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Occurrence of Galilean geometry

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Abstract: The main difference of Galilean geometry is its relative simplicity, for it enables the student to study it in relative detail without losing a great deal of time and intellectual energy. In this paper, we introduce you with new geometric(non-Euclidean) ideas which exist in affine plane and more simple than Euclidean plane.

Keywords: Non-Euclidean Geometry, Galilean Geometry, Affine Plane, Isotropic, Minkowski Space

1. Introduction

The 19th century was a period of rapid development in geometry. In 1854 the eminent German mathematician G. F. B. Riemann formulated, in a famous memoir [1], an extremely general view of geometry which greatly widened its scope. Riemann also noted that there are three related but distinct geometric systems the usual Euclidean geometry, hyperbolic geometry and so-called elliptic geometry which is closer spherical geometry. This list of geometries was extended in 1870 by the German mathematician F. Klein [2] ,[3]. According to Klein there are nine related plane geometries including Euclidean geometry, hyperbolic geometry and elliptic geometry. Klein's views, which were in a way a synthesis of the geometric views of his predecessors and of the work of the English algebraist A. Cayley appeared in 1872 in his Erlanger Program [4]. Klein's broad view of geometry has a universality comparable to that of Riemann.

Thus, just as the fundamental discoveries of Lobachevsky, Bolyai, and Gauss destroyed the exclusive position of Euclidean geometry, so, too, the classical investigations of Riemann and Klein (1854-1872) destroyed the exclusive position of hyperbolic geometry. Nevertheless, even today the term "non-Euclidean geometry" frequently stands for just hyperbolic geometry (less frequently, the plural "non-Euclidean geometries" is used to denote just hyperbolic geometry) and elliptic geometry, and the existence of other geometric systems is known only to specialists.

The views presented in those discussions have long ago lost all scientific significance. Thus, for example, even in Klein's "Non-Euclidean Geometry", we find the assertion that the geometry of our universe must be either Euclidean, hyperbolic, or elliptic; this in spite of the fact that the scientific unsoundness of this viewpoint, at least in its original formulation, followed from Einstein's special theory of relativity of 1905 and, even more decisively, from his general theory of relativity of 1916.

The fact that hyperbolic geometry is linked to the issue of the independence of the parallel axiom and clarifies the role of that axiom in Euclidean geometry, is a strong argument in favor of its pedagogical value. On the other hand, hyperbolic geometry is rather complex—it is definitely more complex than Euclidean geometry—and yet the non-Euclidean nature of a geometry need not imply complexity.

The main distinction of Galilean geometry is its relative simplicity, for it enables the student to study it in relative detail without losing a great deal of time and intellectual energy. Put differently, the simplicity of Galilean geometry makes its extensive development an easy matter, and extensive development of a new geometric system is a precondition for an effective comparison of it with Euclidean geometry. Also, extensive development is likely to give the student the psychological assurance of the consistency of the investigated structure. Another distinction of Galilean geometry is the fact that it exemplifies the fruitful geometric idea of duality. These reasons make me think that one should give serious thought to a mathematics program for teachers' colleges which would include a comparative study of three simple geometries, namely, Euclidean geometry, the geometry associated with the Galilean principle of relativity, and the geometry associated with Einstein's principle of relativity as well as an introduction to the special theory of relativity [5].

Finally, the now popular name "Galilean geometry" is historically inaccurate: Galileo, whose works date from the beginning of the 17^{th} century, did not in fact know this geometry, whose discovery was necessarily preceded by one of the greatest intellectual triumphs of the 19^{th} century the emergence of the idea that many legitimate geometric systems exist. A more accurate name would be "the geometry associated with the Galilean principle of relativity." This name is too long for repeated use and that is why we have decided, somewhat reluctantly, to use the name "Galilean geometry." This name is partially justified by the brilliant clarity and completeness with which Galileo formulated his principle of relativity, which leads directly to the non-Euclidean geometry considered in this article[6],[7].

Let's $\{0,x,y\}$ affine coordinate system is given on two dimensional A_2 – affine plane, and define the given $\vec{X}\{x_1,y_1\}$ and $\vec{Y}\{x_2,y_2\}$ vector product as below

$$(\vec{X}\cdot\vec{Y})_1 = x_1\cdot x_2$$

$$\text{if} \quad (\vec{X}\cdot\vec{Y})_1 = 0 \quad \text{then} \tag{1}$$

$$(\vec{X}\cdot\vec{Y})_2 = y_1\cdot y_2 \quad .$$

2. Galilean Geometry

Definition: A_2 – affine plane which is defined as the vectors' scalar multiplication as in formula (1) is called Γ_2 – Galilean plane.

Let's take the modulus of vector, similar to in Euclidean plane, as scalar product of vector which is defined under square root as below, that is

$$\left|\vec{X}\right| = \sqrt{(\vec{X}\cdot\vec{X})} \tag{2}$$

Well, for the distance between two points $A(x_1,y_1)$ and $B(x_2,y_2)$ is equal to the vector modulus that connects these two points which is $AB = \left|\overrightarrow{AB}\right|$. Since $AB = \{x_2 - x_1; y_2 - y_1\}$,

$$AB_1 = \sqrt{\left(\overrightarrow{AB}\cdot\overrightarrow{AB}\right)_1} = \sqrt{(x_2-x_1)^2} = |x_2 - x_1|$$

$$\text{If} \quad AB_1 = 0 \quad \text{then} \tag{3}$$

$$AB_2 = \sqrt{\left(\overrightarrow{AB}\cdot\overrightarrow{AB}\right)_2} = \sqrt{(y_2-y_1)^2} = |y_2 - y_1| \quad .$$

Plane geometry whose distance is defined as in formula (3) is called as Galilean geometry .

In this article, our fundamental aim is to introduce you with vectors' scalar product in (1) and the geometric ideas that are the reasons of defining the distance between the two points in (3) which are determined.

As is well known, in three dimensional A_3 – affine space, if scalar product of $\vec{X}\{x_1,y_1,z_1\}$ and $\vec{Y}\{x_2,y_2,z_2\}$ is defined as follows

$$\left(\overrightarrow{XY}\right) = x_1x_2 + y_1y_2 - z_1z_2 \tag{4}$$

then it is called ${}^1R_3$ – Minkoswki space. In this space, the modulus of vector is

$$\left|\vec{X}\right| = \sqrt{\left(\vec{X}\cdot\vec{X}\right)} = \sqrt{(x_2-x_1)^2} = \sqrt{x_1^2+y_1^2-z_1^2} \tag{5}$$

positive undefined magnitude.

$$\text{If} \quad \begin{array}{ll} \left(\vec{X}\cdot\vec{X}\right) > 0 \ , & \left|\vec{X}\right| \text{ - has real} \\ \left(\vec{X}\cdot\vec{X}\right) = 0 \ , & \left|\vec{X}\right| \text{ - has zero} \\ \left(\vec{X}\cdot\vec{X}\right) < 0 \ , & \left|\vec{X}\right| \text{ - has imaginary magnitudes.} \end{array}$$

Vectors whose modulus are equal to zero called isotropic vectors. Isotropic vectors form isotropic cone of ${}^1R_3$ – Minkowski space, and it is defined with the following equation

$$x^2 + y^2 - z^2 = 0\ . \tag{6}$$

We have already explained clearly that the distance between two points is equal to the vector modulus which connects these points. By the way, the distance between $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ in ${}^1R_3$ – Minkoswki space is defined as below,

$$AB = \left|\overrightarrow{AB}\right| = \sqrt{\left(\overrightarrow{AB}\cdot\overrightarrow{AB}\right)} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 - (z_2-z_1)^2}$$

If we write the distance for points that are closer to each other as differentiation of coordinates, we get

$$ds^2 = dx^2 + dy^2 - dz^2\ . \tag{7}$$

In this distance, it is possible to see metrica which is non-euclidean and express Einstein's relativity theory.

Now, let's show occurrence of Galilean metric in ${}^1R_3$ – Minkowski space with $\Omega(x,y,y) \subset {}^1R_3$ partial sum.

In fact, for $M_1, M_2 \in \Omega$ we get

$$|M_1M_2| = \sqrt{\left(\overrightarrow{M_1M_2}\cdot\overrightarrow{M_1M_2}\right)} = |x_2 - x_1|$$

But, when $|x_2 - x_1| = 0$ the points $M_1(x_1,y_1,z_1)$ and $M_2(x_2,y_2,z_2)$ are not overlapped. The lenght between them corresponds to the sector which is equal to $|y_2 - y_1|$.

If we accept this sector's length as second distance, it is possible to see that the distance between the two points in increasing $\Omega(x,y,y)$ space which is equivalent to the concept of distance in Galilean geometry.

3. Theorem

Galilean geometry is available in planes which are tangent to isotropic of ${}^1R_3$ – Minkoswki space.

Proof: The equation of isotropic cone in ${}^1R_3$ – Minkowski space is

$$x^2+y^2-z^2=0 \quad .$$

Equation of plane which is tangent to isotropic cone at point (a,b,c) is written as

$$ax+by-cz=0 \, .$$

Since the coordinate of point (a,b,c) belongs to the isotropic cone, then it becomes

$$a^2+b^2-c^2=0 \quad .$$

Let's take any two points like $M(x_1,y_1)$ and $N(x_2,y_2)$ at Π – tangent plane. Since these points belong to tangent plane, then the following equations satisfy

$$ax_1+by_1-cz_1=0\,, \quad ax_2+by_2-cz_2=0\,.$$

By benefiting from the equation $c=\sqrt{a^2+b^2}$, we can get the following expressions

$$z_1=\frac{a}{\sqrt{a^2+b^2}}x_1+\frac{b}{\sqrt{a^2+b^2}}y_1$$

and

$$z_2=\frac{a}{\sqrt{a^2+b^2}}x_2+\frac{b}{\sqrt{a^2+b^2}}y_2 \quad .$$

For the distance between the two points formula in ${}^1R_3$ – Minkowski space, we get

$$|MN|^2=(x_2-x_1)^2+(y_2-y_1)^2-(z_2-z_1)^2$$

But, considering

$$(z_2-z_1)^2=\left[\frac{a}{\sqrt{a^2+b^2}}(x_2-x_1)-\frac{b}{\sqrt{a^2+b^2}}(y_2-y_1)\right]^2$$

if we substitute this expression to its own place, we get the following equation

$$|MN|^2=\left|\frac{b}{\sqrt{a^2+b^2}}(x_2-x_1)+\frac{a}{\sqrt{a^2+b^2}}(y_2-y_1)\right|^2 .$$

The magnitude which is expressed with this equation has $\{x_2-x_1,y_2-y_1\}$ coordinates in plane.

The coordinates of vector gives the length for projection which is in the direction of vector $\left\{\frac{b}{\sqrt{a^2+b^2}},\frac{a}{\sqrt{a^2+b^2}}\right\}$.

If $|x_2-x_1|=0$, then $|MN|^2=\left|\frac{a}{\sqrt{a^2+b^2}}(y_2-y_1)\right|^2$.

It is seen that, the distance between the two points in Π – tangent plane is proportional to the distance between the two points in Galilean geometry. This proves that the geometry in this plane is Galilean geometry.

4. Conclusion

The history of non-Euclidean geometry and especially one of the non-Euclidean which is called Galilean geometry briefly have been explained.

References

[1] Riemann, B., Uber die Hypothesen, welche der Geometrie zu Grunde liegen. Springer, Berlin, 1923.

[2] Klein, F., "Uber die sogenannte Nicht-Euklidische Geometrie," Gesammelte Math Abh I: 254-305, 311-343, 344-350, 353-383, 1921.

[3] Klein, F., Vorlesungen iiber nicht-Euklidische Geometrie. Springer, Berlin, 1928.

[4] Klein, F., Vergleichende Betrachtungen uber neure geometrische Forschungen. Gesammelte mathematische Abhandlungen, Vol. I, 1921, pp. 460-497. (English version is found in Sommerville, D. M. Y., Bibliography of Non-Euclidean Geometry, 2nd ed., Chelsea, New York, 1970.)

[5] Yaglom, I.M. A Simple Non-Euclidean Geometry and Its Physical Basis, by Springer-Verlag New York Inc. 1979.

[6] Vincent Hugh. Using Geometric Algebra to Interactively Model the Geometry of Euclidean and non-Euclidean Spaces. February, 2007.

[7] Артыкбаев А. Соколов Д.Д. Геометрия в целом в плоском пространстве-времени. Ташкент. Изд. «Фан». 1991 г.