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Pensions and Growth: A Cointegration Analysis

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Abstract: This article investigates the long-term relationship between economic growth and old-age provision using time series analysis, particularly the techniques of cointegration. The neoclassical growth model by Solow (1956) provides atheoretical basis for the empirical analysis. The results are based onquarterly data from 1970 to 2013 for the US-economy. In this work, the existence of a cointegrating relation between economic growth and pensions is verified by use of scientifically accepted statistical methods and proven for historical US-data. The empirical analysis confirms that improved technological capabilities constitute a very important determinant of growth in the context of neoclassical theory. The effects within the cointegrated relationship cannot be determined at this point and there is no information if the effect is reciprocal or not. For this purpose, further investigations are necessary and can build on the results presented here.

Keywords: Growth, Old-Age Provision, Pensions, Time Series, Cointegration, Solow Model, Neoclassical Economics

1. Introduction

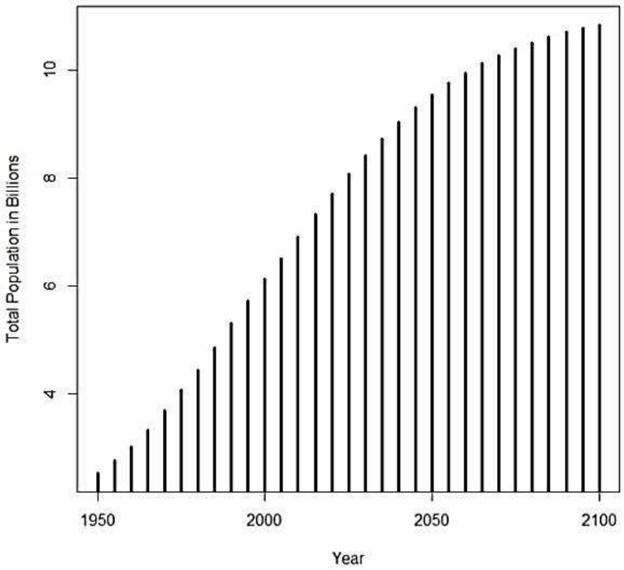
"We cannot continue. Our pension costs and health care costs for our employees are going to bankrupt this city." - Michael Bloomberg. [1] This quote from the year 2009 originates from the then mayor of New York City. It shows the urgency to deal with the economic contexts of aging theory and economic growth. There are other cities or municipalities all over the world, facing similar issues like the city of New York. Meanwhile, there are more than 640 known cases of insolvent cities in the United States of America. In 2007, during the Global Financial Crisis, Detroit, for instance, has slipped into the biggest bankruptcy of a municipality in American history, facing long-term debts amounting to several billion U.S. dollars. Furthermore, not only the current financing conditions, but also the foreseeable trends make the subject interesting for further research. Nowadays, the term demographic change is constantly present in the media. A look at current forecasts is sufficient to recognize the seriousness of the situation. Figures 1 and 2 show the predicted worldwide population dynamics and the percentage of pensioners up to the year 2100.[2] According to these forecasts, about ten billion people will live on the earth at the turn of the century. By then, approximately 21.89 % of the world's population will be older than 65, compared with 8.25 % in 2015. The detachment from the classical age pyramid is caused by many factors, such as low fertility and mortality as well as rising life expectancy.

However, the upcoming pension problem is not solely driven by poor financial circumstances in pension funds and the aforementioned demographic trends. There are umpteen economic determinants which exert influence. No matter, whether one looks at longer training periods, early retirement arrangements, declining wage shares, undeclared work, or emigration: The list of influences is extensive.

Fortunately, in recent decades, an economic perspective has been strengthened. Scientific evidence gains increasing importance in the ongoing debate about the best possible design for old-age security systems. The situation and the development of any social security system are subject to various interrelations, being very different for every stakeholder affected. For the analysis of social security

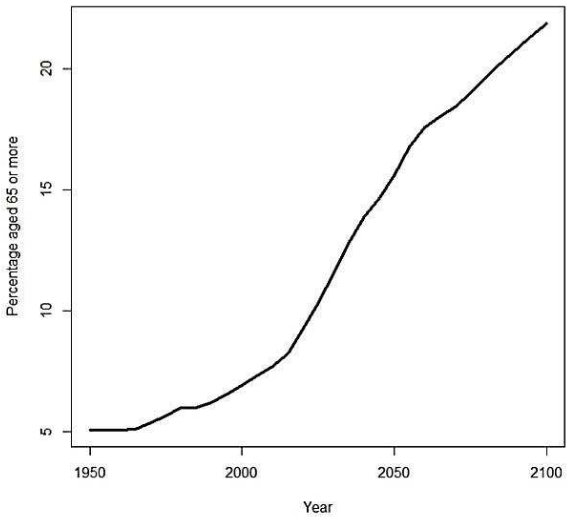
[1] Michael Bloomberg in comments broadcast on NY1 television on April 9th, 2009.
[2] The United Nations World Population Prospects: The 2012 Revision contains estimates since 1950 and projections until 2100 for every country in the world, including estimates and projections of 60 demographic indicators such as birth rates, deaths rates, infant mortality rates and life expectancy. A sample set of summary indicators is provided as part of UN data. The projections are based on a version with a medium fertility rate.

systems and the justification of reform steps, this means that adequate procedures for the respective issues are required. With regard to the pension problem, it must be clarified, which point of view is considered and which incentives arise from that. The disciplines involved may be of legal, social, political, economic, or even sociological nature.



Source: Own illustration based on United Nations, 2013

Figure 1. *Demographic Growth.*



Source: Own illustration based on United Nations, 2013

Figure 2. *Shift of Population Structure.*

This work examines the question, whether a long-term relationship between economic growth and retirement exists. The objective is to detect the influence of old-age security within a growth model to provide a basis for possible policy implications. The paper is organized as follows: Chapter 2 deals with the theoretical and mathematical foundations of growth theory, in this case the Solow model as a basis for empirical studies. The third chapter covers the description and the analysis of the data, the modeling strategy, the empirical results, and at last asummary. Methodological pluralism arises, as soon as different modeling approaches are available within a special field of work. Thus accurate reflections of the modeling strategy are necessary. In connection with this, the choice of the modeling approach and the operationalization is often influenced by the levels of detail, data bases, and time. Here, the applied data is presented, the statistical procedures are explained, the results are shown and ultimately a brief evaluation is carried out. The work ends with a comprehensive conclusion in which the research findings are evaluated in a more general sense and possible further steps are suggested.

2. Theoretical Framework

2.1. Solow Model

In this section, the mathematical growth model is derived. This work refers to Solow (1956), known for its extraordinary relevance even in modern, mathematically characterized growth theory. The modeling demonstrates the relationship between capital accumulation, technological progress, and productivity growth. Since models cannot provide conclusive results without simplifying assumptions, these will be presented briefly. An important admission is that Solow considered a closed economy, meaning that there are no external influences, i.e. no imports or exports. Hence, it is assumed that planned savings are equal to planned investments. This represents the macroeconomic equilibrium condition of the goods market. It eliminates the need to distinguish between gross domestic product (GDP) and gross national income (GNI). In addition to a balanced market for goods, also the factor market is assumed to be in equilibrium. This assumption guarantees that the markets in the modeled economy are efficient. Furthermore, all participants in this economy act rationally. Apart from that, only one homogeneous good is produced with two essential production factors. An investment in research & development (R&D) is not possible.[3] This assumption will be relaxed later on. The model itself can also be solved by considering a Cobb-Douglas production function.[4]

The starting point for the formal solution of the mathematical model is the production function $Y(t)$. This macroeconomic output at time t is a function depending on two arguments.[5] Firstly the capital stock K at time t, and secondly the number of workers L at time t, which may be interpreted as the size of the population. Since it is assumed that a constant shareof the total population is available as workforce, it is only necessary to determine the consideredproportion. On account of that the labor market can be regarded as cleared. The population growth rate n is assumed to be exogenous and constant, which also applies to the savings rate s. Consequently, the production function can

[3] This means, that no endogenous technological progress can be modeled at this point.

[4] For this formal approach, see Jones (2002).

[5] See Solow (1956).

be written as:

$$Y(t) = F(K(t); L(t)) \quad (2.1)$$

The neoclassical production function is linear homogeneous[6] and has positive, but diminishing marginal productivities.[7] These are required properties of the modeling structure.[8] By not considering the time indices, follows:[9]

$$Y = F(K; L) \quad (2.2)$$

Due to the shape of the production function, a part of the income is consumed and the rest of it is saved, respectively invested. This leads inevitably to the following "[...] basic identity at every instant of time:"[10]

$$\dot{K} = sY \quad (2.3)$$

A dot on a variable represents the derivation of the variable with respect to time and therefore reflects the change in this variable over time.[11] To provide more expressiveness, only per capita variables are considered in this model. Accordingly, the production function must be divided by the number of employees L. Hence, the second argument of the function is equal to 1 and must not be considered further. Then equation (2.2) can be written as:

$$y = f(k) \quad (2.4)$$

This means that the per capita output y is a function $f(k)$ with the argument k. It is defined as the capital intensity, i.e. the stock of capital per capita. This function, in turn, has some desirable properties known as "Inada Conditions".[12] The marginal productivities of both factors are positive, but decreasing and infinite in origin, converging to zero. Christiaans (2004) describes the motivation behind these conditions. If they are fulfilled, it exists one $k \in (0; 1)$ such that exactly one stable equilibrium k^* is reached with a time rate of change $\dot{k} = 0$. Those can be written as:

$$f' > 0\,;\; f'' < 0\,;\; f'(\infty) = 0\,;\; f'(0) = \infty \quad (2.5)$$

Without technological progress, the per capita output reaches the equilibrium level, if the capital intensity has reached a constant value, and thus remains the same over time. To determine this value, the variable must be derived logarithmically with respect to time. $\dot{k}$ describes the change in capital intensity over time:

$$\frac{dk}{dt} = \dot{k} \quad (2.6)$$

In this modeling framework savings are, as mentioned above, exogenous. In addition, a likewise exogenous depreciation rate δ is assumed.[13] Therefore, the aggregate savings $S(t)$ correspond to a constant share of income for the period t:[14]

$$S = s\,Y = s\,F(K; L) = I + \delta\,K = \dot{K} + \delta\,K \quad (2.7)$$

The proportion of saved income in a period corresponds to the investments I, combined with the portion of depreciation and amortization δ of the current capital stock. Consequently, the savings of one period corresponds exactly to the change in the capital stock over time $\dot{k}$ and the respective depreciation of this period. Equation (2.7) can now be reorganized and divided by L, resulting again in a per capita expression:

$$\frac{\dot{K}}{L} = s\,f(k) - \delta\,k \quad (2.8)$$

Now, the left side of the equation can be expanded with $\frac{L}{L}$ and subsequently $\frac{\dot{L}K}{L^2}$ can be subtracted from both sides, so that the following equation is obtained:

$$\frac{\dot{K}L}{L^2} - \frac{\dot{L}K}{L^2} = s\,f(k) - \delta\,k - g_L k \quad (2.9)$$

A further simplification, by dissolving and merging, leads ultimately to the "core equation" of the model:

$$\dot{k} = s\,f(k) - (\delta + g_L)k \quad (2.10)$$

Desired is a long-term stable equilibrium. This can be represented by the Solow diagram. How this equilibrium can be achieved will be illustrated in Figure 3.[15] By assumption, the economy starts at point k_0. The difference Δ is the difference between the amount of investment per capita[16] and the capital intensity lost through depreciation and population growth.[17] Δ corresponds to the change in capital per worker. In our initial endowment k_0, the investments are greater than what is needed to keep the amount of capital per capita constant.[18] This means that the capital stock per capita increases, so $\dot{k}$ is positive. Accordingly this difference will be lower in the next period. This mechanism is repeated in subsequent periods, exactly until a capital intensity of k^* is reached. At this point, the temporal change in capital stock per capita is equal to zero. The same principle also applies for a starting capital intensity $k_0 > k^*$. The only difference is a declining capital stock per capita until the equilibrium

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[6] The production function has constant returns to scale, e.g. doubling the input also doubles the output.

[7] Formally: $F_K > 0$; $F_L > 0$; $F_{KK} < 0$; $F_{LL} < 0$.

[8] See Christiaans (2004).

[9] This is merely intended to preserve the clarity and ease of understanding of the mathematical modeling. Nevertheless, it should be taken into account that variables such as output, capital stock, or number of workers are always time-dependent.

[10] Solow (1956), p. 66.

[11] Thus: $\dot{K} = \frac{\partial K}{\partial t}$

[12] See Christiaans (2004).

[13] At this point, deviating from Solow (1956), the modeling of depreciation is considered, as it is frequently done in the secondary literature.

[14] The time indices will again remain unconsidered for reasons of clarity.

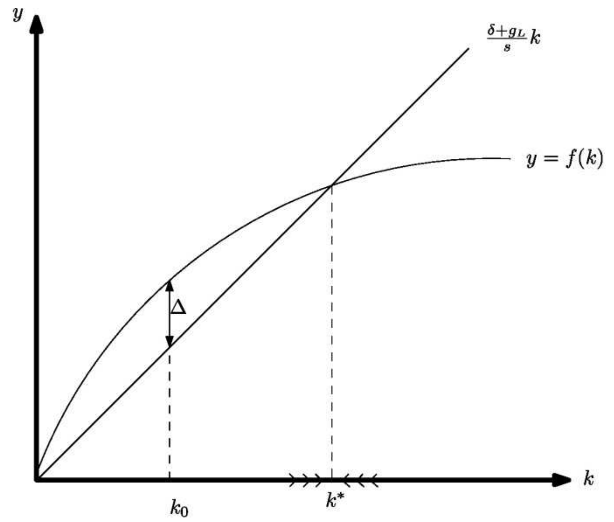
[15] In some textbooks, the production function will be shown in the same diagram. Using this function allows to determine the output per capita and the consumption per capita. It is calculated as the difference between steady state and production function. Since this does not contribute to a further understanding of the context, it will be omitted at this point.

[16] This is the curve labeled $sf(k)$.

[17] This is the curve labeled $(\delta + g_L)k$.

[18] See Jones (2004).

value k^* is reached.

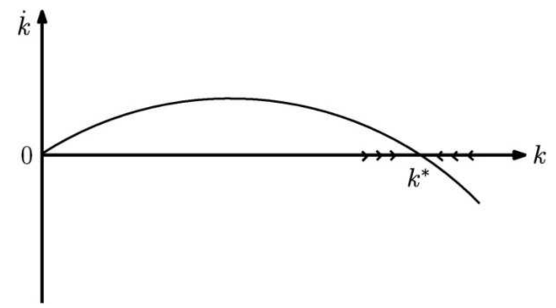


Source: Own illustration based on Solow (1956)

Figure 3. *Solow Diagram.*

A slightly different representation is shown in Figure 4. There, the capital intensity is plotted on the abscissa, instead of the output per capita. Once more, the results from above are comprehensible. On the left side of the equilibrium value of k^*, the capital intensity will increase, on the other side, the capital stock per person will decrease. At equilibrium, the temporal change in capital intensity is equal to zero and the long-term equilibrium is obtained.

In the neoclassical growth equilibrium, all growth rates are constant. This long-term equilibrium is often referred to as "steady state" or state equilibrium growth respectively. If the production function is neoclassical and satisfies the "Inada Conditions", there exists a unique and globally stable long-term equilibrium for starting values $k_0 > 0$. In the basic neoclassical model, income, consumption, and capital stock increase at the rate of population growth plus depreciation. Conversely, if not all growth rates are constant, it is called unbalanced growth. [19] If our exogenous variables are constant,[20] a long-term equilibrium can only occur via an upward shift in the production function. One way to achieve this would be the introduction of technological progress.



Source: Own illustration based on Christiaans (2004)

Figure 4. *Equilibrium Capital Intensity.*

[19] See Christiaans (2004).

[20] In this case, the exogenous variables are: g_L, δ and s.

The simple Solow model can explain the period of adjustment to the long-term equilibrium. However, by taking technological progress into account, an increase of income can be explained. Hereafter, the technological progress is labeled with the variable A. The output function from above will be maintained but is multiplied by the technological progress.

$$Y = A\,F(K, L) \tag{2.11}$$

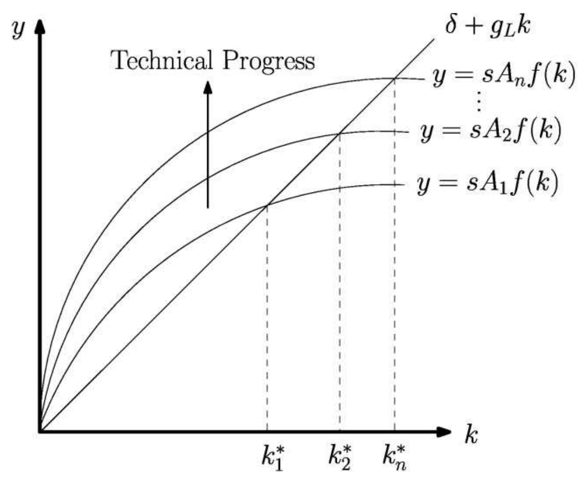
Expressed in a per capita basis, this results in:

$$y = A\,f(k) \tag{2.12}$$

After performing the same processing steps as in the previous section, the following equation results:

$$\dot{k} = s\,f(k) - (\delta\ + g_L)\,k \tag{2.13}$$

Figure 5 presents the Solow model with the consideration of technological innovations. In contrast to Figure 3, it can be seen that there will be upwards shifts in the production function with increasing technological knowledge. It is assumed that the growth rate of technological progress is constant and exogenous. In the figure below three levels of progress have been exemplified. [21] Increasing knowledge induces an upward shift of the production function. The reason is that increasing technological knowledge combined with a constant savings rate implies a higher output. Reasons could be the labor force using the input factors more efficiently, or more advanced machines producing more output with the same amount of energy. Solow (1956) has shown that only in the presence of technological progress sustained capital accumulation is followed by long-term productivity growth at a constant rate of profit.[22] If there is technological progress, the output per capita and capital intensity increase with the growth rate of technological progress.



Source: Own illustration based on Barro und Sala-i-Martin (2004)

Figure 5. *Solow Diagram with Technological Progress.*

[21] The following applies: $A_1 < A_2 < A_n$.

[22] See Arnold (1997).

2.2. Pensions Effects in Neoclassical Models

In the previous chapters, the basics of neoclassical growth theory are explained. To keep the model comprehensible, it is assumed, that the pension system has no impact on the economy. Thus, key macroeconomic variables are independent of the design of the pension system.[23] This corresponds to the situation in a small open economy in which the interest rate equals the world interest rate. At this rate an unlimited amount of capital can be imported and exported. Therefore the internal savings do not affect the accumulation of capital and the level of GDP. This is the result of the economy's ability to borrow unlimited capital from abroad, as long as the marginal product exceeds the real interest rate plus depreciation. In contrast to this stands a closed economy which is examined below. Here the capital supply needs to comply with the capital demand. Increasing the savings in such an economy determines directly the internal accumulation of capital and the level of the social product.[24] Furthermore, there is also a decrease in the interest rate and a rise in real wages due to changes in factor quantities.

The theoretical framework in this chapter represents the neoclassical growth model of Solow (1956). This is also used for the subsequent empirical studies. The practicability of this model is based on the possibilities to derive mathematical relationships and to make qualitative statements. It must not be forgotten that this model has experienced much criticism.[25] Nevertheless, many authors use this model to show the effects of old age on the economy, including Breyer (1990), Homburg (1988) and others.

2.3. Individual Savings

An individual will be considered that lives exactly two periods. The modeling framework is determined by the model of overlapping generations. An individual in this model has to solve the following optimization problem:

$$max\ U_t = U(c_t\,; c_{t+1}) \tag{2.14}$$

This means the consumption will be distributed between two periods, such that the total utility is maximized over the lifetime. It is assumed that the individual is only employed in the first period and that there are no compulsory retirement savings. The corresponding budget constraint is:[26]

$$c_t = w_t - s_t \tag{2.15}$$

$$c_{t+1} = s_t\,(1 + r_t) \tag{2.16}$$

$$w_t = c_t + \frac{c_{t+1}}{(1+r_t)} \tag{2.17}$$

An individual receives the real wage w_t in period t, of which a proportion s_t is saved. These savings will earn interest r_t and will be available in the next period. In contrast, if a compulsory system is introduced, the aforementioned equations change as follows:

$$c_t = w_t\,(1 - b_t) - s_t \tag{2.18}$$

$$c_{t+1} = p_{t+1} + s_t\,(1 + r_t) \tag{2.19}$$

$$w_t + \frac{p_{t+1}}{(1+r_t)} = c_t + \frac{c_{t+1}}{(1+r_t)} \tag{2.20}$$

It is assumed that c_t, c_{t+1} and s_t are expressed in "normal" goods, which means that consumption rises with higher incomes. The introduction of a mandatory pension system forces a redistribution of the active to the passive phase of life. With unchanged savings, the consumption increases in period $t + 1$, while simultaneously decreasing in period t. As a consequence, the respective maximum value will not be reached. The individual will now seek to restore its optimal distribution of consumption. Accordingly, the ratio of $\frac{c_t}{c_{t+1}}$ must be increased. This can only be achieved through a reduction of the savings s_t. Mathematically this can be expressed by the partial derivatives:

$$\frac{\partial s}{\partial b} < 0 \tag{2.21}$$

$$\frac{\partial s}{\partial p} < 0 \tag{2.22}$$

The derivatives state that an introduction or an expansion of this pension system decreases the individual savings.[27] A more extensive representation is to be found in Breyer (1990) or Homburg (1988).[28 29 30]

2.4. Capital Accumulation, Income, and Consumption

In the previous section it was shown that the introduction of a pension system reduces private savings. Thus, no definite statement is made, towards the changes in the total amount of accumulated capital. This is because a funding process may result in additional savings. As part of the funding process, each employee saves for a pension in period $t + 1$ through the contributions paid in period t.

$$p_{t+1} = (1 + r_t) w_t b_t \tag{2.23}$$

As described above, individuals maximize their utility function from equation (2.14). Each individual only draws income during the first period, of which a portion has to be transferred in the second period. Thereby the optimal level of consumption c_{t+1} is determined. If there is no compulsory old-age provision, this transfer can only be made by private savings:

$$c_{t+1} = (1 + r_t)\,s_t \tag{2.24}$$

[23] Key macroeconomic variables are, for example, wage rate, interest rate, capital stock, etc.

[24] This could occur for example through a reform of the pension system.

[25] In the neokeynesian theory, for example, the causal relationship between savings and investments is questioned.

[26] These considerations and derivations follow essentially the theory of intertemporal consumption decisions in Fisher (1930).

[27] Thereby it is irrelevant, which funding method is predominant.

[28] In these and other works, an extensive description of connections between the pension system and other individual decisions take place.

[29] See Breyer (1990).

[30] See Homburg (1990).

However, if a funded system is introduced, a part of the income transfers will be through the compulsory pension system. When comparing the formulas (2.18) and (2.19), the substitutability of private savings is obvious. In both cases, the transfer of income to retirement age will receive the same interest payments, given the amount is identically. With unchanged preferences, one unit of compulsory pension payments replaces exactly one unit of voluntary savings.[31] Homburg (1988) denominated this result as the neutral rate.[32]

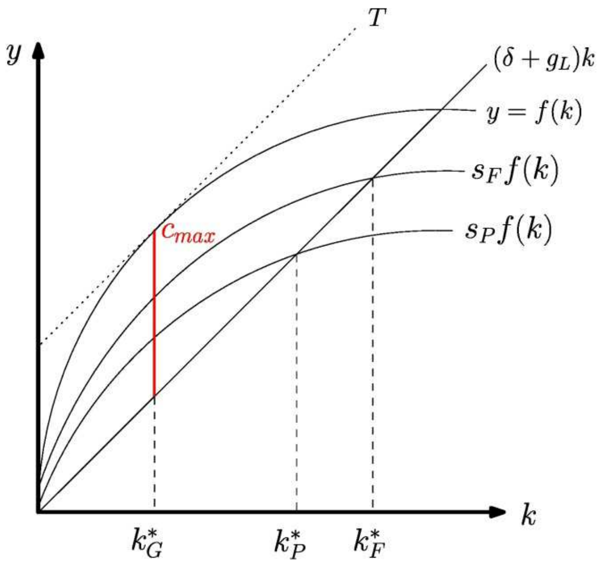
$$p_{t+1} = (1 + n_{t+1})w_{t+1}b \tag{2.25}$$

A policy of constant contribution rates ($b_t = b$) will be assumed. Once again, an economy facing the introduction of a compulsory pensions system will be considered. In the initial state, individuals finance their retirement consumption c_{t+1} by private savings. The introduction of compulsory pensions completely or partially replaces the private savings. Thus, for unchanged preferences, the introduction of pensions will reduce private savings. In contrast to the funded system, the "Pay as you go system" (PAYG) comprises no compensation through capital accumulation for the reduced savings. Thus, the stock of capital in a closed economy will be lower in all subsequent periods, if a PAYG system is introduced.[33]

As mentioned above, the analysis of this section is based on the neoclassical Solow model without technological progress. Since there is a detailed description of the relationships within the model in section 2.1, the following remarks are kept as short as possible. Nevertheless, Figure 5 depicts an economy in steady state.[34] $y = f(k)$ is the per capita production function and k represents the stock of capital per capita. The steady state is characterized by a constant per capita production y^* and a constant capital intensity k^*. The equilibrium condition states that savings must meet the extended amortization. Hence:

$$s_t = (n_t + \delta_t)k_t \tag{2.26}$$

If the parameters are exogenous and constant, economic growth can only occur by including exogenously modeled technological progress. This is expressed by shifting the production function upwards.[35] The starting point of the graph is the production function $y = f(k)$, the straight line $(\delta + g_L)k$ representing the constant depreciation rate, and the savings function $s_P\, f(k)$. In this primordial state there is no public pension system build into the economy and the whole system tends to an equilibrium. In this steady state, there is a capital stock per capita of the amount k_F^*.



Source: Own illustration based on Breyer (1990)

Figure 5. *Solow and Old-Age Provision.*

Introducing a compulsory pension system, that is based on the principle of funding, does not change anything in the graph above. Although private savings are reduced, this loss is compensated by the compulsory savings. Thus, the economy remains in the starting point with the stock of capital per capita k_F^*. However, the introduction of a PAYG system results in a real decrease of savings. Accordingly, the savings function will shift downwards to $s_P\, f(k)$. Due to the constant depreciation, the investments are no longer sufficient to keep the capital stock per capita stable. Therefore there is a decrease in the capital intensity and thus the values converge to a new equilibrium. This new steady state consists of a lower value k_P^* compared to the initial one or the one resulting from a funded system.

However, this result does not include an evaluation of the two pension systems. This is often measured using the level of per capita consumption, not the per capita production or the capital per capita.[36] Accordingly, another point in the graph above is crucial. By assumption, the consumption is calculated as the difference between income and savings, since there are no other options available to the individuals. According to the "Golden Rule of Capital Accumulation",[37] the maximum consumption is found at the point where the marginal product of capital equals the total of depreciation rate and population growth.[38] Formally expressed:

$$MPC = f'(k) = n + \delta \tag{2.27}$$

In Figure 5 this point corresponds to the tangent T of the curve $y = f(k)$, having a slope of $(n + \delta)$. This tangent is parallel to the line $(\delta + g_L)k$ and is shown above in a dotted line. A PAYG system will reduce the capital stock though, however, the benefit from the perspective of individuals will

[31] In reality, various reasons for deviations from the presented theory can exist. For example, if the voluntary savings would be less than the compulsory contributions to the introduced pension system. This results in a higher capital accumulation. Another reason may be differing interest rates, for instance, due to legal requirements.
[32] See Homburg (1988).
[33] See Hauenschild (1999).
[34] G stands for "Golden Rule", F for "Funded System" and P for "PAYG system".
[35] This relationship is shown in Figure 3.
[36] Once again, this becomes clear, when considering the utility function that contains only the consumption of both periods as the determining variables.
[37] See Valdés (1999).
[38] See Breyer (1990).

be maximized due to a higher level of consumption. Accordingly, the utility in an economy without pensions or with a funded system could be increased by introducing a PAYG system. In accordance to Breyer (1990), the following conclusions can be drawn:[39]

If an economy without old-age provision does not meet the "Golden Rule of Capital Accumulation", this optimum condition cannot be achieved by introducing a funded system. There will be no change in the capital stock, because the compulsory savings replaced the voluntary saving amounts.

If the economic capital stock is lower in its initial state than the one determined by the "Golden Rule", the utility per capita can be increased, by introducing higher premium reserve. This can be achieved if the individuals are prevented to reduce their voluntary savings by the level of contributions.[40]

If the economic capital stock is higher in its initial state than the one determined by the "Golden Rule", the utility per capita can be increased by introducing a PAYG system. The reason for this is the implied reduction of the capital stock.

2.5. Factor Prices

Solow (1956) used a production function with the form $Y = F(K, L)$ in his model, notwithstanding Kaldor's stylized facts.[41] According to Kaldor (1961) labor and capital each take up a constant share of the total income of an economy. Subsequently, in the further course the general neoclassical production function is replaced by a Cobb-Douglas production function. The Cobb-Douglas function is represented as follows:

$$Y = AK^{\alpha}L^{1-\alpha} \tag{2.28}$$

A is a factor greater than zero and stands for the productivity of the available technology. α corresponds to the share of labor and $(1-\alpha)$ to the share of capital in relation to total income. Perfect competition is assumed for the factor markets, so that the compensation of the factors corresponds to their respective marginal product.[42] In summary, it can be written as:

$$Capital\ Income = MPC \cdot K = \alpha\, Y \tag{2.29}$$

$$Labor\ Income = MPL \cdot K = (1-\alpha)\, Y \tag{2.30}$$

MPC or MPL is the marginal productivity of the respective input factor. Furthermore, the marginal productivity of capital can be replaced by the real interest rate r and the marginal productivity of labor by the real wage w.[43]

The starting point is a closed economy wherein a pension system is established, which follows the PAYG principle. If the PAYG system is replaced by a funded system, the capital stock of the economy increases. In turn, the enlarged available capital, causes an increase of the total income Y. This increase is disproportionate to the increase in the capital stock.[44] Consequently, there is a reduction in the real interest rate r. Furthermore, an increase in the capital stock induces an augmented real wage. This results from the higher capital per capita and thus higher labor productivity in the new equilibrium.

3. Empirical Analysis

3.1. Description and Analysis of Data

The purpose of this work is to investigate economic effects of pensions in the context of a theoretical growth model. The main idea is to depict all variables of a neoclassical framework and then introducing a parameter that characterizes the amount of old-age provision. The decision, regarding the expedient theory, was made in favor of the widely known Solow model. Thus seven variables have to be considered for the empirical analysis.[45] Consequently, the need for appropriate time series arises, subserving as empirical counterparts for theory. However, there are differences between theoretical and real economies, so that some additional assumptions have to be made.

Table 1. *Data Overview.*

Time Series	Units	Frequency	Adjustment	Solow Term	VECM Term
Real GDP	Billions US-Dollar	Quarterly	Seasonally Adjusted	y	YL
Gross Fixed Capital Formation	Billions US-Dollar	Quarterly	Seasonally Adjusted	k	KL
Total Labor Productivity Growth	Previous Period	Quarterly	Seasonally Adjusted	A	T
Employed Population: Aged 15-64	Persons	Quarterly	Seasonally Adjusted	n	L
Net Equity in Life Insurance and Pension Funds[46]	Billions US-Dollar	Quarterly	Seasonally Adjusted	Not available	PL

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

While some time series like *Real GDP* and *Labor Force* are relatively straightforward to identify with regard to theory, other time series have to be specified as accurately as possible. For example, the capital stock, savings, and

[39] See Breyer (1990).

[40] Examples are credit restrictions during the period of acquisition.

[41] See Bretschger (2004).

[42] The production function has constant returns to scale, since the sum of the output elasticities equals 1.

[43] This approach is based on establishing the marginal product of capital by deducting depreciation. Bretschger (2004) calls this the "net marginal product of capital". However, the depreciation rate of the economy is taken into account. Thus, it follows that $MPC = r + \delta$.

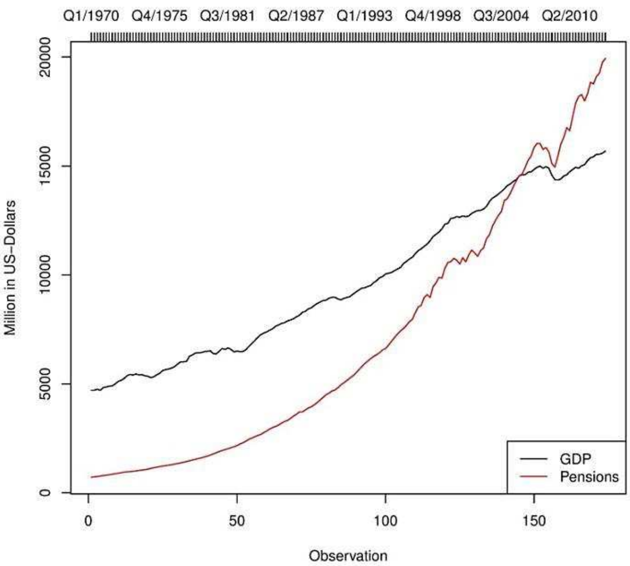
[44] This fact results from the production elasticity α.

[45] These are, as described in detail above, total economic output, capital stock, total savings, population or number of persons in employment, macroeconomic investments, depreciation and technological progress.

[46] Private sector: Households and nonprofit organizations.

investments need to be summarized within a single time series, namely the *Gross Fixed Capital Formation*. This is due to the fact that Solow (1956) assumed a closed economy for his model, whereas the real U.S. economy is open. Similar problems arise with the technological change. As there is no data reproducing technological progress an appropriate surrogate has to be found. According to Fujitsu Research Institute (1998), the total labor productivity can serve as a possible substitute, since technological change is one of the major drivers for higher labor productivity. Thus, all factors included in the theoretical growth model can be replicated.

In a next step, savings for old age needs to be integrated into the empirical analysis. In this approach, the amount of net assets in life insurances and pension funds represents the degree of old-age provision in the private sector. It is assumed that the higher the proportion of assets, the higher the level of retirement savings in the U.S. economy. In Figure 6 the two key variables, GDP and old-age security, are illustrated graphically. Both values increase steadily over time. In the third quarter of 2005 the two lines even cross each other and since then the amount of net assets in life insurances and pension funds exceeds the GDP. The clearly identifiable trends are consistent with both, Kaldor's stylized facts as well as the theoretical considerations about retirement.



Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Figure 6. *Development of the Pensions Variable and GDP.*

Each time series contains 174 observations and represents the period from January 1st, 1970 to April 1st, 2013. The data describes the respective variables for the U.S. economy. All data is taken from the online database FRED of the Federal Reserve Bank of St. Louis and is publicly accessible after short registration. The results of this approach are shown below in more detail. The next section presents the statistical background or rather the empirical procedures.

3.2. Modeling Strategy

"For cointegration, a pair of integrated, or smooth series, must have the property that a linear combination of them is stationary. Most pairs of integrated series will not have the property, so that cointegration should be considered as a surprise when it occurs. In practice, many pairs of macroeconomic series seem to have the property, as is suggested by economic theory." - Clive W. J. Granger [47]

Since the seminal studies by Granger (1981, 1986), Engle and Granger (1987), Stock (1987) and Johansen (1988), cointegration has become one of the most important fields of time series analysis. This concept describes a long-term equilibrium between two or more integrated $I(1)$ time series. These $I(1)$ variables share a common stochastic trend, thus it exists a linear combination of them which is stationary, respectively $I(0)$. As suggested by Granger (2003), consulting economic theory and employing techniques of cointegration is an appropriate approach. In the Solow model the growth rate of capital per capita is equal to the growth rate of income per capita in the steady state:

$$\dot{k} = \dot{y} \tag{3.1}$$

In the long run the relationship between the variables y and k should be stationary or phrased somewhat differently, the variables should be cointegrated. To model a cointegrated system it is useful to employ a VAR model with an error correction term. This is called Vector Error Correction Model (VECM):

$$\Delta \mathrm{y_t} = \prod \mathrm{y_{t-1}} + \nu + \sum_{\mathrm{j=1}}^{\mathrm{p}} \Gamma_{\mathrm{j}} \Delta \mathrm{y_{t-j}} + \varepsilon_{\mathrm{t}} \tag{3.2}$$

with Δy_t as the variables in first differences and ν as a vector of constants. Please note that y is a vector of $I(1)$ variables for the moment, not necessarily the income per capita. $\prod = \alpha\beta'$ denotes the long-run relation with β as the cointegrating vector. The loading matrix α describes the adjustments to this equilibrium. In order to determine the cointegration order r of the VECM, Johansen (1991) suggests the trace test with the hypotheses:

$$H_0: rk(\prod) \leq r\ vs. H_1: rk(\prod) > r \tag{3.3}$$

So, the modeling strategy works as follows: First of all, the order of integration of the considered time series is investigated using a unit root test suggested by Phillips and Perron (1988). If this test indicates $I(1)$ behavior the cointegration order r is determined by the Johansen Trace test for three different models: One simple model with y and k as variables, another with technological progress, and yet one more with a variable for old-age provision. It is important to note, that in the first instance all variables are assumed to be endogenous in this modeling strategy. This assumption will be eased in the following section. The proposed method is applied in many scientific publications such as Hondroyiannis and Papapetrou (2001), Liddle and

[47] Granger (2003), p. 361.

Lung (2010), as well as Guest and Swift (2008), just to mention a few.

3.3. Empirical Results

First of all it is important to consider the trending behavior of the time series for reasons discussed above. Table 2 shows that the variables income per capita YL, capital per capita KL and old-age provision per capita PL seem to be integrated of order 1, thus $I(1)$. As an exception ti this, technological progress should be assumed to be $I(0)$.

Table 2. *Results of Unit Root test.*

	Test Statistic	*1% Critical Value*	*5% Critical Value*	*10% Critical Value*
YL				
Z(rho)	0.971	-20.043	-13.846	-11.097
Z(t)	2.279	-3.486	-2.885	-2.575
KL				
Z(rho)	-0.826	-20.043	-13.846	-11.097
Z(t)	-0.660	-3.486	-2.885	-2.575
PL				
Z(rho)	1.822	-20.043	-13.846	-11.097
Z(t)	4.873	-3.486	-2.885	-2.575
T				
Z(rho)	-181.874	-20.043	-13.846	-11.097
Z(t)	-13.403	-3.486	-2.885	-2.575

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Furthermore, it seems to be appropriate to start with the simple model. In this case, the Solow model without technological progress. As discussed above, Solow (1956) proposed a model which indicates the same growth rates for income per capita and capital per capita in the steady state. To translate this into the empirical framework, YL and KL should be cointegrated. As already mentioned, unit root tests indicate $I(1)$ behavior of YL and KL, so the first condition to use techniques of cointegration analysis is fulfilled. In the next step the order of the lag-length and the order of the cointegration rank should be determined.

Table 3. *Lag-lengths Information Criteria (1).*

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	3811.46	.	.	.	1.2e-22	-44.8172	-44.8022	-44.7803
1	4637.76	1652.6	4	0.000	7.4e-27	-54.4913	-54.4464	-54.3806
2	4642.14	8.7556	4	0.068	7.4e-27	-54.4957	-54.4209	-54.3113
3	4644.41	4.54	4	0.338	7.5e-27	-54.4754	-54.3706	-54.2171
4	4649.47	10.132	4	0.038	7.4e-27	-54.4879	-54.3532	-54.1559

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Table 3 shows that, considering the information criterion by Akaike (1973), a lag-length of two is indicated. Furthermore in order to determine the number of cointegration relations, Table 4 indicates one cointegration relation.

Table 4. *Results of Johanson Trace Test (1).*

maximum rank	parms	LL	eigenvalue	trace ststistic	5% critical value
0	2	4709.0056	.	29.1267	25.32
1	6	4717.9949	0.09870	11.1483	12.25
2	8	4723.569	0.06241	.	.

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Table 5 gives the results for the first VECM. It is important to note that all variables are modeled endogenously, which contradicts the assumption of Solow (1956) that growth is explained exogenously. First of all, the short-run dynamics of the differences of YL (DYL) should be considered. All coefficients of the first lags (LD) and second lags ($L2D$) in the differences of YL and KL are not significant (p-values are higher than 0.05). So the first finding is that growth of income per capita cannot be improved in the short-run by means of investments or higher income per capita. Equally, the autoregressive parts of KL for the differences of KL (DKL) are not significant. In opposite to this, the first and second lags in differences of YL are significant.

Table 5. *VECM (YL,KL).*

	Coef.	Std. Err.	z	P > |z|	[95% Conf. Intervall]
			D_{YL}		
YL					
LD.	.252916	.0780715	0.32	0.746	-.127725 .178309
L2D.	.0244682	.0793076	0.31	0.758	-.130971 .1799083
KL					
LD.	-.127294	.2581655	-0.49	0.622	-.633289 .378701
L2D.	-.1379567	.2564062	0.54	0.591	-.364590 .640503
coint	-.0256285	.0110391	-2.32	0.020	-.047264 -.003992
_cons	4.64e-08	1.22e-07	0.38	0.7004	-193e-07 2.85e-07
			D_{KL}		
YL					
LD.	-.052174	.0230727	-2.26	0.024	-.0973957 -.0069524
L2D.	-.0568296	.023438	-2.42	0.015	-.1027672 -.010892
KL					
LD.	.1437509	.0762963	1.88	0.060	-.0057871 .293289
L2D.	.039801	.757764	0.53	0.599	-.108718 .18832
Coint	-.010518	.0032624	-3.22	0.001	-.0169122 -.0041238
_cons	-1.13e-07	3.60e-08	-3.14	0.002	-1.84e-07 -4.24e-08

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Likewise the long-run relation ($coint$) is significant. Accordingly, the finding of Solow (1956), that KL and YL grow with the same rate in the steady state, can be seen as confirmed by this analysis. In spite of all, the assumption that YL is explained endogenously by KL cannot be clearly verified. In addition DKL can be explained by KL in lags. However autoregressive parts of YL are not and the long-run relation is significant.

Table 6. *In-Sample-Fit (1).*

Equation	Parms	RMSE	R-sq	chi2	P > chi2
D_{YL}	6	5.4e-07	0.2732	61.6421	0.0000
D_{KL}	6	1.6e-07	0.2313	49.35339	0.0000

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

The R^2 (adjusted R^2) will be used to compare the models. Table 6 shows some measured values of the fit of the models for the single variables. This value indicates that the model explains $27.3\ \%$ $(23.1\ \%)$ of the variability of the response data DYL (DKL) around its mean.

Table 7. *Lag-lengths Information Criteria (2).*

Lag	LL	LR	df	P	FPE	AIC	HQIC	SBIC
0	3815.44	.	.	.	1.2e-22	-44.8106	-44.7667	-44.7667
1	4878.79	2126.7	4	0.000	4.5e-28	-57.3034	-57.2435	-57.1559
2	4883.23	8.8719	4	0.064	4.4e-28	-57.3085	-57.2187	-57.0872
3	4886.56	6.661	4	0.155	4.5e-28	-57.3007	-57.1809	-57.0055
4	4891.78	10.454	4	0.033	4.4e-28	-57.3151	-57.1654	-56.9462

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Table 8. *VECM (YL,KL, T).*

	Coef.	Std. Err.	z	P > |z|	[95% Conf. Intervall]
			D_{YL}		
YL					
LD.	.0378567	.0210961	1.79	0.073	-.0034908 .0792043
L2D.	.0116072	.0214388	0.54	0.588	-.0304122 .0536266

	Coef.	Std. Err.	z	P > $\|z\|$	[95% Conf. Intervall]
KL					
LD.	-.0651078	.0696843	-0.93	0.350	-.2016866 .071471
L2D.	.0475412	.0692645	0.69	0.492	-.0882146 .1832971
coint	-.0062673	.0030751	-2.04	0.042	-.0122944 -.0002401
T	7.67e-07	1.65e-08	46.35	0.000	7.35e-07 7.99e-07
_cons	-462e-08	3.66e-08	-1.17	0.244	-1.14e-07 2.90e-08
			D_{KL}		
YL					
LD.	-.052029	.0226733	-2.29	0.022	-.0964679 -.0075902
L2D.	-.0575476	.0230417	-2.50	0.013	-.1027085 -.0123867
KL					
LD.	.1574872	.0748942	2.10	0.035	.0106972 .3042772
L2D.	.0454928	.074443	0.61	0.541	-.1004127 .1913983
coint	-.0060174	.0033051	-1.82	0.069	-.0124952 .0004604
T	4.55e-08	1.78e-08	2.56	0.010	1.07e-08 8.04e-08
_cons	-9.00e-08	3.93e-08	-2.29	0.022	-167e-07 -1.30e-08

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

As a next step another VECM is estimated with technological progress as exogenous variable because this time series has been identified to be $I(0)$. The order of lags is determined as in the model discussed above. Now considering Table 8, the technological progress can improve DYL and DKL also in the short-run. Although the significance of the autoregressive components of the estimates barely changes, the inclusion of technological progress seems to be an important determinant of the growth process. The variable T is significant even at the 1 % level. The inclusion of technological progress implies an ascent of the adjusted R^2 for YL. Accordingly, technological progress seems to be very important when explaining growth of income per capita. Furthermore, both long-term relationships ($coint$) are significant.

Table 9. *In-Sample-Fit (2).*

Equation	Parms	RMSE	R-sq	chi2	P > chi2
D_{YL}	7	1.5e-07	0.9268	2164.051	0.0000
D_{KL}	7	1.6e-07	0.1400	27.83906	0.0001

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Table 10. *Results of Johanson Trace Test (2).*

maximum rank	parms	LL	eigenvalue	trace ststistic	5% critical value
0	30	5570.0548	.	42.8365	42.44
1	36	5582.5657	0.13687	17.8146	25.32
2	40	5589.5462	0.07884	3.8536	12.25
3	42	5591.473	0.02241	.	.

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Table 11. *VECM (YL,KL, T, P) (1).*

	Coef.	Std. Err.	z	P > $\|z\|$	[95% Conf. Intervall]
			D_{YL}		
YL					
LD.	-.0046018	.0200568	-0.23	0.819	-.0439124 .0347087
L2D.	-.0297271	.0201391	-1.48	0.140	-.069199 .0097449
KL					
LD.	-.0478324	.0616841	-0.78	0.438	-.168731 .0730662
L2D.	.0558737	.0604441	0.92	0.355	-.0625945 .174342
PL					
LD.	.0000108	9.23e-06	1.17	0.243	-7.31e-06 .0000289
L2D.	.0000171	9.53e-06	1.79	0.073	-1.62e-06 .0000357
coint2	.0049077	.000828	5.93	0.000	.0032848 .0065306
T	7.49e-07	1.47e-08	50.92	0.000	7.20e-07 7.78e-07
_cons	-102e-07	2.33e-08	-4.36	0.000	-148e-07 -5.61e-08

	Coef.	Std. Err.	z	P > |z|	[95% Conf. Intervall]
			D_{KL}		
YL					
LD.	-.044001	.0244028	-1.80	0.071	-918296 .0038276
L2D.	-.0555845	.024503	-2.27	0.023	-.1036095 -.0075596
KL					
LD.	.1509814	.0750501	2.01	0.044	.003886 .2980769
L2D.	.567175	.07354141	0.77	0.441	-.087421 .2008561
PL					
LD.	-.00002	.0000112	-1.78	0.075	-.000042 2.04e-06
L2D.	-8.93e-06	.0000116	-0.77	0.441	-.0000317 .0000138
coint2	.0019567	.0010075	1.94	0.052	-.0000179 .0039313
T	4.47e-08	1.79e-08	2.50	0.013	9.61e-09 7.98e-08
_cons	-6.12e-08	2.84e-08	-2.16	0.031	-1.17e-07 -5.58e-09

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

However, the purpose of this work is to consider how old age provision effects growth. For this purpose an additional model is estimated. The results of this third analysis are presented below. Lags and the rank of cointegration are determined as above. Again using Johansen's procedure one cointegration relation can be found. Tables 11 and 12 show, that now DYL , DKL , and DPL can be explained by technological progress and the long-run relations. Only DKL is explained by short-run dynamics of KL and YL in lags. Considering the R^2 shown in Table 13 this model explains growth of income best.

Table 12. *VECM (YL,KL, T, P) (2).*

	Coef.	Std. Err.	z	P > |z|	[95% Conf. Intervall]
			D_{PL}		
YL					
LD.	113.8748	166.1835	0.69	0.493	-211.8389 439.5885
L2D.	-262.2382	166.8656	-1.57	0.116	-589.2888 64.81249
KL					
LD.	-425.3177	511.0926	-0.83	0.405	-1427.041 576.4053
L2D.	1117.965	500.8185	2.23	0.026	136.3783 2099.551
PL					
LD.	.0550063	0.0765145	0.72	0.472	-.0949593 .2049719
L2D.	-.0141157	.0789754	-0.18	0.858	-.1689048 .1406733
coint2	29.03228	6.860809	4.23	0.000	15.58534 42.47922
T	.000353	.0001219	2.89	0.004	.000114 .000592
_cons	-.000078	.0001933	-0.40	0.687	-.0004568 .0003008

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

Table 13. *In-Sample-Fit (3).*

Equation	Parms	RMSE	R-sq	chi2	P > chi2
D_{YL}	9	1.3e-07	0.9436	2862.478	0.0000
D_{KL}	9	1.6e-07	0.1516	20.56559	0.0002
D_{PL}	9	.001085	0.2091	45.21982	0.0000

Source: Own illustration based on Federal Reserve Bank of St. Louis (2015)

3.4. Summary

In this section the empirical results are reviewed and interpreted as far as possible. The theoretical framework of growth and old-age provision will not be repeated here in detail.[48] A total of three models is developed, and their implications are considered in more detail:

(I) Solow Model without technological change
(II) Solow Model with technological change
(III) Solow Model with technological change and old-age provision

Informative are primarily the results of the VECM models. The first model provides information on the cointegration relationship between capital and output per capita and the short- and long-run dynamics of the respective variables. With reference to Table 5 the following conclusions can be

[48] At this point reference is made to sections 2 and 3. See Section 2.1 for more details on the Solow model and 2.2 for the effects of old age provision in the neoclassical framework.

drawn: In the short term, only income per capita has a significant influence on the development of capital. Whether these observations are according to the theory of Solow (1956) cannot be determined without further investigation. The theory substantiates that an increase in the capital stock determines the level of production and not vice versa. Since these issues are not in the focus of this work, they will not be investigated in more detail. Much more interesting is the fact that the variable of cointegration ($coint$) is highly significant in both cases. Thus, the model's implication of two identical growing measures seems to be confirmed so far.

The next examined model is a Solow model with technological progress. The autoregressive components remain virtually unchanged. Technological progress, however, is significant in both cases. The long-term relationship of cointegration is also upheld in this case. These observations are highly consistent with the neoclassical modeling framework. Solow (1956) integrated technological progress to model long-term growth in addition to the short-term convergence effects. Ultimately, growth can only be achieved by the exogenously modeled technological progress. Despite all this, the empirical analysis confirms that improved technological capabilities delineate a very important determinant of growth.

The third and last surveyed model integrates a retirement component. In addition to the previously mentioned variables, the amount of net assets in life insurances and pension funds has been incorporated. The short-term dynamics contain no significant results so that further considerations are omitted here. However, the significance of technological progress and the cointegration of YL, KL, and PL is striking. These results are directly related to the issue, whether there is a long-term relationship between economic growth and old-age provision or not. The results of this study clearly indicate a long-term relationship. What this mutual effect looks like cannot be determined exactly at this point. For this purpose further tests are required, which can build upon these previous results.

The main advantage of the used methodology is the possibility to consider long- and short-run dynamics as well as the assumption of exogeneity. However, there are a lot of topics concerning the modeling strategy to be considered in the future: First of all a lot of time series are leptokurtic and/or GARCH-type. These phenomena should be tested and if necessary modelled with appropriate statistical methods. Moreover, it might be most important to model long-run relationships also in a fractional cointegrating system as proposed for example by Shimotsu (2012). This phenomenon occurs when the long-term relationship exhibits long-memory behavior. This might also be the case in macroeconomic time series.

4. Conclusion

This work studied the theoretical foundations of growth theory and old-age provision. These foundations were used to answer the question, whether there is a long-term relationship between retirement and growth or not.

The neoclassical theory, especially Solow (1956), has shown that the problem of a falling rate of profit can be solved by introducing technological progress into the model. However, a sustained growth in productivity is merely possible under the assumption of exogenous technological development. Thus, only the consequences rather than the causes of growth are shown. Furthermore, with respect to the labor force, no distinction is made between skilled or unskilled labor. The problem is the lack of evidence for the role of human capital and the level of technological know-how in the production process. The models with endogenous growth can provide a remedy. But despite all criticism, the scrutinized model is also seen as an approach to explain economic relationships: whether it is the relationship between savings and production volume in the Solow model, the intergenerational interaction of an endogenous saving rate à la Ramsey (1928), Cass (1965) and, Koopmans (1965), or the importance of human capital in the growth process of an economy in the model by Romer (1989). It is likely not possible to consider all stylized facts by Kaldor (1961) in one single model. However, individual components can be investigated from which, if appropriate, an overall picture can be drawn.

Since this work deals with the interaction of economic growth and pensions, the examined relationships of these disciplines are presented subsequently. Basically, the existence of old-age provision in different institutional forms has an impact on the decisions of the individuals. Pensions imply security, which in turn affects the long-term planning of consumption or other economic activities of economic agents. Consequently, the individuals can act differently, since not all risks and contingencies of old-age have to be considered in life planning. Questions arising from this, could for example be, how a pension system affects the accumulation of capital? Or whether, and if to what extent, a sustainable reduction of the contribution rates influences economic development? There are various theories and recently an increase in research findings. Undisputed however, is the direct relationship between economic growth and old-age security in a PAYG system and therein about the relation between employees and pension beneficiaries. Regardless of the form of financing, the employment rate is essential. This in turn is determined by economic growth. In funded systems labor income must be obtained to build up old-age security, so even in this case growth is the influencing variable.

In addition, economic growth increases the general welfare, whereby the scale of distribution between generations is maximized. Moreover, economic growth has an impact on the demographic development. Increasing wealth is generally accompanied by a decline in the fertility and mortality rates. The establishment of an extended family in order to have caregivers for old-age is no longer a motivation and is substituted by other methods, while at the same time life is prolonged e.g. through better health care. In addition to economic growth, also cyclical developments and political behavior can play a crucial role in this context. In the short term influences are for example, changes in taxation, foreign exchange rates, and oil price shocks. Long-term determinants may be changes in the propensity of consumption or saving, trends in productivity or changing wage rates. To consider such processes is important for reform projects which might last several decades.

The importance of savings in terms of economic development however, is somewhat controversial. Even Solow (1956) modeled technological progress as the crucial growth variable, although the supporters of the neoclassical model assume the identity of savings and investments and accordingly macroeconomic savings as the drivers of growth. Today's prevailing view is that investments in human capital are an adequate stimulus for growth. However, no distinction is made with respect to different old-age security systems. At the same time, a decline in the labor force in the neoclassical models results in a decrease of the social product. In the endogenous theory this is not necessarily the case. Romer (1990), for instance, modeled the accumulated human capital as key variable. Regardless of the theoretical model, the demographic trend and the fact that in the future more pensioners have to be supported, remains a problem. Economic growth is therefore a necessity to defuse the resulting distribution conflict. Nevertheless, the current debate about the economic effects of pensions will remain. Despite everything else this work shows that a long-term relationship exists, even though it remains unanswered, whether pensions have a positive or negative effect on economic growth. Obviously the opinions on this differ widely. Further investigations are necessary in order to make more precise statements.

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