Automation, Control and Intelligent Systems
2018; 6(4): 38-46
http://www.sciencepublishinggroup.com/j/acis
doi: 10.11648/j.acis.20180604.11
ISSN: 2328-5583 (Print); ISSN: 2328-5591 (Online)

Non-Fragile Control for Variable Sampling Period Network Control System with Actuator Failure

Xin Tang, Nan Xie*, Bin Xia, Nana Wang

College of Computer Science and Technology, Shandong University of Technology, Zibo, China

Email address:
tx0207@outlook.com (Xin Tang), xienan@sdut.edu.cn (Nan Xie), xiabin@sdut.edu.cn (Bin Xia), 1748462348@qq.com (Nana Wang)

*Corresponding author

To cite this article:
Xin Tang, Nan Xie, Bin Xia, Nana Wang. Non-Fragile Control for Variable Sampling Period Network Control System with Actuator Failure. *Automation, Control and Intelligent Systems*. Vol. 6, No. 4, 2018, pp. 38-46. doi: 10.11648/j.acis.20180604.11

Received: October 8, 2018; **Accepted**: November 19, 2018; **Published**: December 17, 2018

Abstract: The guaranteed cost control problem is studied in this paper for a class of nonlinear discrete-time systems with both time-varying parametric uncertainty and actuator failures. At present, the researches on related fields of networked control systems are relatively mature, and many achievements have been made. But at the same time, most of the researches are focused on linear networked control systems, and the research on nonlinear networked control systems is relatively less. The goal is to design a non-fragile state feedback control law so that the closed-loop system is asymptotically stable and the closed-loop cost function value is no more than a specified upper bound for all admissible uncertainties. Firstly, the system model is established by using the method of variable sampling period. Secondly, the sufficient conditions for the asymptotic stability of the closed-loop system are given by using Lyapunov stability theory. Thirdly, based on the above researches, a non-fragile state feedback controller is designed by using linear matrix inequality (LMI). In the end, through the study of this paper, the cost function of the system under the designed non-fragile guaranteed cost controller does not exceed the given upper bound. This paper considers the actuator failure, and gives the design method of the non-fragile guaranteed cost controller of the nonlinear network control system, and makes a contribution to the field of network control system.

Keywords: Network Control System, Non-Fragile Control, Actuator Failure, Varying Sampling Period, Nonlinear

1. Introduction

In the modern society, the Internet is widely used in various fields. Many high-tech fields and large enterprises, such as resource sharing, automated factories, robots manufactures, advanced aerospace undertakings and electrified transport vehicle manufacturres, depend heavily on the networks. Since the concept of Network Control Systems (NCS) was put forward in the early 1990s, it has attracted people's attention immediately and posed a new challenge to the traditional control theory and application [1].

Networked control system is a distributed feedback control system. It consists of two parts. On the one hand, sensors, controllers and actuators scattered in different geographical locations are connected by the network, they form a closed-loop control system [2-3], which facilitates the connection and data sharing between each component; On the other hand, the related control theory is introduced into network control to control the network system.

Compared with the traditional control system, the NCS has a tremendous impact on the industrial automation and intelligent technology level. Firstly, it has high control efficiency, and can display the real-time operation of the controlled objects, the statistical processes and results. Secondly, precision wise, NCS is more accurate and flexible. And the optimal control can be achieved. Finally, the participation of the network greatly improves the automation degree of the control system and realizes the integration of management and control [4-5]. However, due to the introduction of the network, many challenges are posing, such as time delay and packet loss, bring great difficulties to the analysis and design of the network control system [6-7]. Therefore, the research on network control system has drawn wide attention from scholars both at home and abroad. In recent years, many research results have been obtained on NCS, such as system modeling and stability analysis, robustness and H∞ control analysis, optimal guaranteed performance control, and controller design problems of related systems.

Zhang Jun did some research on time delay, for a linear networked control system with variable delay, the time delay is divided into some intervals by interval non-uniformity method. Different Lyapunov-Krasovskii functional is constructed in each interval, and a triple integral term is introduced. Then a new stability condition for the networked control system is obtained [8]. Considering the delay and uncertainty characteristics of the network control system, Yang Xinwei considered the case where the maximum allowable network delay and the lower limit are not zero. Then he adopted a state feedback controller without predetermined parameters and established a closed-loop networked control system model with state feedback. In addition, he proposed the conditions for robust stability of networked control systems [9]. For systems with time delay and packet loss, Wu Yongjian considered that input delay included both time delay and packet loss. He established a network control system model and proved the stability of the network control system model based on the feedback data delay, which effectively improved the stability of the system. As a result, networked control systems with time delay and packet dropout can run stably [10]. For the study of data packet loss, Shi Feifei established the packet loss as a random Bernoulli sequence and used the designed prediction controller to obtain the controller corresponding to each delay value. Then, the delay compensator selected the corresponding controller to act on the controlled object, established a stochastic system model, and obtained a compensation controller [11]. Aiming at data packet loss, Tian Shuo combined the computational form of the function integral inequality, which effectively improved the control system. In the system control process, the stability and security of the system are fully considered, thus improving the control form of the network control system with delay packet loss [12]. For uncertain time-delay systems with sensor and actuator failures, Zhou Xia models sensor and actuator faults as independent Bernoulli distribution sequences. The fault-tolerant control of sensor and actuator faults is studied, and the design method of stochastic stable fault-tolerant controller is given [13]. Pan Peng selected random sequences to describe actuator failures in the system. An event driven communication channel was established between the sensor and the controller, and the system model was designed. Stability analysis and controller design were also performed [14]. Song Juan studied a class of fault-tolerant control problems with NCS communication delays with time-varying actuator failures. Time-varying actuator failures were modeled as bounded time-varying parameters, and fault-tolerant controllers are designed on this basis [15]. In previous studies, most studies on nonlinear network control systems were based on fixed sampling periods. However, in the network control system, factors such as computer load, network effect, equipment failure and external interference would cause changes in the sampling period of the system. So it was necessary to study the time-varying sampling period network control system. Considering the sampling period uncertainty, Li Yuan modeled it as a discrete system. The period uncertainty was transformed into the matrix uncertainty by Taylor formula, and the system was transformed into a dynamic interval system [16]. Zhao Yan used the method of active variable sampling period to model the nonlinear continuous network control system with multi-packet transmission as a discrete switching system. Sufficient conditions for the closed-loop system to be asymptotically stable were obtained [17]. Aiming at the time-varying sampling period and delay in networked control systems, Fan Jinrong transformed the uncertainty of sampling period and delay into the uncertainty of system structural parameters through matrix Jordan transformation and decomposition, and established the discrete-time convex polyhedron uncertain system model [18].

In networked control systems, since industrial instruments and measurement control components are affected by their own physical and environmental factors, there are parameter disturbances during implementation. Even small disturbances may cause instability of the system. Therefore, the effect of parameter changes on the system performance must be considered in the design of the controller [19]. In recent years, the non-fragile problem of controller parameter uncertainty has attracted wide attention. Ma Weiguo studied the non-fragile guaranteed cost control problem for nonlinear systems with quantized and Markov chain losses. The networked control system was described as a Markov jumping system. Sufficient conditions for the existence of non-fragile guaranteed cost controllers for nonlinear systems with additive and multiplicative disturbances were given in the form of linear matrix inequalities [20]. Aiming at the control input constraints and controller gain disturbances, Gao Xingquan proposed a non-fragile guaranteed cost state feedback control method for a class of norm-bounded parameter uncertain linear systems, and derived a new sufficient condition for solving the constrained non-fragile guaranteed cost control law [21]. Yu Shuiqing studied the non-fragile guaranteed cost control problem for a class of nonlinear networked control systems with stochastic delay and controller gain disturbances [22]. Su Yakun studied the guaranteed cost control problem for a class of uncertain stochastic systems with interval delays. The purpose was to design a non-fragile guaranteed cost control rate and find the upper bound of the cost function [23].

This paper, designs a non-fragile controller for a class of nonlinear networked control systems with variable sampling period. In order to solve the problem of actuator failure, this paper refers to Li Yu's method. The function not only can describe the normal case and outage case but also describe the actuator partial degradation [24]. The sufficient conditions for the asymptotic stability of the closed-loop system are given by using Lyapunov stability theory. Based on the above researches, a non-fragile state feedback controller is designed by using linear matrix inequality (LMI). In the end, through the study of this paper, the cost function of the system under the designed non-fragile guaranteed cost controller does not exceed the given upper bound.

Symbol description: The symbol * indicates the block matrix in a symmetric matrix, A^T is the transpose matrix of A.

2. Problem Description

Consider the following non-linear controlled system:

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t) + f(t,x) \\ y(t) = Cx(t)\end{cases} \quad (1)$$

Where $x(t) \in R^n$、$u(t) \in R^m$ and $y(t) \in R^p$ represent the input state, control input and output state respectively. A, B and C are known matrices of appropriate dimensions. $f(t,x)$ is an in determinate nonlinear term that satisfies the Lipschitz condition:

$$||f(t,x) - f(t,y)|| \le ||\overline{G}(x-y)|| \quad (2)$$

Where $\overline{G}$ is a known constant matrix.

In this system, it is assumed that the state variable $x(t)$ is measurable, and its measurements are first discrete and then transmitted in a single package. $0 < d_k \le d$ expressed the bounded random delay from the sensor to the controller.

In this article, we use a variable sampling period method. If the time delay at sampling time k is d_k, then it is assumed that the variable sampling period T_k at sampling time k is equal to d_k. Therefore, to sample system (1) with variable sampling period T_k, the discretization model is:

$$\begin{cases}x(k+1) = A_k(T_k)x(k) + B_k(T_k)u(k) + \overline{f}(T_k,x) \\ y(k) = Cx(k)\end{cases} \quad (3)$$

where:

$$A_k(T_k) = e^{AT_k}, B_k(T_k) = \int_0^{T_k} e^{As}\,Bds$$

$$\overline{f}(T_k,x) = \int_0^{T_k} e^{As} f(s,x(s))ds \quad (4)$$

$$||f(T_k,x) - f(T_k,y)|| \le ||G(x-y)||$$

$$x(k) = x(t_k), y(k) = y(t_k), t_k = \sum_{l=1}^{k} T_l$$

In order to deal the actuator failure, we refer the function that is proposed by Li Yu [24]. And it is described as the follows.

For control input $u_i, i = 1,2 \dots m$, let u_i^F denotes the signal from the failed actuator. The following failure model is adopted in this paper

$$u_i^F = \alpha_i u_i, i = 1,2 \dots m \quad (5)$$

Where $0 \le \widehat{\alpha_i} \le \alpha_i \le \widetilde{\alpha_i}, i = 1,2 \dots m$ with $\widehat{\alpha_i} \le 1, \widetilde{\alpha_i} \ge 1$.

Define the α, as follows:

In the above model of actuator failure, if $\widetilde{\alpha_i} = \widehat{\alpha_i}$, then it corresponds to the normal case $u_i^F = u_i$; When $\widetilde{\alpha_i} = 0$, it covers the outage case. If $\widetilde{\alpha_i} > 0$ it corresponds to the partial failure case, i.e., partial degradation of the actuator.

Denote:

$$u^F = [u_1^F, u_2^F, \dots u_m^F]^T$$

$$\widetilde{\alpha} = diag\{\widetilde{\alpha_1}, \widetilde{\alpha_2}, \dots \widetilde{\alpha_m}\}$$

$$\widehat{\alpha} = diag\{\widehat{\alpha_1}, \widehat{\alpha_2}, \dots \widehat{\alpha_m}\} \quad (6)$$

$$\alpha = diag\{\alpha_1, \alpha_2, \dots \alpha_m\}$$

α is said to be admissible if α satisfies $\widehat{\alpha} \le \alpha \le \widetilde{\alpha}$.

So (3) can be represented by:

$$x(k+1) = Ax(k) + B\alpha u(k) + \overline{f} \quad (7)$$

For system (7) denote a cost function:

$$J = \sum_{k=0}^{\infty}[x^T(k)Qx(k) + u^T(k)\alpha^T R\alpha u(k)] \quad (8)$$

where $Q > 0$ and $R > 0$ are given weighting matrices.

The purpose of this paper is to design a non-fragile state feedback controller:

$$u(k) = (K + \Delta K)x(k) \quad (9)$$

such that the given closed-loop system:

$$x(k+1) = Ax(k) + B\alpha(K + \Delta K)x(k) + \overline{f} \quad (10)$$

is asymptotically stable , and cost function satisfies $J \le J^*$, J^* is a given constant.

In the state feedback controller (9), K is the nominal controller gain and ΔK is the disturbance of the gain. $\Delta K = DEF, F^T F \le I$. Where D and E are known matrices of appropriate dimensions, F is uncertain parameter matrix.

Lemma 1 [25]: If Q、H、E and R are real matrices of appropriate dimensions, Q and R are symmetrical, $R > 0$, Then $Q + HFE + E^T F^T H^T < 0$ holds for all the matrices F that satisfy $F^T F < R$. If and only if there exists a constant $\varepsilon > 0$ such that $Q + \varepsilon^2 HH^T + \varepsilon^{-2} E^T RE < 0$.

3. Main Results

3.1. Stability Analysis

Theorem 1: For the closed-loop system (7) and the cost function (8), the following matrix inequalities holds if there exists a matrix K, ΔK, $R > 0$ and a scalar $\Gamma > 0$

$$\begin{bmatrix}\Phi_{11} & \Phi_{12} \\ * & P - \Gamma^1 I\end{bmatrix} < 0 \quad (11)$$

where:

$$\Phi_{11} = [A + B\alpha(K + \Delta K)]^T P[A + B\alpha(K + \Delta K)] + \Phi_1$$

$$\Phi_{12} = [A + B\alpha(K + \Delta K)]^T P \quad (12)$$

$$\Phi_1 = -P + \Gamma^1 G^T G + Q + (K + \Delta K)^T \alpha^T R\alpha(K + \Delta K)$$

then system (7) is asymptotically stable and the cost function satisfies:

$$J = \sum_{k=0}^{\infty}[x^T(k)Qx(k) + u^T(k)\alpha^T R\alpha u(k)] < Trace(P) \quad (13)$$

Proof:

Construct Lyapunov function:

$$V(k) = x^T(k)Px(k) \quad (14)$$

Then :

$$\Delta V(K) = V(K+1) - V(K)$$

$$= x^T(k+1)Px(k+1) - x^T(k)Px(k)$$

$$= \phi_2^T P \phi_2 - x^T(k)Px(k) \tag{15}$$

$$\leq \phi_2^T P \phi_2 - x^T(k)Px(k) + \Gamma^1 x^T(k) G^T G x(k) - \Gamma^1 \bar{f}^T \bar{f}$$

$$= \begin{bmatrix} x(k) \\ \bar{f} \end{bmatrix}^T \begin{bmatrix} \phi_{11} - \phi_3 & \phi_{12} \\ * & P - \Gamma^1 I \end{bmatrix} \begin{bmatrix} x(k) \\ \bar{f} \end{bmatrix}$$

where:

$$\phi_2 = [A + B\alpha(K + \Delta K)]x(k) + \bar{f}$$

$$\phi_3 = (K + \Delta K)^T \alpha^T R \alpha (K + \Delta K) + Q \tag{16}$$

Using inequality (11), we get $\Delta V(k) \leq -x^T(k)\phi_3 x(k) < 0$. Due to $Q > 0, R > 0$. Using Lyapunov stability theory, so system (7) is asymptotically stable.

Summing both sides of the above inequality from 0 to ∞, we can get that:

$$V(\infty) - V(0) < -\sum_{k=0}^{\infty} x^T(k)[Q + (K+\Delta K)^T \alpha^T R \alpha (K + \Delta K)]\, x(k) \tag{17}$$

Use the system asymptotically stability and $J = \sum_{k=0}^{\infty}[x^T(k)Qx(k) + u^T(k)\alpha^T R \alpha u(k)]$ yield $J \leq V(0) = x^T(0)Px(0)$.

The upper bound of the system performance index that conclude from theorem 1 depends on the initial state x_0, if x_0 is a zero-mean random variable and satisfies $E = (x_0 {x_0}^T) = I$, then system performance index satisfies:

$$E\{J\} \leq E\{{x_0}^T P x_0\} = \mathrm{Trace}(P) \tag{18}$$

Define:

$$\beta = \mathrm{diag}\{\beta_1, \beta_2, \ldots \beta_m\}, \beta_0 = \mathrm{diag}\{\beta_{10}, \beta_{20}, \ldots \beta_{m0}\} \tag{19}$$

where:

$$\beta_i = \frac{\breve{\alpha}_1 + \hat{\alpha}_1}{2}, \beta_{i0} = \frac{\breve{\alpha}_1 - \hat{\alpha}_1}{\breve{\alpha}_1 + \hat{\alpha}_1} \; i = 1,2 \ldots m \tag{20}$$

$$\alpha = (I + \alpha_0)\beta, |\alpha_0| \leq \beta_0 \leq I \tag{21}$$

where

$\alpha = \mathrm{diag}\{\alpha_{01}, \alpha_{02}, \ldots \alpha_{0m}\}$, $|\alpha_0| = \mathrm{diag}\{|\alpha_{01}|, |\alpha_{02}|, \ldots |\alpha_{0m}|\}$.

3.2. Design of Non-Fragile Guaranteed Controller

On the basis of Theorem 1, the design method of non-fragile guaranteed cost controller is given below.

Theorem 2: For the closed-loop system (7) and the cost function (8), if there exists a matrix $X, Y > 0$ and a scalar $\Gamma > 0, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6 > 0$, the following matrix inequality holds:

$$\begin{bmatrix}
-X + (\varepsilon_3+\varepsilon_4)\beta^2 Y^T Y & 0 & (AX + B\beta Y)^T & 0 & \varepsilon_1 X E^T & XG^T & Y^T\beta^T & 0 & \varepsilon_2 X E^T & X & 0 & 0 & 0 & 0 \\
* & -\Gamma I & \Gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -X & B\beta D + \varepsilon_5 \beta^2 D^T D & 0 & 0 & 0 & 0 & 0 & 0 & B & 0 & B & 0 \\
* & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\Gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -R^{-1} & \beta D + \varepsilon_6 \beta^2 D^T D & 0 & 0 & 0 & I & 0 & 0 \\
* & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 & I \\
* & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -Q^{-1} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \varepsilon_3 I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & \varepsilon_4 I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & \varepsilon_5 I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & \varepsilon_6 I
\end{bmatrix} < 0 \tag{22}$$

Then $u(k) = YX^{-1}x(k)$ is the guaranteed cost controller of system (7) and the cost function satisfies:

$$J=\sum_{k=0}^{\infty}[x^T(k)Qx(k)+u^T(k)\alpha^T R\alpha u(k)]\le \mathrm{Trace}(P) \tag{23}$$

Proof:
Matrix inequality(11) can be transformed into:

$$\begin{bmatrix}\Phi_1 & 0\\ 0 & -\Gamma^1 I\end{bmatrix}+\begin{bmatrix}[A+B\alpha(K+\Delta K)]^T\\ I\end{bmatrix}P\begin{bmatrix}[A+B\alpha(K+\Delta K)]^T\\ I\end{bmatrix}^T<0 \tag{24}$$

Using Schur complement theorem, inequality(24) is equivalent to:

$$\begin{bmatrix}\Phi_1 & 0 & [A+B\alpha(K+\Delta K)]^T\\ * & -\Gamma^{-1}I & I\\ * & * & -P^{-1}\end{bmatrix}<0 \tag{25}$$

It equals to:

$$\begin{bmatrix}\Phi_1 & 0 & (A+B\alpha K)^T\\ * & -\Gamma^{-1}I & I\\ * & * & -P^{-1}\end{bmatrix}+\begin{bmatrix}0\\0\\B\alpha D\end{bmatrix}F\begin{bmatrix}E^T\\0\\0\end{bmatrix}^T+\begin{bmatrix}E^T\\0\\0\end{bmatrix}F^T\begin{bmatrix}0\\0\\B\alpha D\end{bmatrix}^T<0 \tag{26}$$

It follows from the Lemma1 complement that the above inequality is equivalent to:

$$\begin{bmatrix}\Phi_1 & 0 & (A+B\alpha K)^T\\ * & -\Gamma^{-1}I & I\\ * & * & -P^{-1}\end{bmatrix}+\varepsilon_1^{-1}\begin{bmatrix}0\\0\\B\alpha D\end{bmatrix}\begin{bmatrix}0\\0\\B\alpha D\end{bmatrix}^T+\begin{bmatrix}E^T\\0\\0\end{bmatrix}\begin{bmatrix}E^T\\0\\0\end{bmatrix}^T<0 \tag{27}$$

$$\begin{bmatrix}\Phi_1 & 0 & (A+B\alpha K)^T\\ * & -\Gamma^{-1}I & I\\ * & * & -P^{-1}\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \varepsilon_1^{-1}B\alpha D(B\alpha D)^T\end{bmatrix}<0 \tag{28}$$

Using Schur complement theorem, the upper form can be transformed into:

$$\begin{bmatrix}\Phi_1+\varepsilon_1 E^TE & 0 & (A+B\alpha K)^T & 0\\ * & -\Gamma^{-1}I & I & 0\\ * & * & -P^{-1} & B\alpha D\\ * & * & * & \varepsilon_1 I\end{bmatrix}<0 \tag{29}$$

Reusing Schur complement theorem, inequality (29) is equivalent to:

$$\begin{bmatrix}-P+Q & 0 & (A+B\alpha K)^T & 0 & \varepsilon_1E^T & G^T & (K+\Delta K)^T\alpha^T\\ * & -\Gamma^{-1}I & I & 0 & 0 & 0 & 0\\ * & * & -P^{-1} & B\alpha D & 0 & 0 & 0\\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0\\ * & * & * & * & -\varepsilon_1 I & 0 & 0\\ * & * & * & * & * & -\Gamma I & 0\\ * & * & * & * & * & * & -R^{-1}\end{bmatrix}<0 \tag{30}$$

The above mentioned:$\Delta K=DFE$, the above inequality can be written as:

$$\begin{bmatrix}-P+Q & 0 & (A+B\alpha K)^T & 0 & \varepsilon_1E^T & G^T & K^T\alpha^T+(DFE)^T\alpha^T\\ * & -\Gamma^{-1}I & I & 0 & 0 & 0 & 0\\ * & * & -P^{-1} & B\alpha D & 0 & 0 & 0\\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0\\ * & * & * & * & -\varepsilon_1 I & 0 & 0\\ * & * & * & * & * & -\Gamma I & 0\\ * & * & * & * & * & * & -R^{-1}\end{bmatrix}<0 \tag{31}$$

The above inequality can be written as:

$$\begin{bmatrix} -P+Q & 0 & (A+B\alpha K)^T & 0 & \varepsilon_1 E^T & G^T & K^T\alpha^T \\ * & -\Gamma^{-1}I & I & 0 & 0 & 0 & 0 \\ * & * & -P^{-1} & B\alpha D & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\Gamma I & 0 \\ * & * & * & * & * & * & -R^{-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha D \end{bmatrix} F \begin{bmatrix} E^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} E^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha^T D^T \end{bmatrix} < 0 \tag{32}$$

Using Lemma1, (32)is equivalent to:

$$\begin{bmatrix} -P+Q & 0 & (A+B\alpha K)^T & 0 & \varepsilon_1 E^T & G^T & K^T\alpha^T \\ * & -\Gamma^{-1}I & I & 0 & 0 & 0 & 0 \\ * & * & -P^{-1} & B\alpha D & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\Gamma I & 0 \\ * & * & * & * & * & * & -R^{-1} \end{bmatrix} + \varepsilon_2^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha D \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha D \end{bmatrix}^T + \varepsilon_2 \begin{bmatrix} E^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0 \tag{33}$$

At this point, we use the Schur theorem to get the following inequality:

$$\begin{bmatrix} -P & 0 & (A+B\alpha K)^T & 0 & \varepsilon_1 E^T & G^T & K^T\alpha^T & 0 & \varepsilon_2 E^T & I \\ * & -\Gamma^{-1}I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -P^{-1} & B\alpha D & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\Gamma I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R^{-1} & \alpha D & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0 \tag{34}$$

The left and right sides of (34) are respectively multiplied by the diagonal $\mathrm{diag}\{P^{-1}, \Gamma I, I, \dots, I\}$ and let $X = P^{-1}, Y = KX$, the following inequality is obtained.

$$\begin{bmatrix} -X & 0 & (A+B\alpha Y)^T & 0 & \varepsilon_1 XE^T & XG^T & Y^T\alpha^T & 0 & \varepsilon_2 XE^T & X \\ * & -\Gamma I & \Gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -X & B\alpha D & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\Gamma I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R^{-1} & \alpha D & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0 \tag{35}$$

Take (21) in the inequality,(35) can be written as:

$$\begin{bmatrix} -X & 0 & [AX+B(I+\alpha_0)\beta Y]^T & 0 & \varepsilon_1 XE^T & XG^T & Y^T[(I+\alpha_0)\beta]^T & 0 & \varepsilon_2 XE^T & X \\ * & -\Gamma I & \Gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -X & B(I+\alpha_0)\beta D & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\Gamma I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R^{-1} & (I+\alpha_0)\beta D & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0 \tag{36}$$

It equals to:

$$\begin{bmatrix} -X & 0 & (AX+B\beta Y)^T & 0 & \varepsilon_1 XE^T & XG^T & Y^T\beta^T & 0 & \varepsilon_2 XE^T & X \\ * & -\Gamma I & \Gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -X & B\beta D & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\Gamma I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R^{-1} & \beta D & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -Q^{-1} \end{bmatrix} + \begin{bmatrix} 0\\0\\B\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \alpha_0 \begin{bmatrix} \beta Y^T\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T + \begin{bmatrix} \beta Y^T\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \alpha_0^T \begin{bmatrix} 0\\0\\B\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T +$$

$$\begin{bmatrix} 0\\0\\0\\0\\0\\0\\I\\0\\0\\0 \end{bmatrix} \alpha_0 \begin{bmatrix} \beta Y^T\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T + \begin{bmatrix} \beta Y^T\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \alpha_0^T \begin{bmatrix} 0\\0\\B\\0\\0\\0\\I\\0\\0\\0 \end{bmatrix}^T + \begin{bmatrix} 0\\0\\0\\\beta D^T\\0\\0\\0\\0\\0\\0 \end{bmatrix} \alpha_0^T \begin{bmatrix} 0\\0\\B\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T + \begin{bmatrix} 0\\0\\B\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \alpha_0 \begin{bmatrix} 0\\0\\0\\\beta D^T\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T + \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\\beta D^T\\0\\0 \end{bmatrix} \alpha_0^T \begin{bmatrix} 0\\0\\0\\0\\0\\0\\I\\0\\0\\0 \end{bmatrix}^T + \begin{bmatrix} 0\\0\\B\\0\\0\\0\\I\\0\\0\\0 \end{bmatrix} \alpha_0 \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\\beta D^T\\0\\0 \end{bmatrix}^T < 0 \quad (37)$$

Using Lemma1, the above inequality can be written as :

$$\begin{bmatrix} -X & 0 & (AX+B\beta Y)^T & 0 & \varepsilon_1 XE^T & XG^T & Y^T\beta^T & 0 & \varepsilon_2 XE^T & X \\ * & -\Gamma I & \Gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -X & B\beta D & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\Gamma I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -R^{-1} & \beta D & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & * & * & -Q^{-1} \end{bmatrix} + \varepsilon_3^{-1} \begin{bmatrix} 0\\0\\B\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\B\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T + \varepsilon_3 \begin{bmatrix} \beta Y^T\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} \beta Y^T\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T +$$

$$\varepsilon_4^{-1} \begin{bmatrix} 0\\0\\0\\0\\0\\0\\I\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\B\\0\\0\\0\\I\\0\\0\\0 \end{bmatrix}^T + \varepsilon_4 \begin{bmatrix} \beta Y^T\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} \beta Y^T\\0\\0\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T + \varepsilon_5^{-1} \begin{bmatrix} 0\\0\\0\\\beta D^T\\0\\0\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\0\\\beta D^T\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T + \varepsilon_5 \begin{bmatrix} 0\\0\\B\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\B\\0\\0\\0\\0\\0\\0\\0 \end{bmatrix}^T + \varepsilon_6^{-1} \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\\beta D^T\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\0\\0\\0\\0\\0\\\beta D^T\\0\\0 \end{bmatrix}^T + \varepsilon_6 \begin{bmatrix} 0\\0\\B\\0\\0\\0\\I\\0\\0\\0 \end{bmatrix} \begin{bmatrix} 0\\0\\B\\0\\0\\0\\I\\0\\0\\0 \end{bmatrix}^T < 0 \quad (38)$$

For the formula (38), we use the Schur theorem to get the following inequality (39):

$$\left[\begin{array}{ccccc} -X+(\varepsilon_3+\varepsilon_4)\beta^2 Y^T Y & 0 & (AX+B\beta Y)^T & 0 & \varepsilon_1 XE^T \\ * & -\Gamma I & \Gamma I & 0 & 0 \\ * & * & -X & B\beta D+\varepsilon_5\beta^2 D^T D & 0 \\ * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & -\varepsilon_1 I \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{array}\right.$$

$$\begin{bmatrix} XG^T & Y^T\beta^T & 0 & \varepsilon_2 XE^T & X & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & B & 0 & B & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\Gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -R^{-1} & \beta D + \varepsilon_6 \beta^2 D^T D & 0 & 0 & 0 & I & 0 & 0 \\ * & * & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 & I \\ * & * & * & -\varepsilon_2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \varepsilon_3 I & 0 & 0 & 0 \\ * & * & * & * & * & * & \varepsilon_4 I & 0 & 0 \\ * & * & * & * & * & * & * & \varepsilon_5 I & 0 \\ * & * & * & * & * & * & * & * & \varepsilon_6 I \end{bmatrix} < 0 \qquad (39)$$

The proof is completed.

4. Conclusion

In work process of the actual network control system, there will inevitably be some fault conditions, such as controlled system failure, actuator failure, sensor failure and so on. Among them, actuator failure is the main cause of system control performance failure. Therefore, the fault-tolerant research in this paper is mainly for the case of actuator failure. This paper studies non-fragile guaranteed cost control of a nonlinear networked control system with variable sampling period and actuator failure. In this paper, a system fault model with parameter α is established by digitizing the actuator fault. The actuator fault is described as a fault matrix within a bounded continuous number interval. Based on this assumption, the Lyapunov function of the controlled system is designed, and the sufficient conditions for the asymptotic stability of the system are obtained by using Lyapunov stability theorem. The design method of the system's non-fragile guaranteed cost controller is given by linear matrix inequality technique. Through the research of this paper, it is concluded that the system can run stably under the condition of the actuator failure in the actual working process. At the same time, the condition that the system meets the guaranteed cost state is given. And the method of designing non-fragile controllers for maintaining guaranteed performance with the system is also given.

References

[1] ANTSAKLIS P, BAILLIEUL J. Special issue on Technology of Networked Control Systems [J]. Proceedings of the IEEE, 2007, 95(1): 5-8.

[2] You K Y, Xie L H. Survey of Recent Progress in Networked Control Systems [J]. ACTA AUTOMATICA SINICA, 2013, 39(2): 101−118.

[3] Ren J X, Liu L M. Survey of Networked Control Systems [J]. Industrial Control Computer, 2016, 29(8): 91-92.

[4] Wei L. Analysis of the Development of Networked Control Systems [J]. Technology Outlook, 2016 (05): 14-14.

[5] Dong Z J. Modeling and Stability Analysis of Networked Control Systems [D]. North China Electric Power University, 2015.

[6] Walsh G C, Beldiman O, Bushnell L G. Asymptotic Behavior of Nonlinear Networked Control Systems [J]. IEEE Trans. On Automatic Control, 2001, 46(7): 1093-1097.

[7] Zhang W, Branicky M S, Phillips S M. Stability of Networked Control Systems [J]. IEEE Control System Magazine, 2001, 21(1): 84-99.

[8] Zhang J, Luo D Y, Sun M P. A New Stability Condition for Networked Control Systems with Time-Varying Delays Based on Interval Inequality [J]. ACTA ELECTRONICA SINICA, 2016, 44(1): 54-59.

[9] Yang X W, Guo C X. Stability Design of Networked Control Systems with Uncertain Time Delay [J]. Modern Electronic Technology, 2016, 39 (16): 10 -13.

[10] Wu Y J, Chen Y D, Zhou Z. Research on Stability of Networked Control Systems Based on Feedback Data Delay [J]. Journal of XinYu University, 2017, 22 (3): 31-34.

[11] Shi F F, Huang L, Han J W. Time Delay Compensation for Networked Control Systems with Packet Loss [J]. Journal of Harbin University of Science and Technology, 2017, 22 (3): 36-41.

[12] Tian S. Control Study of Networked Control Systems with Time Delay Packet Loss [J]. Applied Mechanics and Materials, 2016(22): 23-23.

[13] Zhou X, Zhong S M, Kang W, Hu Y G. Random Fault-Tolerant Design of A Class of Networked Control Systems with Stochastic Time-Delay [J]. Journal of Fuyang Normal University, 2013, 30 (3): 1-3.

[14] Pan P, Jiang S, Pan F. Event-Triggered Robust Control for Networked Control Systems with Actuator Failures and Time-varying Transmission Delays [J]. International Journal of Intelligent Computing and Cybernetics, 2015, 8(2): 172-186.

[15] Song J. Fault-Tolerant Control of Networked Control Systems with Time-Varying Actuator Faults [J]. Space Control, 2016 (5): 3-8.

[16] Li Y, Zhang G S, Sun K Q. Guaranteed Cost Control for Networked Control Systems with Time-Varying Bounded and Variable Sampling [J]. Journal of Shenyang Chemical Engineering University, 2016, 30 (4): 378-384.

[17] Zhao Y, Wang Y L. Nonlinear NCS Fault Detection with Variable Sampling Period and Multi-Packet Transmission [J]. Computer measurement and control, 2016, 24 (5): 55-58.

[18] Fan JR, Fang H J. Robust fault-tolerant control for networked systems with variable sampling period [J]. Control Engineering, 2013, 20 (5): 859-863.

[19] Lan Y P, Chen Q L, Hu X C, Liu Y F. Research on Non-Fragile Robust Control of Maglev Permanent Magnet Linear Motor Control System [J]. Journal of Electrical Technology, 2016, 31 (7): 26-32.

[20] Ma W G, Yang Z. Non-Fragile Guaranteed Cost Control for Networked Nonlinear Systems [J]. Computer Engineering and Design, 2014 (10): 3363-3366.

[21] Gao X Q, Hu Y F. Robust Non-Fragile Guaranteed Cost Control with Control Input Constraints [J]. Journal of Shenyang University of Technology, 2015, 37 (5): 571-576.

[22] Yu S Q. Analysis and Synthesis of Networked Control Systems. Xi'an Electronic and Science University, 2014.

[23] Su Y K, Ye F T. Design of a Non-Fragile Guaranteed Cost Controller for Stochastic Systems with Interval Delay [J]. Journal of Bohai University (NATURAL SCIENCE EDITION), 2016, 37 (1): 62-69.

[24] Yu L. "An LMI Approach to Reliable Guaranteed Cost Control of Discrete-time Systems with Actuator Failure."Applied Mathematics & Computation, 2005 (162.3): 1325-1331.

[25] Xie L H. Output Feedback H∞ Control of Systems with Parameter Uncertainty [J]. International Journal of Control, 1996, 63(4): 741-750.