in this work are:

To find the relation between an orientation in a moving frame s relative to the corresponding one in a universal frame S. The concept of the body-observer triangle [12] in TUST is used to get the sought relation.

Utilize the found relation to explain the stellar aberration phenomenon through identifying S and s by heliocentric and geocentric frames respectively. The formula obtained for the aberration angle, which is different from Bradley's [2] and the relativistic expressions [14], is in accord with experimental results as it predicts a value for the aberration constant just $0.006''$ less than its generally accepted value, and it determines the aberration angle and the star's apparent position at each instant of the year.

The concept of aberration correction vector is introduced and used to determine the apparent motion of a given star in the celestial sphere, as well as, its associated vision directions at any time.

The observed apparent elliptical orbits of distant celestial objects find in our approach a simple mathematical derivation.

The concept of graded inertial frames is introduced and the direction's change of light beams between them is employed to quantify aberration in an Earth's satellite (or any planet's satellite) relative to a momentary apparent position of the star from Earth (from the planet).

The transformation matrix between a geocentric frame and a satellite's non-rotating frame is set up and utilized to transform celestial positions measurements between the two frames. In particular, the temporary vision direction of a star from Earth is adopted as a temporary fixed direction throughout one revolution to which the apparent direction from the satellite is referred.

To the best of our knowledge, this work is the first

to determine the aberration angles and star's vision directions as functions of their latitudes and time, and to specify the minimum value of the aberration angle pertaining to a given star,

to spell out the significance of graded inertial frames and apply it to tackle aberration as observed from an Earth's satellite in a similar way to its treatment in a sun's satellite, namely the planet Earth. In earlier works [13, 15] the Earth's satellite motion was assumed to be in the ecliptic, whereas our present satellite is moving in an arbitrary plane (of course, through the Earth's center). The existence of graded inertial frames makes it understood why aberration as observed from Earth depends to a great degree of accuracy on its velocity in the heliocentric frame.

to set up the Earth-satellite transformation matrix together with the associated connective matrices, and thus set one-to-one correspondences for distant celestial positions between all satellites.

2. Direction's Change Between Two Frames

The straight path followed by a light's ray in vacuum in any inertial frame S is determined by two points, or by a point and a direction. We are concerned here with distances much larger than wavelength, and the word point in the above context may stand for a small ring through which light passes and whose dimensions are small in comparison with the length of the straight segment connecting the two points. It is also clear in the latter context that the experimental equivalent of a segment connecting two points is a telescope.

Consider a beam of light propagating in the universal frame S parallel to the direction $\boldsymbol{e}$. The path of a narrow beam (say a ray) may be determined in S by two points $B \in S$ and $O \in S$; we assume the ray propagates along the vector $\boldsymbol{BO} \parallel \boldsymbol{e}$. If s is an inertial frame moving at a velocity $\boldsymbol{v} = v\boldsymbol{i}$ in S, then what would be the orientation of the segment BO if looked at in the moving frame s from its end O? (or shortly, how does $\boldsymbol{BO}$ appear in s?). The s observers find the answer [13] through sending a pulse of light from a point $b \in s$ when at $B \in S$ and receiving it at $O \in S$ by an s observer who happens to be at $O \in S$ when light arrives, call it $o \in s$ (in language of TUST, the observer which is conjugate to O [10]). When the pulse arrives at $o \in s$, the source $b \in s$ is at a point $b' \in S$. In the moving frame s light makes the trip $(b \in s \to o \in s)$ whose path $\boldsymbol{bo}$ coincides when light is received by o (at O) with the straight segment $\boldsymbol{b'O}$ in the frame S. Reverting to the body-observer triangle (*Figure* 1) [12] we find that, when light is received, the vector $\boldsymbol{BO}$ is seen in s tilted towards the frame s' velocity $\boldsymbol{v}$ by an angle δ given by the equation,

$$\sin\delta = \beta \sin\theta\,, (\beta = v/c), \tag{1}$$

where $\theta \equiv \angle(\boldsymbol{v}, \boldsymbol{OB}) \in [0, \pi]$ is the angle between $\boldsymbol{OB}$ and the velocity $\boldsymbol{v}$ of s relative to S. Phrasing it in different words, the conjugate observers $O \in S$ and $o \in s$ see the ray along the "conjugate" coplanar directions $(-\boldsymbol{e})$ and $(-\boldsymbol{e}_L)$ respectively, where $\boldsymbol{e}$ and $\boldsymbol{e}_L$ are unit vectors of the paths $\boldsymbol{BO}$ and $\boldsymbol{b'O}$ respectively.

The body-observer triangle affirms the following relation

$$\gamma(\boldsymbol{e}_L + \beta\boldsymbol{i})t = T\boldsymbol{e}, \ \left(\gamma = 1/\sqrt{1-\beta^2}\right), \tag{2}$$

between the conjugate directions, the respective initial and final position vectors $cT\boldsymbol{e}$ and $\gamma ct\boldsymbol{e_L}$ in S of the light's source $b \in s$, and the vector velocity of the moving frame. Taking the cross product of both sides of (2) by $\boldsymbol{e}$ yields

$$(\boldsymbol{e_L} + \beta\boldsymbol{i}) \times \boldsymbol{e} = 0, \tag{3}$$

which in turn yields equation (1).

The sides' lengths of the body-observer triangle appearing in (*Figure* 1) can be expressed in terms of the initial distance $|\boldsymbol{BO}| = cT = R$ of the source from the receiver:

$$|\boldsymbol{b'O}| = E(\beta,\theta)R, \tag{4}$$

$$|\boldsymbol{Bb'}| = \beta E(\beta,\theta)R, \tag{5}$$

where the "Euclidean factor" $E(\beta,\theta)$ is given by [10-13]

$$E(\beta,\pi-\theta) \equiv \frac{\beta cos\theta+\sqrt{1-\beta^2 sin^2\theta}}{1-\beta^2} = \frac{\Gamma(\beta,\theta)}{\sqrt{1-\beta^2}}. \tag{6}$$

The entity $\Gamma(\beta,\theta)$ is called the "scaling factor". Using (4), (6), and the identity $\Gamma(\beta,\theta)\Gamma(\beta,\pi-\theta)=1$, we write (2) in the form

$$-\boldsymbol{e_L} = (1-\beta^2)E(\beta,\theta)(-\boldsymbol{e}) + \beta\boldsymbol{i}, \tag{7}$$

which determines the relation between the vision directions $(-\boldsymbol{e_L})$ and $(-\boldsymbol{e})$ of the same ray in s and S respectively. Equation (7) shows that given the vision direction of an object in one frame determines its vision direction in the other, and that the vision direction $(-\boldsymbol{e_L})$ in the moving frame s is tilted from its counterpart $(-\boldsymbol{e})$ in S towards its velocity $\boldsymbol{v} = c\beta\boldsymbol{i}$.

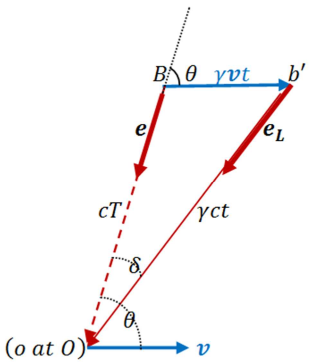

Figure 1. *Body - observer triangle. The same beam is seen along* $\boldsymbol{OB}$ *in S and along* $\boldsymbol{ob'}$ *in the moving frame s. The angle between the two paths is the aberration angle* $\delta = \angle(\boldsymbol{OB},\boldsymbol{ob'}) = \angle(\boldsymbol{e},\boldsymbol{e_L})$. *The velocity of s in S is denoted by* $\boldsymbol{v}$. $\theta = \angle(\boldsymbol{OB},\boldsymbol{v})$ *is the angle between* $\boldsymbol{v}$ *and the initial position vector of the light's source.*

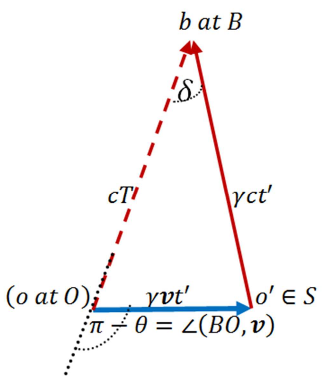

Figure 2. *Seeing* $\boldsymbol{OB}$ *from* $b \in s$ *at* $B \in S$.

Remarks

1. When looked from s at the vector $\boldsymbol{OB}$ (i.e. from B) (*Figure* 2), it is seen to coincide with $\boldsymbol{o'B}$ which makes with $\boldsymbol{OB}$ the same angle δ, given by (2.1). Here, a pulse of light emanates from $o \in s$ when at $O \in S$ and heads to $B \in S$ where it is received by $b \in s$. By this moment, the source of light o is at $o' \in S$ and the path of the pulse, which is ob in s, coincides with $\boldsymbol{o'B}$ in S.

2. In aim to focus on our goal, which is the deflection of a light path when observed from a geocentric frame from its orientation in the heliocentric frame, we depicted in *Figure* 1 (and in *Figure* 3b) the views of S and s when s is the moving frame. The full picture is that s can be considered stationary while S is moving at velocity $(-\boldsymbol{v})$ in s, and the light's path $\boldsymbol{bo}$ in s is deflected by $(-\delta)$ when observed from S (*Figure* 3a) [10-13].

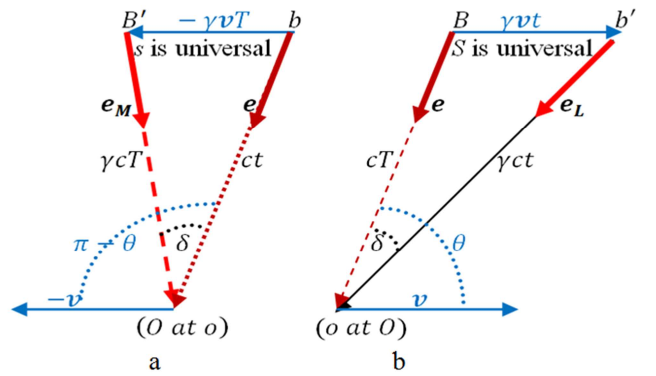

Figure 3. *a. The pulse's path is* $\boldsymbol{bo}$ *in s and* $\boldsymbol{B'o}$ *in S when s is stationary, b. The pulse's path is* $\boldsymbol{BO}$ *in S and* $\boldsymbol{b'O}$ *in s when S is stationary.*

3. Ray's Direction in a Moving Frame – Aberration Angle

Consider a beam of light propagating in the universal frame S parallel to the direction $\boldsymbol{e}$. The path's direction of a narrow beam, or a ray, is determined in S by a vector $\boldsymbol{BO} \parallel \boldsymbol{e}$, where B and O are points in S, say two small rings through which the ray passes. We showed in section 2 that the path of every light's ray in the moving frame s (determined also by two rings) is tilted towards the direction of the frame s' velocity $\boldsymbol{v}$. A segment $\boldsymbol{BO_1}$ of the beam's path in S will be seen in the moving frame s to coincide with $\boldsymbol{b'_1O_1}$, and the segment $\boldsymbol{BO}$ is seen in s to coincide with $\boldsymbol{b'O}$ when light from $b \in s\ at\ B \in S$ arrives at O (*Figure* 4). We assume that only a part of the beam travelling in the direction $\boldsymbol{e}$ in S is picked at O_1 while the remaining part continues its path to O. At the latter instant, the s' path which coincided earlier with $\boldsymbol{b'_1O_1}$ is now coinciding with $\boldsymbol{b'o'_1}$. Note that receiving the same ray (from $b \in s\ at\ B \in S$) by s observers at different points of $\boldsymbol{BO}$ takes place at consecutive times. For instance, light arrives at O_1, $\gamma\,|\mathrm{b}'_1\boldsymbol{O_1}|/c$ seconds after arriving at B, and it is received at O, $\gamma\,|\boldsymbol{o'_1O}|/c$ seconds after being received at O_1. At the instant of the final reception (which takes place at O) all partial paths in s, considered above, are aligned on the same vision line in s which coincides with $\boldsymbol{Ob'}$ in S. At any point, say $B \in S$, of the ray one could have chosen the segment which precedes B to observe the ray's direction from s, and the result would be: the ray's direction in s is already tilted from its direction in S.

Aberration plane: The path of the incoming ray (say $\boldsymbol{BO}$) in S and the vector velocity $\boldsymbol{v}$ of s define a plane in S, and in s, which we call the aberration plane the ray. Because the latter plane contains the ray's path in s, the aberration plane of a ray is that which contains the moving frame's velocity vector and the path of the ray in either frame. An incoming beam of light originating from a distant star α can be seen by a telescope in s with ocular and objective lenses r and o respectively, only if the telescope $\boldsymbol{ro}$ is set along the direction $\boldsymbol{ob'}$ which is tilted from the direction $\boldsymbol{OB}$ towards the direction of motion, by the aberration angle δ (*Figure* 5). In contrast, the same star α is seen by a telescope in S if oriented along $\boldsymbol{OB}$.

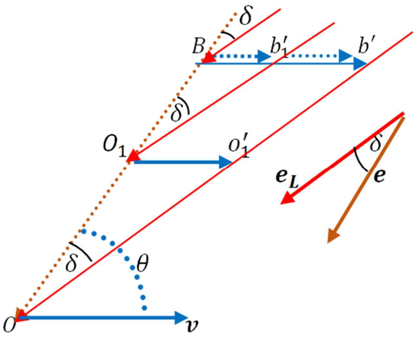

Figure 4. *The beam's path (in brown) which is determined in S by $O \in S$ and the direction **e**, is seen (in red) in s tilted by the same angle δ regardless of the point from which it is observed.*

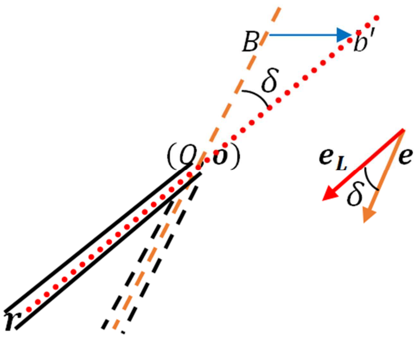

Figure 5. *The orientation of a telescope $\boldsymbol{ro}$ in the moving frame s, that allows visioning a distant celestial object α, is tilted from the object's vision line direction in S by the aberration angle δ. The dashed black double lines represent an imaginary telescope in the frame S.*

4. Stellar Aberration

Consider a "distant" star α, in the sense that the radius of the earth's orbit is negligible in comparison with the distance between the sun and the star α. In this context, the phrase "the vicinity of the sun at some instant T_0" will mean [15] the region of space containing the sun and Earth and whose dimensions remain negligible in comparison with the distance between the sun and the star α throughout a long period of time. In the stationary inertial frame $S \equiv NXYZ$ with origin at the sun's center "N" and the coordinate plane XY in the ecliptic, the motion of α, as seen from the sun's vicinity, has no observable effect on its location in S during a relatively long period of time (centuries), and in particular, on the angle Θ, between $N\alpha$ and the ecliptic (ecliptic latitude), which remains almost unchanged. The Z-axis of the heliocentric frame S is perpendicular to the ecliptic, the X-axis passes through the Earth's center at vernal equinox, and the Y-axis direction is in accordance with S being right-handed. The zero of timing may be chosen at some vernal equinox. The latter special heliocentric frame is called an "ecliptic frame" [1, 16].

Rays from α received by all S observers in vicinity of the sun are practically parallel, and the star α appears to all these observers at the same ecliptic latitude Θ. Let $s \equiv Exyz$ be a geocentric frame whose axes are parallel to S' axes. The results obtained in s are naturally valid in all geocentric frames that results from s through rotations or translations, such as equatorial and horizontal frames (see [1] for definitions).

All S-observers (in vicinity of the sun) see the rays from the star α throughout the year coming along the direction $\boldsymbol{e} = -\boldsymbol{e}'$, where $\boldsymbol{e}'$ is the unit vector of $\boldsymbol{N\alpha}$. For a geocentric observer, which is moving around the sun, the direction of the received ray is tilted from its direction in S in the momentary aberration plane by the aberration angle $\delta(t)$, where

$$\sin\delta(t) = \frac{v}{c}\sin\theta(t) \tag{8}$$

and $\theta(t)$ is the angle between the fixed direction $\boldsymbol{N\alpha}$ in S and Earth's momentary orbital velocity $\boldsymbol{v}(t)$ around the Sun. Consequently, a star α can be seen by a terrestrial telescope $\boldsymbol{ro}$ only if it is tilted from the fixed direction $\boldsymbol{N\alpha}$ in S towards Earth's velocity by the aberration angle δ. The vector velocity $\boldsymbol{v} = v\boldsymbol{e}_t$ of the earth around the sun, with $\boldsymbol{e}_t$ is the unit tangent vector to Earth's orbit, rotates approximately uniformly in S and in the geocentric frame s, with an angular velocity $\omega = 2\pi\, rad/year$; its magnitude v is almost constant. There follows that the aberration plane containing this rotating vector, the fixed direction $\boldsymbol{e}'$, and the tilted direction $\boldsymbol{ro}$, rotates about $\boldsymbol{N\alpha}$ in S and about $\boldsymbol{E\alpha}$ in s with angular velocity $\omega = 2\pi\, rad/year$. Assuming Earth's orbit is approximately circular, which is equivalent to an almost constant magnitude v of the orbital velocity, the components of the unit vector $\boldsymbol{e}'$ of the negative direction of the incoming ray from the distant star (Θ, Φ), in S or s, are:

$$\boldsymbol{e}' = (\cos\Theta\cos\Phi, \cos\Theta\sin\Phi, \sin\Theta) \tag{9}$$

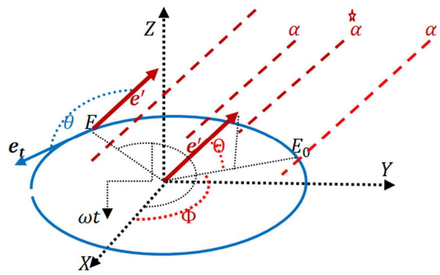

Figure 6. ***Nα*** *and* ***Eα*** *are essentially parallel because the star α is too far. The true position of α is along the same direction **e'** in the heliocentric and geocentric frames S and s.*

The components of the earth's orbit tangent vector $\boldsymbol{e}_t$ are

$$\boldsymbol{e}_t = (\cos(\tfrac{1}{2}\pi + \omega t)), \sin(\tfrac{1}{2}\pi + \omega t), 0)$$
$$= (-\sin\omega t, \cos\omega t, 0). \tag{10}$$

The cosine of the angle $\theta \equiv \angle(\boldsymbol{e}', \boldsymbol{e}_t)$ between Earth's vector velocity and the negative direction of the incoming ray is

$$\cos\theta = \boldsymbol{e}'.\boldsymbol{e_t} = \cos\Theta \sin(\Phi - \omega t). \quad (11)$$

Identifying $sin\,\delta$ by δ, we get from (8)

$$\delta^2 \approx \sin^2\delta = \left(\frac{v}{c}\right)^2 (1 - \cos^2\Theta \sin^2(\omega t - \Phi))$$

$$= \left(\frac{v}{c}\right)^2 (1 - \cos^2\Theta \sin^2\varphi), \quad (12)$$

where $\varphi = \omega t - \Phi = \angle(\boldsymbol{NE_0}, \boldsymbol{NE})$. Because Earth is nearest to the star (Θ, Φ) when it is at the position E_0 of longitude Φ in the ecliptic, φ measures the angle between the current position vector of the Earth in its orbit and its position vector when closest to the star α. Moreover, the velocity of Earth in its closest position to the star is perpendicular to the plane which contains the sun's center, Earth's center, and the star α. At the latter position the incoming ray is perpendicular to Earth's velocity: $\boldsymbol{e_t}.\boldsymbol{e}' = 0$.

For a given distant star the ecliptic latitude Θ is fixed, and the relation (12) determines the aberration angle δ in terms of the earth's position in its orbit in the ecliptic, or equivalently, at any instant throughout the year. The Earth's velocity vector reverses direction each half a year which yields the angle

$$2\delta = 2\,\frac{v}{c}(1 - cos^2\Theta \sin^2\varphi)^{1/2} \quad (13)$$

between two lines of sight separated by this period.

The aberration constant κ [1] is by definition the maximum apparent annual displacement of a star or a celestial object due to Earth's orbital motion around the sun; its generally accepted measured value [1] is $\kappa = 20.49551"$. By (13) the aberration angle attains its maximum value

$$\delta_{emax} = v/c \quad (14)$$

for $\sin\varphi = 0$, which corresponds to $\varphi = 0\ or\ \varphi = \pi$, and its minimum value

$$\delta_{emin} = \frac{v}{c}\sin\Theta = \delta_{emax}\sin\Theta \quad (15)$$

for $\varphi = \frac{1}{2}\pi\ or\ \varphi = \frac{3}{2}\pi$. Substituting in (14) by $c \approx 299792.458\ km/s$ and $v \approx 29.78\ \frac{km}{s}$ [1, 17] we obtain the value

$$\delta_{emax} \approx \frac{v}{c} = \frac{29.78}{299792.458km} rad$$

$$\times 57.2958 \frac{deg}{rad} \times 3600 \frac{sec}{deg} = 20.4894"$$

for the aberration constant, which is about $0.0061''$ less than its generally accepted value quoted above.

The obtained value of κ was gotten under the assumption that Earth's orbit is circular, while in reality the eccentricity of the elliptical Earth's orbit varies in cycle extending to hundreds thousands of years [1] from $0.0034\ to\ 0.058$; it is currently 0.0167. Moreover, the Earth's spinning motion was not taken into account.

Aberration Observed from a Planet: Provided the orbits are approximately circular and the observed celestial object is very far from the sun, the aberration constants δ_{emax} and δ_{pmax} for Earth and another solar planet p are related by [13]

$$\frac{\delta_{emax}}{\delta_{pmax}} = \frac{v_e}{v_p} = \sqrt{\frac{r_p}{r_e}}, \quad (16)$$

where $v_p\ and\ r_p$ $(v_e and\ r_e)$ are the planet's (the earth's) velocity and distance from the sun respectively. Equation (16) states that the aberration constant is inversely proportional to the square root of the planet distance from the sun. The first equality in (16) follows from two equation of the form (14) for Earth and for a planet; the second follows from the equality of centripetal and gravitational accelerations of a planet: $v^2/r = GM/r^2$, where G is the gravitational constant and M is the Sun's mass.

Recalling that the telescope in s is tilted by δ towards Earth's orbital velocity vector, the observed ecliptic latitude of the star is highest for $\varphi = \frac{1}{2}\pi$ where Earth would be receding from the star and lowest for $\varphi = \frac{3}{2}$ where it would be approaching it. The ecliptic latitude of the star at $\varphi = 0$ or $\varphi = \pi$ remains unchanged because the telescope is tilted horizontally by δ_{emax}. The latter longitudes correspond to the nearest and furthest positions of Earth from the star respectively.

Two Special Cases

The Ecliptic Poles: If the star α is at either ecliptic pole, $(\Theta = \pm\pi/2\,)$, the line of sight to α would be perpendicular to the ecliptic, and the Earth's velocity would be perpendicular to the incoming beam of light $(\theta = \angle(\boldsymbol{v}, \boldsymbol{e}') = \pi/2)$. In this case, both equations (8) and (12) reduce to $\delta = v/c$.

The Ecliptic: If the distant star α is in the ecliptic, then setting $\Theta = 0$ in (12) yields

$$\delta \approx \sin\delta = \frac{v}{c}\cos(\omega t - \Phi). \quad (17)$$

The plane of aberration in this case is the ecliptic plane itself, and the star appears to oscillate harmonically in the celestial plane, perpendicular to the line of sight, about its longitude Φ with a period of one year. The aberration angle attains its maximum absolute value (14) at $\varphi \equiv \omega t - \Phi = 0, \pi$ (closet and furthest locations from the star), and vanishes $(\delta = 0)$ for $\varphi = \pi/2\,, 3\pi/2$. I.e. a maximum absolute value for δ occurs when the line of sight to the star is perpendicular to Earth's velocity and a minimum value (= zero) when it is along it.

A Final Remark: Because Earth revolves almost uniformly around the sun in the ecliptic, observations of the celestial sphere should have an approximate rotational symmetry about $NZ\ (or\ EZ)$. In particular, entities pertaining to aberration of a celestial object may depend on its latitude but not on its longitude, despite the latter may appear as a zero level for specifying Earth's positions on its orbit, i.e., in functions of the term $\varphi \equiv \omega t - \Phi$ which measures the angle between the position vectors of Earth at an instant t and its nearest position to a star of longitude Φ.

5. Airy Telescope Experiment

According to this experiment [18] one expects that if the telescope $\boldsymbol{ro}$ is filled with water, the aberration angle will increase by about one third. This expectation is based on the fact that the speed of light in water is about $3c/4$ which should yield a new aberration angle $\delta' \approx v/(3c/4)\sin\theta = (4/3)\,\delta$.

This expectation is refuted by the fact that the ray which enters the telescope is already tilted by an angle δ from its direction in the frame S in the direction of Earth's velocity. Thus the incoming ray has a normal incidence on the telescope, which wouldn't change if the telescope is filled with water. Like most physical observables, a ray's direction is dependent on the frame in which the measurements are performed, and its outcome in s is true as much as is its outcome in S.

The latter argument meets with Pauli's who considers in his book "Relativity Theory" [19] the negative result of Airy's experiment as self-evident when looked at from the rest frame of Earth s. In s, the received waves from the star should have normal incidence on the telescope which points to the apparent position of star. This fact persists even if the telescope is filled with water, and Airy Experiment is reduced to the clear fact that for a zero angle of incidence the refraction angle is also zero.

6. Aberration Correction Vectors

The vision direction (or apparent direction) is the momentary direction along which the star is seen from Earth. Being in the aberration plane, its unit vector $\boldsymbol{A}$ is a linear combination of the fixed direction $\boldsymbol{e}'$ and the unit vector of Earth's velocity, $\boldsymbol{e_t}(t)$. The aberration correction vector $\boldsymbol{\delta}(\mathrm{t}) = \boldsymbol{A}(\mathrm{t}) - \boldsymbol{e}'$ is of length δ given by (12), lies in the aberration plane, and makes an acute angle with earth's velocity $v\boldsymbol{e_t}$ [13]. Being in the aberration plane, $\boldsymbol{\delta}$ is a linear combination of $\boldsymbol{e}'$ and $\boldsymbol{e_t}$, and it can be expressed in the bases $(\boldsymbol{ijk})$ of unit vectors of the frame $s \equiv Exyz$ as follows:

$$\boldsymbol{\delta} = \varrho\boldsymbol{e}' + \sigma\boldsymbol{e_t} = (\varrho\cos\Theta\cos\Phi - \sigma\sin\omega t)\boldsymbol{i}$$
$$+(\varrho\cos\Theta\sin\Phi + \sigma\cos\omega t)\boldsymbol{j} + \varrho\sin\Theta\,\boldsymbol{k}, \quad (18)$$

where we have used the expressions (9) and (10) for vectors of concern. A component of $\boldsymbol{\delta}$ along $\boldsymbol{e}'$ wouldn't contribute to a change in its direction, and hence we may take the correction $\boldsymbol{\delta}$ perpendicular to the star's fixed direction $\boldsymbol{e}'$. The parameters ϱ and σ are thus determined by the conditions, $\boldsymbol{\delta} \perp \boldsymbol{e}'$ and $|\boldsymbol{\delta}| = \delta$. Utilizing (13) we get after little calculations,

$$\varrho = (v/c)\sin(\omega t - \Phi)\cos\Theta,\ \sigma = (v/c), \quad (19)$$

and hence

$$\boldsymbol{\delta}(t) = \frac{v}{c}\{[\cos^2\Theta\cos\Phi\sin(\omega t - \Phi) - \sin\omega t]\boldsymbol{i}$$
$$+[\cos^2\Theta\sin\Phi\sin(\omega t - \Phi) + \cos\omega t]\boldsymbol{j}$$
$$+\sin\Theta\cos\Theta\sin(\omega t - \Phi)\,\boldsymbol{k}\}. \quad (20)$$

The aberration vector $\boldsymbol{\delta}$ is what should be added to the direction $\boldsymbol{e}'$ to obtain the vision direction $\boldsymbol{A}(t) = \boldsymbol{\delta}(t) + \boldsymbol{e}'$ along which the star is seen in s. The vision direction $\boldsymbol{A}$ is normalized to the second order in δ, for, $A^2 = e^2 + 2\,\boldsymbol{\delta}.\boldsymbol{e}' + \delta^2 = 1 + \delta^2 = 1 + 0(\delta^2)$. In order to see the star, our telescope should point along the vision vector $\boldsymbol{A}(t) = \boldsymbol{\delta}(t) + \boldsymbol{e}'$.

It is to be noted that the star's fixed direction $\boldsymbol{e}'$ is specified from start by two vision vectors separated by half a year,

$$\boldsymbol{e}' = \tfrac{1}{2}[\boldsymbol{A}(t_0) + \boldsymbol{A}(t_0 + \tfrac{1}{2}year)] \quad (21)$$

The longitudinal and latitudinal angles defined by this vector are the star α ecliptic coordinates (Θ, Φ) which we already started with.

Special Earth's Positions Relative to the Star:

1. For a given star (Θ, Φ), Φ is a constant, and hence

$$\Delta\varphi \equiv \Delta(\omega t - \Phi) = \pi \leftrightarrow \Delta t = \pi/\omega = \tfrac{1}{2}year. \quad (22)$$

The symbol $(\leftrightarrow)$ means the two statements are equivalent. Utilizing (22) we obtain from (20),

$$\boldsymbol{\delta}\big(t + \tfrac{1}{2}year\big) = -\boldsymbol{\delta}(t), \quad (23)$$

which affirms that the aberration correction vector $\boldsymbol{\delta}$ has opposite directions for locations on Earth's orbit separated by half a year.

2. In particular, for $\varphi = 0$ *and* $\varphi = \pi$, or equivalently, $\omega t_1 = \Phi$ and $\omega t_2 = \Phi + \pi$, equation (20) yields

$$\delta(\varphi = 0) = \tfrac{v}{c}(-\sin\Phi\,\boldsymbol{i} + \cos\Phi\,\boldsymbol{j}) = -\delta(\varphi = \pi). \quad (24)$$

The normal to the plane $\Phi =$ constant is

$$(-\sin\Phi, \cos\Phi, 0) = (-\sin\omega t_1, \cos\omega t_1, 0), \quad (25)$$

which by (24) is along $\delta(\varphi = 0)$, and by (10) is the orbit's tangent vector at the nearest position to the star α. Similar fact holds for the furthest location from α, and we deduce that at the nearest and furthest locations from α the aberration correction vector is perpendicular to the aberration plane, and in particular, to the line of sight to α. Therefore, a telescope at either of these locations is tilted (horizontally) towards the Earth's velocity by $\delta = v/c$.

3. For $\varphi = \tfrac{1}{2}\pi$ and $\varphi = \tfrac{3}{2}\pi$ we have

$$\delta\left(\tfrac{1}{2}\pi\right) = \tfrac{v}{c}[(-\sin^2\Theta\cos\Phi, -\sin^2\Theta\sin\Phi, \sin\Theta\cos\Theta) = -\delta\left(\tfrac{3}{2}\pi\right) \quad (26)$$

For $0 < \Theta < \frac{1}{2}\pi$, the z-component would be positive for $\varphi = \tfrac{1}{2}\pi$ and negative for $\varphi = \tfrac{3}{2}\pi$, which means that the telescope should be tilted upward at the first location and downward at the second. If $-\frac{1}{2}\pi < \Theta < 0$, the telescope should be tilted opposite to the z-axis at $\varphi = \tfrac{1}{2}\pi$ and towards it at $\varphi = \tfrac{3}{2}\pi$, which correspond again for a telescope in the southern ecliptic hemi- sphere to be tilted up and down respectively.

7. Apparent Elliptic Trajectory

In this section we prove that distant celestial objects appear to draw small elliptic trajectories, which is a well-known observational fact. Utilizing $\omega t = (\omega t - \Phi) + \Phi$, we write (6.3) in the form

$$\boldsymbol{\delta}(t) = \frac{v}{c}[-\sin^2\Theta\cos\Phi\sin(\omega t - \Phi) + \cos(\omega t - \Phi)\sin\Phi]\boldsymbol{i}$$

$$+\frac{v}{c}[-\sin^2\Theta\sin\Phi\sin(\omega t - \Phi) - \cos(\omega t - \Phi)\cos\Phi]\boldsymbol{j}$$

$$+\frac{v}{c}\sin\Theta\cos\Theta\sin(\omega t - \Phi)\,\boldsymbol{k}. \quad (27)$$

It is useful to decompose the aberration correction vector into a horizontal (east-west) and "vertical" (north-south) components,

$$\boldsymbol{\delta} = \boldsymbol{\delta_h} + \boldsymbol{\delta_v}, \quad (28)$$

where

$$\boldsymbol{\delta_h} = \frac{v}{c}\cos(\omega t - \Phi)\,\boldsymbol{e_h}, \quad (29)$$

$$\boldsymbol{\delta_v} = \frac{v}{c}\sin\Theta\sin(\omega t - \Phi)\,\boldsymbol{e_v}, \quad (30)$$

and where $\boldsymbol{e_h}$ and $\boldsymbol{e_v}$ are the unit vectors

$$\boldsymbol{e_h} = \sin\Phi\,\boldsymbol{i} - \cos\Phi\,\boldsymbol{j}, \quad (31)$$

$$\boldsymbol{e_v} = -\sin\Theta\cos\Phi\,\boldsymbol{i} - \sin\Theta\sin\Phi\,\boldsymbol{j} + \cos\Theta\,\boldsymbol{k}. \quad (32)$$

The vectors $\boldsymbol{e_h}$ and $\boldsymbol{e_v}$ exist everywhere except at the ecliptic poles where Φ becomes undefined. The horizontal and vertical components change sinusoidally with the same frequency ω but with different amplitudes, (v/c) and $(v/c)\sin\Theta$ respectively (we assume $\Theta \geq 0$); the horizontal component is $\pi/2$ ahead in phase from the vertical one, so whenever one component attains an extremum the other vanishes.

It can be checked that the triad of unit vectors $\{\boldsymbol{e'}, \boldsymbol{e_h}, \boldsymbol{e_v}\}$ is mutually orthogonal, and consequently so is the set $\{\boldsymbol{e'}, \boldsymbol{\delta_h}, \boldsymbol{\delta_v}\}$; the horizontal and vertical correction vectors are both perpendicular to the direction of the incoming beam in *S*. From the parametric representation (29) and (30), the aberration correction vector displays the ellipse

$$\frac{\delta_h^2}{(v/c)^2} + \frac{\delta_v^2}{(v/c)^2\sin^2\Theta} = 1. \quad (33)$$

Likewise, the vision vector $\boldsymbol{A} = \boldsymbol{e'} + \boldsymbol{\delta}$ draws the same ellipse, and the observed star appears to follow a small elliptical trajectory in the celestial sphere with horizontal major axis of length v/c, vertical minor axis of length $(v/c)\sin\Theta$, and center (Θ, Φ).

Being free of the star's longitude Φ, the ellipse (33) may refer to the apparent trajectory of any star of latitude Θ and arbitrary longitude. Indeed, under the assumption of a circular orbit for Earth, all trajectories with the same Θ may coincide on one another through rotations about *Ez*. The elliptic trajectories approach circles near the poles and horizontal straight segments near the ecliptic's equator.

8. Aberration in Graded Inertial Frames

When *S* and *s* are equally inertial the star is seen in each frame when taken universal along the same direction $\boldsymbol{e'}$ [13]. If *S* (*s*) is universal, which implies that the other frame is moving, the line of sight to the star in *s* (*S*) is tilted in the direction of the velocity of *s* relative to *S*, namely $\boldsymbol{v}$, (*S* relative to *s*, namely $-\boldsymbol{v}$,) from $\boldsymbol{e'}$ by the aberration angle δ $(-\delta)$, (*Figure* 3).

The vision direction in the geocentric frame *s* is determined with reference to a fixed direction in the heliocentric frame *S*. It is known however that our sun revolves about the center of the galaxy [1] at about 8 times the earth's orbital velocity and makes a full round in about 230 million years (*a galactic year*). As observed from the sun, the aberration angles of an extragalactic object is given by an equation like (8) with *v* is replaced by 8*v*. Moreover, provided the sun's motion is planar and approximately circular, the corresponding aberration correction vector would be given by equation like (27) in which v is replaced by $8v$, $\omega \approx 2\pi\, rad/230$ million years $\approx 2.73 \times 10^{-8} rad$ per year, Θ is the latitude of the extragalactic object with respect to the Sun's plane of motion in the Galaxy, and Φ is its longitude relative to some starting point on the sun's orbit. The vision vector of an extragalactic object traces during a *galactic year* an ellipse that is similar to (33) but 8 times larger in both dimensions. The vision vector $\boldsymbol{A}(t)$ which points to the apparent position of the extragalactic object in the heliocentric frame *S* is practically fixed at a direction $\boldsymbol{A}_0$ for thousands of years, and the annual aberration (observed from Earth) determines the apparent position of the extragalactic object relative to $\boldsymbol{A}_0$ (symbolized earlier by $\boldsymbol{e'}$).

In reality no frame is exactly inertial, but there are *graded inertial frames* in which one frame is more inertial than another. For instance, the set of non-rotating frames with origins at the center of mass of, the galaxy (*G*), the solar system (*S*), the earth-moon system (*s*), an Earth's satellite (*sat*), is graded in the sense that: the motion of *S* in G can be counted uniform for thousands of years, whereas the duration of uniformity of Earth's motion in *S* ranges from minutes to hours depending on the required degree of accuracy, and finally the motion of *sat* in *s* is close to uniform only for seconds. The apparent position $\boldsymbol{e'}$ of a distant celestial object α in the heliocentric frame *S* works, for hundreds of years, as a true fixed direction to which is referred the apparent position $\boldsymbol{A_E}(t)$ of the same object α in the geocentric frame *s*. Moreover, the apparent position $\boldsymbol{A_E}(t)$ in *s* can be considered for few hours in the neighborhood of any instant t_i as a constant position $\boldsymbol{A_i} = \boldsymbol{A_E}(t_i)$ to which we refer the apparent position of α in an Earth's satellite. The deviation of the motion of a frame from uniformity during the period at which it is taken inertial is either undetectable, or its effect on the results of the experiment is negligible.

9. Observing Aberration from an Earth's Satellite

We argue here that apparent positions of a celestial object as observed from Earth can be adopted as transient fixed positions when observing the same object from a satellite.

Suppose that a satellite (a space station for instance) is orbiting the earth in a circular orbit without spin, and let Π be the plane of its motion in the geocentric frame s. We suppose that the satellite's orbit is low so that the revolution period τ is small enough to legitimate considering s during this period as inertial. During the orbital period τ of the satellite, which is about 93 minutes for the international space station (ISS) the geocentric frame $s \equiv Exyz$ is almost inertial, and the change in the apparent position of our distant star α is small $(41"\times 93/365\times 24\times 60\approx 0.0072")$; its apparent position is almost fixed in s during half a period. It follows that at any chosen instant of time t_i there exists an approximately fixed direction $\boldsymbol{A_E}(t_i) = \boldsymbol{A}_i$ in s along which the star α is seen during a short period of time $|t - t_i| \leq \tau/2$.

Imagine that our satellite is stationary at some point of its orbit and a telescope is mounted there to observe the star α. The velocity of the telescope would be extremely close to the orbital velocity of Earth and it would have the same orientation of a telescope on Earth when pointing to the star α. I.e., the apparent position of α as seen from the satellite would be the same as its apparent current position $\boldsymbol{A_E}(t_i) = \boldsymbol{A}_i$ from Earth. Now, the revolution of the satellite around Earth will add to $\boldsymbol{A}_i$ a correction that depends on the velocity of revolution. The rest of this section is devoted to pursue the latter issue.

The parallelism between aberration as seen from Earth and that seen from an Earth's satellite is clear. The satellite replaces Earth, Earth replaces the Sun, the satellite's plane of motion Π replaces the ecliptic, the pair (Θ, Φ) is replaced by the latitude and longitude (Θ', Φ') with respect to Π (and a direction ox' in it), the geocentric frames s and a frame attached to the satellite that do not rotate relative to s, call it a "*satcentric*" frame $sat \equiv ax'y'z'$, replace the heliocentric and geocentric frames S and s respectively, and finally, the transient vision direction $\boldsymbol{A_i}$ of the star in s replaces the fixed vision direction $\boldsymbol{e}'$ is S.

The satcentric frame $sat \equiv ax'y'z'$ may be duplicated by a copy that have parallel axes but with origin at Earth's center, $esat \equiv Ex'y'z'$. The axes of *sat* or *esat* are chosen as follows: Ez' (az') is perpendicular to the satellite's plane of rotation Π, Ex' (ax') is in the coordinate plane Exz, i.e. the intersection of the planes Π, and Exz, and Ey' (ay') points to the direction that yields *esat (sat)* right-handed. The transformation from *sat* to s is implemented by means of an orthogonal matrix M with determinant +1 that embodies the given latter data. Straight forward calculations yield,

$$M = \begin{pmatrix} \frac{\sqrt{1-\zeta^2-\eta^2}}{\sqrt{1-\eta^2}} & \frac{\zeta\eta}{\sqrt{1-\eta^2}} & \zeta \\ 0 & -\sqrt{1-\eta^2} & \eta \\ \frac{-\zeta}{\sqrt{1-\eta^2}} & \frac{\eta\sqrt{1-\zeta^2-\eta^2}}{\sqrt{1-\eta^2}} & \sqrt{1-\zeta^2-\eta^2} \end{pmatrix}. \quad (34)$$

The entries of the third column are the components of the unit normal to the plane Π, which is the unit vector $\boldsymbol{k}'$ of the axis z', in the system $Exyz$:

$$M\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}\zeta\\ \eta\\ \sqrt{1-\zeta^2-\eta^2}\end{pmatrix} \equiv \begin{pmatrix}\cos p\cos q\\ \cos p\sin q\\ \sin p\end{pmatrix}. \quad (35)$$

In the third column, (p, q) are the latitude and longitude angles of $\boldsymbol{k}'$ in the geocentric ecliptic system $Exyz$. In terms of (p, q) the matrix (34) can be written in the form

$$M = \begin{pmatrix} \frac{\sin p}{\sqrt{1-\cos^2 p\sin^2 q}} & \frac{\cos^2 p\cos q\sin q}{\sqrt{1-\cos^2 p\sin^2 q}} & \cos p\cos q \\ 0 & -\sqrt{1-\cos^2 p\sin^2 q} & \cos p\sin q \\ \frac{-\cos p\cos q}{\sqrt{1-\cos^2 p\sin^2 q}} & \frac{\cos p\sin p\sin q}{\sqrt{1-\cos^2 p\sin^2 q}} & \sin p \end{pmatrix} \quad (36)$$

Another way of writing the latter matrix utilizes the equality

$$\frac{\sin p}{\sqrt{1-\cos^2 p\sin^2 q}} = \frac{1}{\sqrt{1+\cot^2 p\, cos^2 q}}. \quad (37)$$

The transformation from s to *sat* is carried out by means of the transpose matrix M^t. If $\boldsymbol{A}$ is a vector in s, its expression in the frame *esat* or in *sat,* will be $M^t\boldsymbol{A}$.

Since our concern is confined to directions, which are vectors, the transformation between s and *esat* or the satcentric frame *sat*, is governed by the matrix M and do not involve the displacement vector

$$\boldsymbol{Ea} = h(\cos\omega' t\, \boldsymbol{i}' + \sin\omega' t\, \boldsymbol{j}'), \quad (38)$$

in which h is the distance between the satellite and Earth's centre and $\omega'$is the angular velocity of the satellite in s.

We iterate that the vectors

$$\boldsymbol{A_E}(t_i) = \boldsymbol{A}_i \; (i = 1, 2, 3, \ldots, t_{i+1} = t_i + \tau) \quad (39)$$

in s are all transient and each works as a fixed direction in *sat* only for the period $|t - t_i| \leq \tau/2$. The matrix M^ttransforms the latter set of vectors to the set of vectors

$$M^t\boldsymbol{A}_i \; (i = 1, 2, 3, \ldots, t_{i+1} = t_i + \tau) \quad (40)$$

in *sat.* The angles (Θ', Φ') of the star α, with respect to Π and the x' -axis in it, are either directly measured, or calculated through the transform $M^t\boldsymbol{e}$ of the vector $\boldsymbol{e}$. The vision's direction to α from the satellite is the sum of $M^t\boldsymbol{A_i}$ and the aberration correction vector $\boldsymbol{\delta}'$. Setting $t' = t - t_0$, where $t_0$is the instant marking a passage of the satellite from Ex'(i.e. from the plane Exz), which is also the passage of ax' from Earth's center, we get in *sat*, in parallel to the formulas for $\boldsymbol{\delta}(t)$ in s, $\boldsymbol{\delta}'(t') = \boldsymbol{\delta}'_{\boldsymbol{h}}(t') + \boldsymbol{\delta}'_{\boldsymbol{v}}(t')$, where

$$\boldsymbol{\delta}'_{\boldsymbol{h}}(t') = (v'/c)\, cos(\omega' t' - \Phi')\, \boldsymbol{e}'_h, \quad (41)$$

$$\boldsymbol{\delta}'_{\boldsymbol{v}}(t') = (v'/c)\sin\Theta'\sin(\omega' t' - \Phi')\, \boldsymbol{e}'_v, \quad (42)$$

and where $\boldsymbol{e}'_{\boldsymbol{h}}$ and $\boldsymbol{e}'_{\boldsymbol{v}}$ are the unit vectors

$$\boldsymbol{e}'_{\boldsymbol{h}} = sin\,\Phi'\,\boldsymbol{i}' - cos\,\Phi'\,\boldsymbol{j}' \quad (43)$$

$$\boldsymbol{e}'_v = -\sin\Theta'\cos\Phi'\,\boldsymbol{i}' - \sin\Theta'\sin\Phi'\,\boldsymbol{j}' + \cos\Theta'\boldsymbol{k}'. \quad (44)$$

In (41) and (42), $\omega' = 2\pi/\tau$ is the angular velocity of the satellite ($v' = h\omega'$ is its orbital velocity). The instantaneous vision direction $\boldsymbol{A}'_i$ from our satellite to α in its i^{th} round is

$$\boldsymbol{A}'_i = M^t\boldsymbol{A}_i + \boldsymbol{\delta}' \quad (45)$$

Note that the correction vector, which is periodic in time with a period τ, is the same in all rounds; it is independent of i. The telescope in *sat* should point in the direction $\boldsymbol{A}'_i$ in order to see the star α.

The aberration angle, which is the angle between the vision direction $\boldsymbol{A}'_i$ and the temporarily fixed direction $M^t\boldsymbol{A}_i$, is given by

$$\sin\delta' = \frac{v'}{c}\sin\theta'(t'), \quad (46)$$

where,

$$\theta'(t') = \angle(M^t\boldsymbol{A}_i, \boldsymbol{v}'(t')) = \angle(\boldsymbol{A}_i, M\boldsymbol{v}'(t')) \quad (47)$$

is the angle between the star's vision direction and the satellite's velocity at an instant t'. In *esat*

$$\boldsymbol{v}'(t') = v'\boldsymbol{e}'_t = v'(-\sin\omega' t'\,\boldsymbol{i}' + \cos\omega' t'\,\boldsymbol{j}') \quad (48)$$

where $\boldsymbol{e}'_t$ is the unit tangent vector to the satellite's orbit and v' is its constant velocity magnitude.

In parallel to (13) we may derive from (46),

$$\delta' \approx \sin\delta' = \frac{v'}{c}\sqrt{1 - cos^2\Theta'\sin^2\varphi'} \quad (49)$$

where $\varphi' = \omega' t' - \Phi'$. When half a period elapses the velocity of the satellite reverses direction, and with it the aberration angle, resulting in an angle $2\delta'$ between the corresponding two lines of sight from the satellite to α. The aberration angle takes its maximum $\delta'_{max} \approx v'/c$ (the aberration constant in the satellite) for $\varphi' = 0, \pi, 2\pi, \ldots$ and its minimum $\delta'_{smin} = (v'/c)\sin\Theta'$ for $\varphi' = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi$ .....

The star α is observed in *sat* to follow during a period τ the apparent elliptic path

$$\frac{(\delta'_h)^2}{(v'/c)^2} + \frac{(\delta'_v)^2}{(v'/c)^2\sin^2\Theta'} = 1 \quad (50)$$

with a major horizontal and minor vertical axes of lengths v'/c and $(v'/c)\sin\Theta'$ respectively. It is clear for a telescope in the satellite pointing to the star α, that the word "vertical" corresponds to tilting along az' while "horizontal" to tilting perpendicularly to az' and to the line of sight $a\alpha$.

The relation between the aberration constants in Earth and in a satellite is

$$\kappa'/\kappa = v'/v. \quad (51)$$

For a satellite, like ISS, whose speed is 7.66 *km/sec* our theory predicts the result

$$\kappa' = \kappa\frac{7.66}{29.78} \approx 20.4894'' \times 0.2572 \approx 5.2702'', \quad (52)$$

which is to be compared with experiment.

Satellites Connective Matrices

The transformation matrix between two satellites sat_1 and sat_2 revolving about Earth in circular orbits in the planes Π_1 and Π_2 respectively is called the connective matrix and denoted by M_{12}; it is obtained as follows. Let

$$\boldsymbol{k}_1 = (\cos p_1\cos q_1, \cos p_1\sin q_1, \sin p_1)^t \quad (53)$$

$$\boldsymbol{k}_2 = (\cos p_2\cos q_2, \cos p_2\sin q_2, \sin p_2)^t \quad (54)$$

be the unit normals to the planes Π_1 and $\Pi_2$respectively, and denote the transformation matrices from sat_1 and sat_2 to s by $M_1(p_1, q_1)$ and $M_2(p_2, q_2)$ respectively. It is evident that the transformation matrix from the satellite sat_1 to the satellite sat_2 is

$$M_{12}(p_1, q_1; p_2, q_2) = M_2^t(p_2, q_2)M_1(p_1, q_1). \quad (55)$$

The transformations from sat_2 to sat_1 are carried out by the matrix $M_{21} = M_{12}^{-1} = M_1^tM_2$. Earth may be given the index (0) and the connective matrix from a satellite sat_i to Earth is $M_{i0} = M_i$.

Connective matrices make all geometric information regarding the celestial sphere which are obtained in one satellite immediately available to observers on Earth and in all other satellites. For instance, if the current vision position of a distant star α in sat_1 is $\boldsymbol{P}'^{\alpha}_1$, then its apparent position from Earth is $M_1(\boldsymbol{P}'^{\alpha}_1 - \boldsymbol{\delta}_1)$ and its apparent position in sat_2 is

$$M_2^tM_1(\boldsymbol{P}'^{\alpha}_1 - \boldsymbol{\delta}_1) + \boldsymbol{\delta}_2 = M_{12}(\boldsymbol{P}'^{\alpha}_1 - \boldsymbol{\delta}_1) + \boldsymbol{\delta}_2, \quad (56)$$

where $\boldsymbol{\delta}_1$and $\boldsymbol{\delta}_2$ are the correction vectors in sat_1 and sat_2 respectively.

Finally, it is worth it to mention that we could have handled aberration in a satellite through referring its motion to the ecliptic heliocentric frame *S*.

10. Conclusion

The TUST theory provides a neat explanation of the stellar aberration phenomenon that highlights its independence of the light's source velocity, and provides a formula for the aberration angle at each instant of the year. The introduction of the aberration correction vector enables us to find an approximate formula for the apparent position of any celestial object at any instant of time. The introduced concept of graded inertial frames makes it understood why annual stellar aberration depends, to a great degree of accuracy, only on the velocity of Earth relative to the heliocentric frame, and legitimizes the approximate treatment of aberration when observed from a satellite in parallel with its treatment when observed from Earth. The derived satellites connective matrices publicize any geometric measurement regarding the distant celestial sphere that is obtained in one satellite to observers on Earth and in all satellites.

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