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Special Case Algorithm for N-jobs M-machines Flow Shop Scheduling Problems

Geleta Tadele Mohammed

Department of Mathematics, College of Natural Sciences, Arba Minch University, Arba Minch, Ethiopia

Email address:
geletatadele@gmail.com, geleta.tadele@amu.edu.et

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Abstract: In this paper we have developed an algorithm for n-jobs with m-machine flow shop scheduling problems in order to handle variety of data type than Geleta [3]. In other hands, as in Geleta [3] it main objective is how to obtain optimal sequence of jobs with aim of minimum squared value of lateness.

Keywords: N-jobs M-machine Flow Shop Scheduling Problems, Squared Lateness, Completion Time and Processing Time Multiple

1. Introduction

We extended the work of change [1], Ikram [2] and Geleta [3] to special class of n-jobs m-machine scheduling problems it to handle other data type. The algorithm seems the same with that of Geleta's but it is shifted accordingly to other data type. Still it does not work for all types of data.

As in Geleta [3],throughout this paper, A_{ij} represents the processing time of job j on machine i and the order of machines is fixed and it is $A_1 \to A_2 \to A_3 \to \cdots \to A_{(m-1)} \to A_m$,with the completion time of job done on i^{th} position place is given by

$$C_i = \left[\sum_{j=1}^{i} A_{1j}\right] + A_{2i} + A_{3i} + \cdots + A_{mi}, \text{ for } i=1:\text{ n} \qquad (1)$$

Problem formulation and assumption

In addition to the assumption of Geleta [3], I want to have only the following:

$$Max\ \{A_{1i}\} \le Min\{A_{2i}\} \qquad (2a)$$

$$Max\{A_{2i}\} \le Min\{A_{3i}\} \qquad (2b)$$

$$Max\ \{A_{3i}\} \le Min\{A_{4i}\} \qquad (2c)$$

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$$Max\ \{A_{(m-1)i}\} \le Min\{A_{mi}\} \qquad (2d)$$

And if $A_{1x} < A_{1y}$, then the following conditions holds true:

$$A_{1x}A_{2y} \le A_{1y}A_{2x} \qquad (3a)$$

$$A_{1x}A_{3y} \le A_{1y}A_{3x} \qquad (3b)$$

$$A_{1x}A_{4y} \le A_{1y}A_{4x} \qquad (3c)$$

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$$A_{1x}A_{my} \le A_{1y}A_{mx} \qquad (3d)$$

Since all jobs spend more amount of time on the last machine, then $d_j = kA_{mj}, j = 1: n$, where k is processing time multiple. This is because of the condition (2a) to (2d).

Let σ denotes an arbitrary sequence, and $[j]$ represents jobs occupying j^{th} position of σ . If $L_{[j]}, C_{[j]}$ and $d_{[j]}$ represents lateness, completion time and assigned due date of the job in the j^{th}, then our objective function is to minimize

$$L^2 = \sum_{i=1}^{n} L^2{}_{[i]} = \sum_{i=1}^{n} (C_{[i]} - d_{[i]})^2 \qquad (4)$$

Since the completion time of the j^{th} for any sequence σ is:
$C_{[j]} = \left[\sum_{i=1}^{j} A_{[1i]}\right] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]}$, for j =1: n

and by using $d_{[j]}= \mathrm{k}A_{[1j]}, j = 1:n$ equation (4) becomes:

$$L^2=\sum_{j=1}^{n}[\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]} - \mathrm{k}A_{[1j]}]^2 \quad (5)$$

This is a function of k and that have to be minimized.

By exactly the same procedure as in Geleta [13], we have the following optimal processing time multiple k* andoptimal due date:

$$\mathrm{k}^*=\frac{\sum_{j=1}^{n}A_{[1j]}\sum_{i=1}^{j}A_{[1i]}+\sum_{j=1}^{n}A_{[1j]}A_{2j}+\sum_{j=1}^{n}A_{[1j]}A_{[3j]}+\sum_{j=1}^{n}A_{[1j]}A_{[4j]}+\ldots+\sum_{j=1}^{n}A_{[1j]}A_{[mj]}}{\sum_{j=1}^{n}[A_{[1j]}]^2} \quad (6a)$$

Hence, by using the value of k^* we assign the value of due dates of each to be

$$d_{[j]}= \mathrm{k}^*A_{[mj]}, j = 1:n \quad (6b)$$

Theorem (Minimization of L^2 under a certain condition):

Our objective function $L^2 = \sum_{j=1}^{n}[\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]} - \mathrm{k}A_{[1j]}]^2$ can be minimized by applying the rule that job x should be done before job y if the following conditions are satisfied:

$$A_{1x} < A_{1y} \quad (6c)$$

$$A_{1x}A_{2y} \le A_{1y}A_{2x} \quad (6d)$$

$$A_{1x}A_{3y} \le A_{1y}A_{3x} \quad (6e)$$

$$A_{1x}A_{4y} \le A_{1y}A_{4x} \quad (6f)$$

.

.

.

$$A_{1x}A_{my} \le A_{1y}A_{mx} \quad (6g)$$

Proof:

Since $L^2 = \sum_{j=1}^{n}[[\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]} - \mathrm{k}A_{[1j]}]^2$, the extending this expression we can obtain that

$$L^2 = \sum_{j=1}^{n}\left\{[\left[\sum_{i=1}^{j}A_{[1i]}\right] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]}]^2 + \left[\mathrm{k}A_{[1j]}\right]^2 - 2\mathrm{k}A_{[1j]}\left(\left[\sum_{i=1}^{j}A_{[1i]}\right] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]}\right)\right\}$$

By having re-arrangement of summation inside the bracket we have,

$$L^2=\sum_{j=1}^{n}[[\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]}]^2 + \sum_{j=1}^{n}\left[\mathrm{k}A_{[1j]}\right]^2 - 2k\sum_{j=1}^{n}[A_{[1j]}([\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]})] \quad (7)$$

From the third term of Equation (7), we obtain:

$\sum_{j=1}^{n}[A_{[1j]}([\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]})]=$ $\sum_{j=1}^{n}A_{[1j]}\sum_{i=1}^{j}A_{[1i]} + \sum_{j=1}^{n}A_{[1j]}A_{2j} + \sum_{j=1}^{n}A_{[1j]}A_{[3j]} + \sum_{j=1}^{n}A_{[1j]}A_{[4j]} + \ldots + \sum_{j=1}^{n}A_{[1j]}A_{[mj]}$, which is constant and independent of the sequence of the jobs change [1].

And also, the middle term $\sum_{j=1}^{n}\left[\mathrm{k}A_{[1j]}\right]^2$ is constant (because of it is the sum of square quantity) and independent of sequence of the jobs.

Now, the remaining thing to prove is that $\sum_{j=1}^{n}[[\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]}]^2$ can be minimized by applying the conditions given above in the theorem?

The answer is yes!

Let σ_1 be a sequence of jobs in which job x and y are arranged in a position k and k+1 respectively, and σ_2 be a sequence of the jobs in which job x and y are arranged in apposition of k+1 and k respectively.

$$\text{Let } f(\sigma_1) =\sum_{j=1}^{n}[[\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]}]^2 \quad (8)$$

And

$$f(\sigma_2) =\sum_{j=1}^{n}[[\sum_{i=1}^{j}A_{[1i]}] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]}]^2 \quad (9)$$

Now, by expanding equation (9), we obtain

$$f(\sigma_1) =\left[A_{[11]} + A_{[21]} + A_{[31]} + \cdots + A_{[m1]}\right]^2 + \left[A_{[11]}+A_{[12]} + A_{[22]} + A_{[32]} + \cdots + A_{[m2]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]} + A_{[23]} + A_{[33]} + \cdots + A_{[m3]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]} + A_{[24]} + A_{[34]} + \cdots + A_{[m4]}\right]^2+\ldots+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]} + \cdots + A_{[1(k-1)]}+A_{[1x]} + A_{[2x]} + A_{[3x]} + \cdots + A_{[mx]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]} + \cdots + A_{[1(k-1)]}+A_{[1x]} + A_{[1y]} + A_{[2y]} + A_{[3y]} + \cdots + A_{[my]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]} + \cdots + A_{[1(k+2)]} + A_{[2(k+2)]} + A_{[3(k+2)]} + \cdots + A_{[m(k+2)]}\right]^2+\ldots +\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]} + \cdots + A_{[1n]} + A_{[2n]} + A_{[3n]} + \cdots + A_{[mn]}\right]^2 \quad (10)$$

$$f(\sigma_2)=\left[A_{[11]}+A_{[21]}+A_{[31]}+\cdots+A_{[m1]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[22]}+A_{[32]}+\cdots+A_{[m2]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[23]}+A_{[33]}+\cdots+A_{[m3]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+A_{[24]}+A_{[34]}+\cdots+A_{[m4]}\right]^2+\ldots+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1(k-1)]}+A_{[2(k-1)]}+A_{[3(k-1)]}+\cdots+A_{[m(k-1)]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1(k-1)]}+A_{[1y]}+A_{[2y]}+A_{[3y]}+\cdots+A_{[my]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots A_{[1(k-1)]}+A_{[1y]}+A_{[1x]}+A_{[2x]}+A_{[3x]}+\cdots+A_{[mx]}\right]^2+\left[A_{[11]}+A_{[12]}+A_{[13]}+\cdots+A_{[1(k+2)]}+A_{[2(k+2)]}+A_{[3(k+2)]}+\cdots+A_{[m(k+2)]}\right]^2+\ldots+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1n]}+A_{[2n]}+A_{[3n]}+\cdots+A_{[mn]}\right]^2 \quad (11)$$

$$\text{Now,} f(\sigma_1)-f(\sigma_2)=\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1(k-1)]}+A_{[1x]}+A_{[2x]}+A_{[3x]}+\cdots+A_{[mx]}\right]2+\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1(k-1)]}+A_{[1x]}+A_{[1y]}+A_{[2y]}+A_{[3y]}+\cdots+A_{[my]}\right]^2$$
$$-\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1(k-1)]}+A_{[1y]}+A_{[2y]}+A_{[3y]}+\cdots+A_{[my]}\right]$$
$$-\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots A_{[1(k-1)]}+A_{[1y]}+A_{[1x]}+A_{[2x]}+A_{[3x]}+\cdots+A_{[mx]}\right]^2 \quad (12)$$

Then by simplifying equation (12) we get

$$f(\sigma_1)-f(\sigma_2)$$
$$=A_{[1x]}\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1(k-1)]}+A_{[1x]}+A_{[1y]}+A_{[2y]}+A_{[3y]}+\cdots+A_{[my]}\right]-A_{[1y]}\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots A_{[1(k-1)]}+A_{[1y]}+A_{[1x]}+A_{[2x]}+A_{[3x]}+\cdots+A_{[mx]}\right]$$

$$=A_{[1x]}A_{[11]}+A_{[1x]}A_{[12]}+A_{[1x]}A_{[13]}+\ldots+A_{[1x]}A_{[1(k-1)]}+A_{[1x]}A_{[1x]}+A_{[1x]}A_{[1y]}+A_{[1x]}A_{[2y]}+A_{[1x]}A_{[3y]}+\ldots+A_{[1x]}A_{[my]}-A_{[1y]}A_{[11]}-A_{[1y]}A_{[12]}-A_{[1y]}A_{[13]}-A_{[1y]}A_{[14]}-A_{[1y]}A_{[1(k-1)]}-A_{[1y]}A_{[1x]}-A_{[1y]}A_{[2x]}-A_{[1y]}A_{[3x]}-A_{[1y]}A_{[4x]}-\cdots-A_{[1y]}A_{[mx]}$$

$$=\left[A_{[1x]}\right]^2-\left[A_{[1y]}\right]^2+\left(A_{[1x]}-A_{[1y]}\right)\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1(k-1)]}\right]+\left[A_{[1x]}A_{[1y]}+A_{[1x]}A_{[2y]}+A_{[1x]}A_{[3y]}+A_{[1x]}A_{[4y]}+\cdots+A_{[1x]}A_{[my]}-A_{[1y]}A_{[1x]}-A_{[1y]}A_{[2x]}-A_{[1y]}A_{[3x]}-A_{[1y]}A_{[4x]}-\cdots-A_{[1y]}A_{[mx]}\right]$$

$$=\left[A_{[1x]}\right]^2-\left[A_{[1y]}\right]^2+\left(A_{[1x]}-A_{[1y]}\right)\left[A_{[11]}+A_{[12]}+A_{[13]}+A_{[14]}+\cdots+A_{[1(k-1)]}\right]+\left[A_{[1x]}A_{[1y]}-A_{[1y]}A_{[1x]}\right]+\left[A_{[1x]}A_{[2y]}-A_{[1y]}A_{[2x]}\right]+\left[A_{[1x]}A_{[3y]}-A_{[1y]}A_{[3x]}\right]+\left[A_{[1x]}A_{[4y]}-A_{[1y]}A_{[4x]}\right]+\cdots+\left[A_{[1x]}A_{[my]}-A_{[1y]}A_{[mx]}\right]$$

Now, if $A_{1x}<A_{1y}$

$$A_{1x}A_{2y}\le A_{1y}A_{2x}$$
$$A_{1x}A_{3y}\le A_{1y}A_{3x}$$
$$A_{1x}A_{4y}\le A_{1y}A_{4x}$$
$$.$$
$$.$$
$$.$$
$$A_{1x}A_{my}\le A_{1y}A_{mx},$$

Then, $f(\sigma_1)<f(\sigma_2)$.

Thus, the interchanging of job x and y reduces the value of L^2. Hence, job x should be done before job y. Therefore, the conditions stated in the above theorem are satisfied.Now, by repeatedly applying the above rule, L^2 can be minimized by arranging jobs depending on their processing time on the first machine as S.P.T rule.

2. Algorithms to Get Optimal Sequence

Step 1: Verify that the conditions given from (2a)-(2d) and (3a) – (3d). If all of these conditions holds true proceed to the next step. Else stop.

Step 2: Determine the values of k^* using by the formula (6a).

Step 3: By using shortest processing time rule on the last machine A_m determine the optimal sequence of jobs.

Step 4: Finally, find L^2 for the obtained optimal sequences of jobs.

Recommendation: One can create other algorithm that works for all types of data is my recommendation for other researchers.

3. Example

For the following 3-jobs 3-machine flow shop scheduling problem, find the optimal sequence of jobs such that L^2 is minimum.

Jobs	**Machines**		
	A_1	A_2	A_3
1	$A_{11}=3$	$A_{21}=6$	$A_{31}=12$
2	$A_{12}=5$	$A_{22}=8$	$A_{32}=10$
3	$A_{13}=4$	$A_{23}=7$	$A_{33}=11$

Solution:

Here, what we have to do is that, according to the above algorithm, we have to find:-

a. k^*
b. Due-dates of each job.
c. An optimal sequence.

Step 1:

Clearly, $max\{3{,}5{,}4\} = 5 < min\{6{,}8{,}7\} = 6$

$$max\{6{,}7{,}8\} = 8 < min\,\{12{,}10{,}11\} = 10$$

And also, let say job 1 is x and job 2 is y.
Then, $A_{1x}A_{2y}$= (3) (8) =24

$$A_{1x}A_{3y}\text{= (3) (10) =30}$$

$$A_{1y}A_{2x}\text{= (5) (6) =30}$$

$$A_{1y}A_{3x}\text{= (5) (12) =60}$$

This implies that

$$\begin{cases} A_{1x}A_{2y} = (3)(8) = 24 < A_{1y}A_{2x} = 30 \\ A_{1x}A_{3y} = (3)(10) = 30 < A_{1y}A_{3x} = 60 \end{cases}$$

Therefore, all conditions for step 1 are satisfied.

Step 2:

$$\text{Since, } k^* = \frac{\sum_{j=1}^{n} A_{[1j]} \sum_{i=1}^{j} A_{[1i]} + \sum_{j=1}^{n} A_{[1j]}A_{2j} + \sum_{j=1}^{n} A_{[1j]}A_{[3j]} + \sum_{j=1}^{n} A_{[1j]}A_{[4j]} + \ldots + \sum_{j=1}^{n} A_{[1j]}A_{[mj]}}{\sum_{j=1}^{n} [A_{[1j]}]^2} \ldots \quad (13)$$

Then, because of our problem is for n=3=m, k^* becomes,

$$k^* = \frac{\sum_{j=1}^{3} A_{[1j]} \sum_{i=1}^{j} A_{[1i]} + \sum_{j=1}^{3} A_{[1j]}A_{2j} + \sum_{j=1}^{3} A_{[1j]}A_{[3j]}}{\sum_{j=1}^{3} [A_{[1j]}]^2}$$

$$= \frac{A_{11}(A_{11}) + (A_{11}+A_{12})(A_{11}+A_{12}) + (A_{11}+A_{12}A_{13})(A_{11}+A_{12}A_{13}) + A_{11}A_{21} + A_{12}A_{22} + A_{13}A_{23} + A_{11}A_{31} + A_{12}A_{32} + A_{11}A_{33}}{{A_{11}}^2 + {A_{12}}^2 + {A_{13}}^2}$$

$$= \frac{(3)(3)+(3+5)(3+5)+(3+5+4)(3+5+4)+(3)(6)+(5)(8)+(4)(7)+(3)(12)+(5)(10)+(3)(11)}{(3)^2+(5)^2+(4)^2} = \frac{424}{50}$$

$$= 8.48$$

Now, by using this values of k^* we can assign the due dates of each job as follows:

Jobs	Machine A_3	Due-date ($d_{[j]}$= $k^* A_{[3j]}$)
1	12	101.76
2	10	84.8
3	11	93.28

Step 3:

As indicated in the above algorithm, we arrange jobs as per shortest processing time rule on machine 3(A_3). And also, the same result we obtain, if we arrange jobs by earliest due date rule.

Therefore, by using both of them (one of them is enough), we obtain the optimal sequence of jobs 2-3-1.

Step 4:

Determination ofL^2. Here, $L^2$is listed in the following table for all possible sequences of jobs we have.

Sequences of jobs(σ)	Processing time multiple k^*	Squared value of lateness L^2
1-2-3	8.48	13983.976
1-3-2	8.48	14187.376
3-1-2	8.48	13975.976
3-2-1	8.48	13687.776
2-3-1	8.48	13361.776
2-1-3	8.48	11254.736

Note that, L^2 is calculated by $L^2 = \sum_{j=1}^{n} \left[\sum_{i=1}^{j} A_{[1i]}\right] + A_{[2j]} + A_{[3j]} + \cdots + A_{[mj]} - k^* A_{[3j]}]^2$, because of $n = m = 3$ for our problem, then it becomes

$$L^2 = \sum_{j=1}^{3} \left[\sum_{i=1}^{j} A_{[1i]}\right] + A_{[2j]} + A_{[3j]} - k^* A_{[3j]}]^2$$

$$= (A_{11} + A_{21} + A_{31} - k^* A_{11})^2 + (A_{11} + A_{12} + A_{22} + A_{32} - k^* A_{32})^2 + (A_{11} + A_{12} + A_{13} + A_{23} + A_{33} - k^* A_{33})^2$$

For instance, for the sequence of jobs2-3-1:-

Jobs	Machines		
	A_1	A_2	A_3
2	$A_{11} = 5$	$A_{21} = 8$	$A_{31} = 10$
3	$A_{12} = 4$	$A_{22} = 7$	$A_{32} = 11$
1	$A_{13} = 3$	$A_{23 = 6}$	$A_{33} = 12$

$$L^2 = (5 + 8 + 10 - (8.48)10)^2 + (5 + 4 + 7 + 11 - (8.48)11)^2 + (5 + 4 + 3 + 6 + 12 - (8.48)12)^2$$

$$= (23 - 84.8)^2 + (27 - 93.28)^2 + (30 - 101.76)^2$$

$$= (-61.8)^2 + (-66.28)^2 + (-71.76)^2$$

$$= 3819.24 + 4393.0384 + 5149.4976$$

$$= 13361.776$$

Similarly, for the sequence of jobs1-3-2 we have,

Jobs	Machines		
	A_1	A_2	A_3
1	$A_{11} = 3$	$A_{21} = 6$	$A_{31} = 12$
3	$A_{12} = 4$	$A_{22} = 7$	$A_{32} = 11$
2	$A_{13} = 5$	$A_{23 = 8}$	$A_{33} = 10$

$$L^2 = (3 + 6 + 12 - (8.48)12)^2 + (3 + 4 + 7 + 11 - (8.48)11)^2 + (3 + 4 + 5 + 8 + 10 - (8.48)10)^2$$

$$= (21 - 101.76)^2 + (25 - 93.28)^2 + (30 - 84.8)^2$$

$$= (-80.76)^2 + (-68.28)^2 + (-54.8)^2$$

$$= 6522.1776 + 4662.1584 + 3003.04$$

$$= 14187.376$$

For the sequence of jobs1-2-3 we have,

Jobs	Machines		
	A_1	A_2	A_3
1	$A_{11} = 3$	$A_{21} = 6$	$A_{31} = 12$
2	$A_{12} = 5$	$A_{22} = 8$	$A_{32} = 10$
3	$A_{13} = 4$	$A_{23=7}$	$A_{33} = 11$

$$L^2=(3+6+12-(8.48)12)^2+(3+5+8+10-(8.48)10)^2+(3+5+\ 4+7+11-(8.48)11)^2$$

$$=\ (21-101.76)^2+(26-84.8)^2+(30-93.28)^2$$

$$=(-80.76)^2+(-58.8)^2+(-63.28)^2$$

$$= 6522.1776+3457.44+4004.3584$$

$$= 13983.976$$

For the sequence of jobs 2-1-3 we have,

Jobs	Machines		
	A_1	A_2	A_3
2	$A_{11} = 5$	$A_{21} = 8$	$A_{31} = 10$
1	$A_{12} = 3$	$A_{22} = 6$	$A_{32} = 12$
3	$A_{13} = 4$	$A_{23=7}$	$A_{33} = 11$

$$L^2=(5+8+10-(8.48)10)^2+(5+3+6+12-(8.48)12)^2+(5+3+\ 4+7+11-(8.48)11)^2$$

$$=\ (23-84.8)^2+(31-101.76)^2+(44-93.28)^2$$

$$=(-61.8)^2+(-70.76)^2+(-49.28)^2$$

$$= 3819.24+5006.9776+2428.5184$$

$$= 11254.736$$

For the sequence of jobs 3-1-2 we have,

Jobs	Machines		
	A_1	A_2	A_3
3	$A_{11} = 4$	$A_{21} = 7$	$A_{31} = 11$
1	$A_{12} = 3$	$A_{22} = 6$	$A_{32} = 12$
2	$A_{13} = 5$	$A_{23=8}$	$A_{33} = 10$

$$L^2=(4+7+11-(8.48)11)^2+(4+3+6+12-(8.48)12)^2+(4+3+\ 5+8+10-(8.48)10)^2$$

$$=\ (22-93.28)^2+(25-101.76)^2+(30-84.8)^2$$

$$=(-71.28)^2+(-76.76)^2+(-54.8)^2$$

$$= 5080.8384+5892.0976+3003.04$$

$$= 13975.976$$

For the sequence of jobs3-2-1 we have,

Jobs	Machines		
	A_1	A_2	A_3
3	$A_{11} = 4$	$A_{21} = 7$	$A_{31} = 11$
2	$A_{12} = 5$	$A_{22} = 8$	$A_{32} = 10$
1	$A_{13} = 3$	$A_{23=6}$	$A_{33} = 12$

$$L^2=(4+7+11-(8.48)11)^2+(4+5+8+10-(8.48)10)^2+(4+5+\ 3+6+12-(8.48)12)^2$$

$$=\ (22-93.28)^2+(26-84.8)^2+(30-101.76)^2$$

$$=(-71.28)^2+(-58.8)^2+(-71.76)^2$$

$$= 5080.8384+3457.44+5149.4976$$

$$= 13687.776$$

Now, from the above table the optimal sequence of the given jobs is 2-3-1 with L^2 =11254.736 $(Jobx = 2\ isdonejoby = 1)$. This completes our example.

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