Topological Structure of Riesz Sequence Spaces

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Abstract: In this paper, to be the Riesz matrix is symbolized by R, it is defined the spaces $l_\infty(R), c(R)$ and $c_0(R)$, where for instance $c_0(R) = \left\{ x = (x_k) \in w \left| \lim_n \frac{1}{P_n} \sum_{k=1}^{n} p_k x_k = 0 \right. \right\}$ and computed its duals (α-dual, β-dual and γ-dual). Furthermore, it is investigated topological structure of $c_0(R)$ and determined necessary and sufficient conditions for a matrix C to map $c_0(R)$, or c_0 into c_0 or $c_0(R)$.

Keywords: Topological Sequence Space, Banach Spaces, α-Dual, β-Dual

1. Introduction

The study of sequence spaces is in fact a special case of the more general study of function spaces if the domain is restricted to the set of natural numbers N. The set w of all functions from the natural numbers N to the field K of real R or complex numbers C, can be turned into a vector space. In other words, let w be the set of all (real-or) complex valued sequences $\{x_n\}_{n \in N}$, i.e., $x_n \in C$ under the operations of pointwise addition and scalar multiplication given by

$$\{x_n\}_{n \in N} + \{y_n\}_{n \in N} = \{x_n + y_n\}_{n \in N}$$

and $\lambda\{x_n\}_{n \in N} = \{\lambda x_n\}_{n \in N}$, for every $x_n, y_n \in C$ and scalar λ form a vector space over C. Any subspace x of w is then called a sequence space. In other words, a sequence space is a vector space whose elements are infinite scalar sequences of real or complex numbers and is closed under the coordinate wise addition and scalar multiplication. If it is closed under coordinate wise multiplication as well, it is called a sequence algebra. Sequence spaces when equipped with a linear topology form topological vector spaces, [1].

The spaces l_∞, c and c_0 are the linear spaces of complex bounded, convergent and null sequences respectively, i.e., $l_\infty = \{x = (x_k) \in w: \sup_k|x_k| < \infty\}$, $c = \{x = (x_k) \in w: \lim_k x_k \text{ exists}\}$, $c_0 = \{x = (x_k) \in w: \lim_k x_k = 0\}$.

The multipliers from X into Y are given by

$$X^Y = \{y \in w | xy \in Y, \forall x \in X\}$$

for $X, Y \subset w$, where xy is the coordinate wise product, i.e., $xy = \{x_k y_k\}_{k \in N}$. We notates

$$(X^Y)^Z = X^{YZ} = \{y \in w | xy \in Z, \forall x \in X^Y\}$$

for X, Y, Z $\subset$ w. A sequence space X is called Y-space if X = $X^{Y Y}$. Classical α-, β- and γ- duals of X are given by X^l, X^{cs} and X^{bs}, respectively, where

$$l = \left\{ x = (x_k) \in w \left| \sum_k |x_k| < \infty \right. \right\},$$

$$cs = \left\{ x = (x_k) \in w \left| \sum_k x_k \text{ is convergent} \right. \right\},$$

$$bs = \left\{ x = (x_k) \in w \left| \sup_n \left| \sum_k x_k \right| < \infty \right. \right\}.$$

These are Banach spaces with their natural norms. We know that $\varphi \subset X^\alpha \subset X^\beta \subset X^\gamma$. If X $\subset$ Y then $Y^\zeta \subset X^\zeta$ and for every X we have $X^\zeta = X^{\zeta\zeta\zeta}$, X $\subset X^{\zeta\zeta}$, where ζ is one of the α-, β- or γ-duals. [2-4].

Let X and Y be two sequence spaces and let $A = (a_{nk})$ be an infinite matrix of complex numbers a_{nk} where $n, k \in$ N. Then we say that A defines a matrix transformation from X

into Y and we denote it by writing $A: X \rightarrow Y$ if for every sequence $x = (x_k) \in X$, the sequence $Ax = \{(Ax)\}$, the A-transform of x, is in Y, where

$$(Ax)_n = \sum_k a_{nk} x_k, \left(n \in N, x \in D_{00}(A)\right) \qquad (1)$$

and by $D_{00}(A)$ denotes the subspace of w consisting of $x \in w$ for which the sum exists as a finite sum. By $(X: Y)$, we denote the class of all matrices A such that $A: X \rightarrow Y$. Thus $A \in (X: Y)$ if and only if the series on the right side of (1) convergent for each $n \in N$ and each $x \in X$ and we have $Ax = \{(Ax)_n\}_{n \in N} \in Y$ for all $x \in X$. For an arbitrary sequence space X, the matrix domain X_A (or $X(A)$) of an infinite matrix A in X is defined by $X_A = \{x \in w: Ax \in X\}$, which is a sequence space.

Kızmaz [5] defined the sequence spaces

$$l_\infty(\Delta) = \{x = (x_k)|\Delta x \in l_\infty\},$$

$$c(\Delta) = \{x = (x_k)|\Delta x \in c\},$$

$$c_0(\Delta) = \{x = (x_k)|\Delta x \in c_0\},$$

where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$, and showed that these are Banach spaces with norm $\|x\| = |x_1| + \|\Delta x\|_\infty$. After then Et [6] defined the sequence spaces $l_\infty(\Delta^2), c(\Delta^2), c_0(\Delta^2)$ where $\Delta^2 x = (\Delta^2 x_k) = (\Delta x_k - \Delta x_{k+1})$, and showed that these are Banach spaces with norm $\|x\|_1 = |x_1| + |x_2| + \|\Delta^2 x\|_\infty$. Recently Pooja [7] study on mth difference sequence spaces and Khan [8] works on some inclusion relations between the difference sequence spaces defined by sequence of moduli.. In addition, similar topics are studied and genaralized by some author [9-10].

2. Main Results

Let $p = (p_k)$ and (P_n) be given $p_0 > 0$, $p_k \geq 0$ $(\forall k, n \in N)$, $P_n = \sum_{k=0}^n p_k$. Let the matrix $R = R_p = (R, p) = (R, p_n) = r_{nk}$ defined by

$$(r_{nk}) = \begin{cases} \dfrac{p_k}{Pn}, & k \leq n \\ 0, & otherwise \end{cases}$$

is called a Riesz matrix (associated with the sequence p) and

$$(r^{-1}{}_{nk}) = \begin{cases} (-1)^{n-k} \dfrac{P_k}{p_n}, & k \leq n \leq k+1 \\ 0, & k+1 < n \end{cases}$$

is called inverse matrix of Riesz matrix. Also, for $Rx = y$, we get

$$y_n = \frac{1}{P_n} \sum_{k=1}^n p_k x_k$$

and

$$x_n = \frac{1}{p_n}(P_n y_n - P_{n-1} y_{n-1}) \ \text{ or } \ x_n = \sum_{k=n-1}^n (-1)^{n-k} \frac{P_k}{p_n} y_k.$$

Now we define

$$l_\infty(R) = \left\{x = (x_k): \sup_n \left|\frac{1}{P_n} \sum_{k=1}^n p_k x_k\right| < \infty\right\},$$

$$c(R) = \left\{x = (x_k): \lim_n \left|\frac{1}{P_n} \sum_{k=1}^n p_k x_k - l\right| = 0, l \in R\right\},$$

$$c_0(R) = \left\{x = (x_k): \lim_n \left(\frac{1}{P_n} \sum_{k=1}^n p_k x_k\right) = 0\right\}.$$

These spaces are sequence spaces and are normed spaces with norm $\|x\|_{E(R)} = \sup|R_n x|$ where E is one of the sequence spaces l_∞, c, c_0 respectively.

Teorem 2.1. Let X be a normed space and $x = (x_k)$ be a sequence in X. We also define the operator

$$T: c_0(R) \rightarrow c_o$$

$$x \rightarrow Tx = Rx = \frac{1}{P_n} \sum_{k=1}^n p_k x_k$$

Then T is a linear isometry.

Proof. Let $\forall x, \ x' \in c_0(R)$ and $\alpha \in K$. The equality

$$T(\alpha x + x') = R(\alpha x + x') = \alpha Rx + Rx' = \alpha T x + Tx'$$

holds. From this, we get that the operator T is a linear. If the equality $Tx = Tx'$ holds, then we have

$$\frac{1}{P_n} \sum_{k=1}^n p_k(x_k - x'_k) = 0.$$

By induction method, we have $x_n = x'_n$ for all $n \in N$. Therefore the operator T is injective. Furthermore, because the operator T is bijective, we get immediately that $c_0(R) \cong c_0$.

Theorem 2.2. $\left(E(R), \|.\|_{E(R)}\right)$ are normed spaces where E is one of the sequence space l_∞, c, c_0.

Proof. Let E be c_0 sequence space and $\alpha \in K$.

(i) $\|x\|_{c_0(R)} = 0 \Leftrightarrow \sup_n \left|\frac{1}{P_n} \sum_{k=1}^n p_k x_k\right| = 0$

$$\Leftrightarrow \sup \frac{1}{P_n} \sum_{k=1}^n p_k x_k = 0$$

$$\Leftrightarrow x = 0 \text{ for all } n \in N.$$

(ii) $\|ax\|_{c_0(R)} = \|aRx\|_{c_0} = sup \left|\frac{1}{P_n} \sum_{k=1}^n a p_k x_k\right|$

$$= |a|.\|Rx\|_{c_0} = |a|.\|x\|_{c_0(R)}$$

(iii) For $x, x' \in c_0(R)$,

$$\|x + x'\|_{c_0(R)} = \|R(x + x')\|_{c_0}$$

$$= \sup_n \left|\frac{1}{P_n} \sum_{k=1}^n p_k(x_k + x'_k)\right|$$

$$\leq \sup_n \left| \frac{1}{P_n} \sum_{k=1}^{n} p_k x_k \right| + \sup_n \left| \frac{1}{P_n} \sum_{k=1}^{n} p_k x'_k \right|$$
$$\leq \|Rx\|_{c_0} + \|Rx'\|_{c_0}$$
$$\leq \|x\|_{c_0(R)} + \|x'\|_{c_0(R)}.$$

From (i), (ii) and (iii) we immediately get $\left(c_0(R), \|.\|_{c_0(R)}\right)$ is a normed spaces. Similarly we can prove for other sequence spaces.

Teorem 2.3. The normed spaces $\left(c_0(R), \|.\|_{c_0(R)}\right)$ is a Banach space.

Proof. Let (x^n) be a Cauchy sequence in $c_0(R)$, where $x^n = (x^n) = (x_1^n, x_2^n, \cdots) \in c_0(R)$, for each $n \in \mathbb{N}$. Then for each $\varepsilon > 0$, there exists $N = N(\varepsilon)$, such that for all $n, m \geq N$,

$$\|x^n - x^m\|_{c_0(R)} = \|R(x^n - x^m)\|_{c_0}$$
$$= \|Rx^n - Rx^m\|_{c_0} < \varepsilon.$$

Therefore we obtain that Rx^n is a Cauchy sequence in c_0 whence by $(c_0, \|.\|_{c_0})$ the Banach space, it converges to (Rx) say, i.e., there exists $\lim Rx^n = Rx \in c_0$. Now we must show that $x \in c_0(R)$. We have

$$\lim Rx^n = Rx \in c_0 \iff \|Rx^n - Rx\|_{c_0} \to 0.$$

Thus, we get

$$\|Rx^n - Rx\|_{c_0} = \|R(x^n - x)\|_{c_0} = \|(x^n - x)\|_{c_0(R)} \to 0.$$

So, it is corrected that $x^n \to x \in c_0(R)$. This implies $\left(c_0(R), \|.\|_{c_0(R)}\right)$ is a Banach space.

We shall quote some lemmas which are needed in proving our theorems concerning the α, β, γ duals of the Riesz sequence spaces.

Lemma 2.1. Let R be a Riesz matrix. Then $R \in (c_0, c_0)$ if and only if
(i) $\lim_n r_{nk} = 0$,
(ii) $\sup \sum_k |r_{nk}| < \infty$.

Lemma 2.2. Let R be a Riesz matrix. Then $R \in (c, c_0)$ if and only if
(i) $\lim_n r_{nk} = 0$,
(ii) $\sup \sum_k |r_{nk}| < \infty$,
(iii) $\lim \sum_k r_{nk} = 0$.

Lemma 2.3. Let R be a Riesz matrix. Then $R \in (c_0, c)$ if and only if
(i) $\lim_n r_{nk}$ exists,
(ii) $\sup \sum_k |r_{nk}| < \infty$.

Lemma 2.4. Let R be a Riesz matrix. Then $R \in (c_0, l_\infty)$ if and only if
(i) $\sup \sum_k |r_{nk}| < \infty$.

Lemma 2.5. Let R be a Riesz matrix. Then $R \in (c_0, l)$ if and only if
(ii) $\sup \sum_{n=1}^{\infty} |\sum_k r_{nk}| < \infty$.

Teorem 2.4. Let R be a Riesz matrix and $P_n \to \infty \ (n \to 0)$ where $P_n = \sum_{k=0}^{n} p_k$ for each $n \in \mathbb{N}$, then the inclusion $c_0 \subset c_0(R)$ holds.

Proof. Let R be a Riesz matrix and $x = (x_k)$ be a sequence in c_0. Then the following statement is obvious.

$$x \in c_0 \Rightarrow Rx \in c_0 \text{ and } Rx \in c_0 \iff R \in (c_0, c_0).$$

By Lemma 2.1, we have $R \in (c_0, c_0) \iff$
(i) $\lim_n r_{nk} = 0$,
(ii) $\sup \sum_k |r_{nk}| < \infty$.
Now we show the correctness of these statements.

$$\lim_n r_{nk} = \lim \frac{p_k}{P_n} = p_k \lim \frac{1}{P_n} = p_k . 0 = 0,$$

and

$$\sup \sum_k |r_{nk}| = \sup \sum |\frac{p_k}{P_n}| = 1 < \infty.$$

This implies $R \in (c_0, c_0)$ such that for each $x \in c_0(R)$. This completes the proof.

Theorem 2.5. Let R be a Riesz matrix and $\sup |P_n| < \infty$, then the inclusion $c_0(R) \subset c_0$ holds.

Proof. Let R^{-1} be inverse of Riesz matrix and $x = (x_k)$ be a sequence in $c_0(R)$. We need to find that $x \in c_0$ or equivalent we need to find that $R^{-1} \in (c_0, c_0)$. By Lemma 2.1, we can easily show the following statements.

$$\lim_n r^{-1}{}_{nk} = \lim (-1)^{n-k} \frac{P_k}{p_n} = 0,$$

$$\sup \sum_k |r^{-1}{}_{nk}| = \sup \sum_k |(-1)^{n-k} \frac{P_k}{p_n}| = 1 < \infty.$$

This implies $R^{-1} \in (c_0, c_0)$ such that for each $x \in c_0$. Hence the proof is completed.

Theorem 2.6. Let R be a Riesz matrix. Then we have
(i) $c_0 \subset l_\infty(R)$ if and only if $\sup \sum_k |r_{nk}| < \infty$,
(ii) The inclusion $c \subset c_0(R)$ is invalid.

Proof. (i) If the inclusion $c_0 \subset l_\infty(R)$ holds, then the necessary and sufficient condition is that $R \in (c_0, l_\infty)$. By Lemma 2.4, we must show that

$$\sup \sum_k |r_{nk}| < \infty.$$

It is seen by Theorem 2.4. Hence, we say the inclusion $c_0 \subset l_\infty(R)$ holds.

(ii) If the inclusion $c \subset c_0(R)$ holds, then the necessary and sufficient condition is that $R \in (c, c_0)$. By Lemma 2.2, one can show that the validity of the options (i) and (ii). However, the part of (iii) is not provided, i.e., $\lim \sum_k r_{nk} = 1 \neq 0$. Thus, we obtain that the inclusion $c \subset c_0(R)$ is invalid.

Theorem 2.7. Let R be a Riesz matrix and $x = (x_k)$ be a sequence in $c_0(R)$, $y = (y_k)$ be a sequence in c_0. Then we have

(i)

$$(c_0(R))^\alpha = \left\{ a = (a_k) \in w : \sup \sum_{n=1}^\infty \left| \sum_k (-1)^{n-k} a_n \frac{P_k}{p_n} \right| < \infty \right\},$$

(ii)

$$(c_0(R))^\beta = (c_0(R))^\gamma = \left\{ a = (a_k) \in w : \left(\Delta \left(\frac{a_k}{p_k} \right) p_k \right) \in l, \left(a_n \frac{P_n}{p_n} \right) \in l_\infty \right\}$$

Proof. (i) Let us take any $a = (a_k) \in w$. We easily derive with

$$ax = (a_k x_k) = \left(a_k \sum_{i=1}^k (-1)^{k-i} \frac{P_i}{p_k} y_i \right) = \sum_{i=1}^k (-1)^{k-i} a_k \frac{P_i}{p_k} y_i = (Ty)_k$$

For all $k \in N$, where $T = (t_{nk})$ is defined by

$$t_{nk} = \begin{cases} (-1)^{n-k} a_n \dfrac{P_k}{p_n}, n-1 \le k \le n \\ 0, otherwise \end{cases}$$

We also define the linear operator

$$T : c_0 \to l$$

$$y \to Ty = ax \in l.$$

By Lemma 2.5, we get
(i) $a \in (c_0(R))^\alpha \Leftrightarrow ax \in l, \forall x \in (c_0)R,$

$$\Leftrightarrow \forall Rx = y \in c_0, Ty \in l$$

$$\Leftrightarrow T \in (c_0, l) \Leftrightarrow \sup \sum_{n=1}^\infty \left| \sum_k t_{nk} \right| < \infty$$

$$\Leftrightarrow \left\{ a = (a_k) \in w : \sup \sum_{n=1}^\infty \left| \sum_k (-1)^{n-k} a_n \frac{P_k}{p_n} \right| < \infty \right\}.$$

(ii) Let $x \in c_0(R)$ and $a \in w$. Then $y = Rx \in c_0$, We get

$$\sum_{k=1}^n a_k x_k = \sum_{k=1}^{n-1} \left(a_k \frac{P_k}{p_k} - a_{k+1} \frac{P_k}{p_{k+1}} \right) y_n + a_n \frac{P_n}{p_n} y_n = (Cy)_n$$

and we take $C = c_{nk}$ matrix,

$$c_{nk} = \begin{cases} a_k \dfrac{P_k}{p_k} - a_{k+1} \dfrac{P_k}{p_{k+1}}, 0 \le k \le n-1 \\ a_n \dfrac{P_n}{p_n}, k = n \\ 0, otherwise \end{cases}$$

and so $\sum_{k=1}^n a_k x_k = (Cy)_n$. Then we have

$$a \in (c_0(R))^\beta \Leftrightarrow ax \in cs, \forall x \in c_0(R) \Leftrightarrow \sum_{k=1}^n a_k x_k \in c, \forall x \in c_0(R)$$

$$\Leftrightarrow (Cy)_n \in c, \forall y \in c_0 \Leftrightarrow C \in (c_0, c).$$

By Lemma 2.3, we get

$$(c_0(R))^\beta = \left\{ a = (a_k) \in w : \left(\Delta \left(\frac{a_k}{p_k} \right) p_k \right) \in l \text{ and } \left(a_n \frac{P_n}{p_n} \right) \in l_\infty \right\}$$

On the other hand, $(c_0(R))^\gamma$ can be found in a similar way.

Theorem 2.8. Let (X, Y) denote the set of all infinite matrices C which map X into Y. Then we have $C \in (c_0(R), c_0)$ if and only if
(i) $D^{(m)} \in (c_0, c)$,
(ii) $D \in (c_0, c_0)$
where the matrix D depends on the matrix C, ($\forall m \in N$).
Proof. If $x \in c_0(R)$ then $y = Rx \in c_0 \Rightarrow x = R^{-1}y$, we have

$$C_n^{(m)} x = \sum_{k=1}^m c_{nk} x_k$$

$$= \sum_{k=1}^m c_{nk} \left(\frac{1}{p_k} (P_k y_k - P_{k-1} y_{k-1}) \right)$$

$$= \sum_{k=1}^{m-1} \left(\frac{c_{nk}}{p_k} - \frac{c_{nk+1}}{p_k} \right) P_k y_k + \frac{c_{nm}}{p_m} P_m y_m = D_n^{(m)} y$$

Now we also define the matrix:

$$d_{nk}^{(m)} = \begin{cases} \left(\dfrac{c_{nk}}{p_k} - \dfrac{c_{nk+1}}{p_k} \right) P_k, k < m \\ \dfrac{c_{nm}}{p_m} P_m, k = m \\ 0, otherwise \end{cases}$$

Hence, by Lemma 2.1 and Lemma 2.3, we get
(i) $D^{(m)} \in (c_0, c) \Leftrightarrow$
(1) $\sup \sum_k |d_{nk}| = \sup \sum_k \left| \frac{c_{nk}}{p_k} - \frac{c_{nk+1}}{p_k} \right| P_k < \infty$ and

$$\left|\frac{c_{nm}}{p_m}P_m\right| < \infty$$

    (2) $\lim_n d_{nk} = \lim_n \left|\frac{c_{nk}}{p_k} - \frac{c_{nk+1}}{p_{k+1}}\right| P_k$ exists.

(ii) $D \in (c_0, c_0)$ $\Leftrightarrow$

    (1) $\quad sup \sum_k |d_{nk}| = sup \sum_k \left|\frac{c_{nk}}{p_k} - \frac{c_{nk+1}}{p_k}\right| P_k < \infty$ and $\left|\frac{c_{nm}}{p_m}P_m\right| < \infty$

    (2) $\lim_n d_{nk} = \lim_n \left|\frac{c_{nk}}{p_k} - \frac{c_{nk+1}}{p_{k+1}}\right| P_k = 0$.

Hence, it is clear that C $\in$ (c$_0$(R), c$_0$). Conversely, this is obtained by the same kind of argument.

Theorem 2.9. Let C be a matrix. Then we have $C \in \left(c_0, c_0(R)\right)$ if and only if

(i) $sup \sum_{k=1}^{\infty} \left|\frac{1}{P_n} \sum_{i=1}^{n} p_k c_{ik}\right| < \infty$,

(ii) $\lim \frac{1}{P_n} \sum_{i=1}^{n} p_i c_{ik} = 0$.

Proof. Let $C \in (c_0, \ c_0(R))$. For each $x \in c_0$, Cx exists and we have $Cx \in c_0(R) \Rightarrow RCx \in c_0 \Rightarrow RC \in (c_0, \ c_0)$.

If D is a matrix RC, then we can take $d_{nk} = \frac{1}{P_n} \sum_{i=1}^{n} p_i c_{ik}$.

By Lemma 2.1, there is a matrix C that provides the following statements such that

$$sup \sum_{k=1}^{\infty} \left|\frac{1}{P_n} \sum_{i=1}^{n} p_k c_{ik}\right| < \infty \ \text{ and } \ \lim \frac{1}{P_n} \sum_{i=1}^{n} p_i c_{ik} = 0.$$

This is the desired result. In addition, the sufficient condition is trivial.

3. Conclusion

In this paper, to be the Riesz matrix is symbolized by R, we define the spaces $l_\infty(R), c(R)$ and $c_0(R)$ and compute duals (α-dual, β-dual and γ-dual) of $c_0(R)$. Also, it is shown that if the matrix R are a regular matrix and $sup|P_n| < \infty$ then $c_0 = c_0(R)$. Finally, we investigate topological structure of $c_0(R)$ and determine necessary and sufficient conditions for a matrix C to map $c_0(R)$, or c_0 into c_0 or $c_0(R)$.

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