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Wear and Reliability Life of Large Modulus Gear Rack

Wang Decheng[1], Chen Li[1, *], Cheng Peng[1, 2], Liu Hongqi[1, 2], Shao Chenxi[1, 2]

[1]China Academy of Machinery Science & Technology, Beijing, China
[2]China Productivity Center for Machinery, Beijing, China

Email address:
black_chenli@hotmail.com (Chen Li)
[*]Corresponding author

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Abstract: In this paper, based on the Archard wear theory, numerical simulation method and Hertz contact theory, the numerical simulation model of gear rack wear is established under the influence factor of load and hardness for the wear condition of large modulus gear rack. The simulation calculation of its wear process and wear life is realized through the Matlab software. Results show: Along the tooth profile, wear rate of tooth root is greater than the crown. Maximum wear position is decided by comprehensive influence of contact stress and slip distance. The analysis calculation of large module gear rack's wear process and life can be solved through the method of combining experiment and numerical simulation for good engineering application.

Keywords: Gear Rack, Wear, Reliability Life, Numerical Simulation

1. Introduction

With high transmission torque and motion accuracy, the large modulus straight tooth cylindrical gear rack structure is more and more widely used in the field of large heavy duty machinery and engineering.

The polygon effect of gear rack will increase with the increase of the modulus, which lead to the increase of the instantaneous transmission ratio fluctuation. So its reliable operation and service life prediction have been a major concern and in-depth study from design to use [1].

At present, it's widely used to study on wear life through the large number of simulation tests. The cost of the test is enormous, the simulation validity and accuracy of the time varying wear factors are also limited. In addition, the kinds of wear theory is only to expressed the initial or end wear. And the detailed and in-depth study of dynamic influencing factors in wear process is very little [2, 3].

In the paper [4], using Archard and Hertz theory the wear process of gear tooth profile is calculated without the time varying factors such as surface hardness, wear depth and so on. Jiang Qinyu et al. [5–8] study on the wear of cam, gear and other spare parts with the numerical simulation technology, but the effect of stress is only considered in most studies, and the coincidence degree of the engineering practice is not satisfactory [9–12].

The large modulus gear rack structure of the ship lift as the object of study, the wear model is established based on Archard and Hertz theory. With the measured data of hardness under different wear depth, the whole wear process is numerical simulated considering the influence of contact stress and hardness. The wear process simulation analysis and reliability life prediction of the large module gear rack are researched.

2. Analysis of the Gear Rack Wear Condition Parameters

The gear rack drive of this ship lift is the open transmission. The main failure form is wear. As we known, the wear process includes running-in wear stage, stable wear stage and severe wear stage whose wear rate is descending, invariant and ascending. In order to ensure the normal function of the friction parts, it must be in stable wear stage after running-in stage. The tooth profile change caused by wear can lead to shock, vibration and noise of gear rack, so its life is shorten. It will cause the fastening of the transmission mechanism is easy to loosen, and so much so that can lead to major accidents by the critical parts fatigue failure. Therefore, it is necessary to study on the influence of wear for the rack life.

2.1. Radius of Curvature at Different Meshing Positions

The contact stress of gear teeth is changed in the meshing process of double teeth or single tooth meshing area of involute cylindrical gear rack. With the gear rotation, the curvature radius is also changed which leads to the change of synthetical curvature radius. Because the rack tooth profile is a straight line, the synthetical curvature radius R_x can be calculated according to the geometric relation, which is any point curvature radius on gear tooth profile:

$$R_x = \sqrt{|r_x^2 - r_b^2|} \tag{1}$$

where r_x is the distance from gear center to any point x on rack tooth profile, r_b is the base circle radius of gear. Taking a length x on the contact profile L, tooth crown E as starting point, according to the geometric relation, the distance from any point x to gear center is:

$$r_x = r_1 - \left(\frac{L}{2} - x\right) \cdot cos\alpha \ x \in [0, L] \tag{2}$$

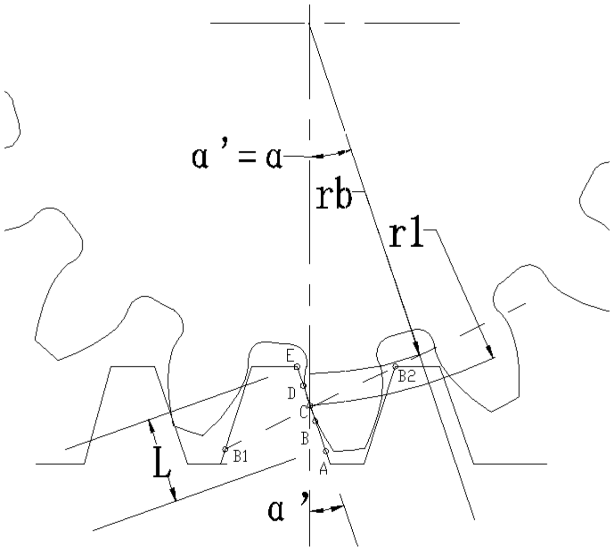

Figure 1. *Schematic diagram of gear rack meshing area.*

According to the gear rack meshing condition, as Figure 1, let A is the starting point of meshing, single tooth meshing area is (B, D), double teeth meshing area are (A, B) and (D, E). According to the continuous transmission requirement, known as the gear rack actual coincidence degree is $\varepsilon \in (1, 2)$, AE = L, double teeth meshing area is $x \in [0, DE) \cap (\mathrm{BE}, \mathrm{L})$, single tooth meshing area is $x \in [DE, BE]$.

Therefore, by Eqs. (1) and Eqs. (2) the curvature radius R_x on any position of gear rack contact profile is:

$$R_x = \sqrt{\left|\left(r_1 - (\frac{L}{2} - x) \cdot cos\alpha\right)^2 - r_b^2\right|} \ x \in [0, L] \tag{3}$$

2.2. Maximum Contact Stress

By using Hertz contact theory the contact stress σ of each point is calculated in the contact area of the tooth profile. Cause the width of contact area is smaller than the length of the whole tooth profile, the curvature radius change of width direction is ignored in this case. Therefore the maximum contact stress of the contact area (line) is:

$$\sigma = \sqrt{\frac{F\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}{\pi B\left[\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right]}} = \sqrt{\frac{T/(R_x \cdot r_1)}{\pi b\left[\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right]}} \tag{4}$$

where E_1/E_2 is the elastic modulus of gear/rack materials, μ_1/μ_2 is the poisson ratio of gear/rack materials, b is the gear width, R_1/R_2 is the contact curvature radius of gear/rack on the tooth profile, $R_1 = R_x$, $R_2 \to \infty$, α is the pressure angle of the contact point, F is the load of gear rack, $F = T/r_1$, T is the torque of gear rack, r_1 is the dividing circle radius of gear.

It is assumed the load is shared by contact teeth in the double teeth meshing area, so the maximum contact stress exists only in the single tooth meshing area of the gear rack. By Eqs. (3) and Eqs. (4), in the single tooth meshing area (DE, BE) the maximum contact stress is:

$$\sigma_{max} = \max(\varphi(x))$$
$$x \in \left[\frac{(\varepsilon-1)}{2} \times L, \frac{(3-\varepsilon)}{2} \times L\right] \tag{5}$$

2.3. Relationship Between Contact Surface Hardness and Depth

In practical application of engineering, the key parts of the important equipment are used to be surface treated for well hardness and other performance. Based on the analysis of the hardness and depth data with the same batch and process, the function fitting relation of hardness and depth is established. The recurrence formula of contact surface hardness is:

$$H_n = \mathrm{f(h)} + H_0 \tag{6}$$

where H_n is the contact surface hardness after n wear, H_0 is the initial surface hardness, h is the wear depth, $f(h)$ is the hardness and depth fitting function.

Thus the different depth hardness can be calculated by the surface hardness. The calculation accuracy is decided by the approximation of the fitting function $f(h)$. In case of no internal defect, the gear rack hardness is generally decreased continuously from surface to the center. So the nonlinear square method is used to fit their functional relation. Its accuracy is the highest which is verified by the measured data of an practical example.

3. Wear Life Calculation Model Based on Archard Model

Archard wear model was proposed by British professor J. F. Archard in 1953. It is suitable for research of elastomer wear characteristics that is similar to abrasion mechanism of the large module gear rack pair. So it can be used to predict the wear life:

$$\frac{V}{S} = \frac{k \cdot F_n}{H} \tag{7}$$

where V is the Volume wear, S is the slip distance, F_n is the normal acting force, k is the dimensionless wear coefficient, H is the material Brinell hardness. In Archard model, the wear coefficient is defined as the probability of producing debris, but in fact it contains all the factors that affect the wear except load F_n, slip distance S and material hardness H. There is a large number of experiments and researches for the Archard wear model by many scholars at home and abroad. Therefore, there is a lot of reference materials for wear coefficient value.

3.1. Calculation Model of Wear Depth

If the gear rack contact area is S_W:

$$\begin{cases} V = S_W \cdot h \\ \sigma = \frac{F_n}{S_W} \end{cases} \tag{8}$$

So Eqs. (7) can be written as:

$$h = \frac{k \cdot S \cdot \sigma}{H} \tag{9}$$

From the above, the functional relation about wear depth and coefficient, slip distance, contact stress and hardness is given. A generous of experiments have been confirmed the proportional relation of rack wear, slip distance and surface hardness. Under the condition of limited load range the wear volume is proportional to the load. Therefore, the wear depth h_1 after first single side meshing and initial surface hardness H_0 are:

$$\mathrm{h}_1 = \frac{k \cdot L \cdot \sigma_{max}}{H_0} \tag{10}$$

By Eqs. (6) the wear depth h_2 after second single side meshing and the contact surface hardness H_1 after the first meshing are:

$$\begin{cases} \mathrm{h}_2 = \frac{k \cdot L \cdot \sigma_{max}}{H_1} + h_1 \\ H_1 = \mathrm{f}(h_1) + H_0 \end{cases} \tag{11}$$

After the n times meshing, the computational model of cumulative wear depth is:

$$h_n = \frac{k \cdot L \cdot \sigma_{max}}{f(h_{n-1}) + H_0} + h_{n-1} \tag{12}$$

3.2. Wear Life Calculation

Rack wear life is the operate time under the condition of that rack wear is less than the wear failure threshold. According to the tooth surface wear failure criterion, the maximum allowable wear depth h_{max} can be calculated. With the Matlab program termination end condition set to $h \geq h_{max}$, the maximum allowable working engagement times n_{max} is available, the large modulus gear rack wear life is:

$$N = \frac{n_{max}}{n_L} \tag{13}$$

where n_L is the unit time meshing times of gear rack under the normal operating condition.

4. Numerical Simulation of Reliability Life

First, for calculation of gear rack reliability life, the accurate failure criterion is required. AGMA, DIN and JB/T5664 are presented the failure criterion of gear rack tooth wear. But in the engineering application, the actual application requirements should be considered, such as the correct mesh, tooth surface hardness and roughness and so on, for reliability and service life.

Therefore, in this paper, the correct mesh of the large module gear rack should be the failure criterion in operation process. It is the requirement of coincidence degree. When it is greater than the maximum transverse coincidence degree or less than the allowable coincidence degree, there is the gear rack failure. So the full probability of the gear rack correct mesh is the reliability:

$$R = P[(\varepsilon - \varepsilon_{amax}) < 0 \cup (\varepsilon - [\varepsilon]) > 0] \tag{14}$$

where ε_{amax} is the maximum transverse coincidence degree, $[\varepsilon]$ is the allowable coincidence degree for continuous drive requirement.

Monte Carlo method is a numerical calculation to solve the approximate solution of engineering problems. Through the statistical sampling test or stochastic simulation about the random variable, the statistics of function are estimated and described. The contact profile is considered as a random variable under the correct meshing condition in this study. Then the wear probability distribution and reliability of wear life are given. The specific simulation steps are as follows:

(1) Using the Matlab software to generate the correct meshing contact profile of the gear rack with a given sample capacity N of the known distribution.

(2) With the allowable wear depth threshold, the corresponding wear life data is calculated by the model.

(3) According to the life data to draw a histogram for judging and assuming which kind of distribution it is.

(4) Calculating the mean, variance and distribution function, the test of distribution fit is carry out by Kolmogorov-Smirnov method.

(5) The reliability is available by its known life.

5. Scheme and Example of Numerical Simulation

According to the characteristics of large modulus, small amount, heavy load and so on, the accurate life reliability of the components can be calculated under the real condition by the comprehensive analysis method of mathematical model and actual measurement data. In this paper, the complete and effective simulation analysis method is established based on the known model parameters and the experimental data of some key performance parameters. The specific scheme and example are as follows.

5.1. Input Parameters and Calculation Block Diagram

This numerical simulation process is written by the Matlab software. The required input parameters include: gear rack modulus m, number of gear teeth z, modification coefficient x, pressure angle *alpha*, gear and rack material elastic modulus E_1 and E_2, density ρ_1 and ρ_2, Poisson ratio μ_1 and μ_2, driving torque T, initial contact length L, wear coefficient k, working meshing times n, allowable maximum wear depth $h_{\max}$, initial surface hardness H_0, gear rack surface hardness and corresponding depth (soft tooth surface). The calculation block diagram of wear numerical simulation is as follows:

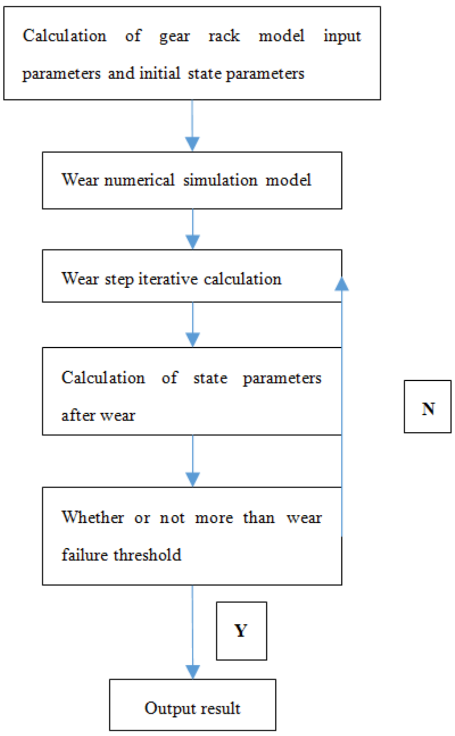

Figure 2. *Calculation block diagram of gear rack wear life.*

5.2. An Example

As known relevant parameters of a ship lift's large modulus gear rack:

gear rack modulus *m=63*, number of gear teeth *z=16,* modification coefficient *x=0.5*, pressure angle *alpha=20°*, gear and rack material elastic modulus E_1*=209000 MPa* and E_2*=202000 MPa*, density ρ_1=ρ_2=7850, Poisson ratioμ_1 =0.271 and μ_2,=0.3, driving torque *T=453000 KN*mm*, initial contact length *L=120 mm*, wear coefficient $k = 0.3 \times 10^{-6}$, working meshing times $n = 1.0 \times 10^4$, (considering the tooth surface wear failure criterion, the requirement of continuous drive and the effective hardened layer depth of tooth surface) allowable maximum wear depth h_{max}*=1.5 mm*, tooth surface quench hardening, initial surface hardness H_0=596 HB, depth of quenching is 6 mm, the hardness and depth measurement data as shown in Figure 3:

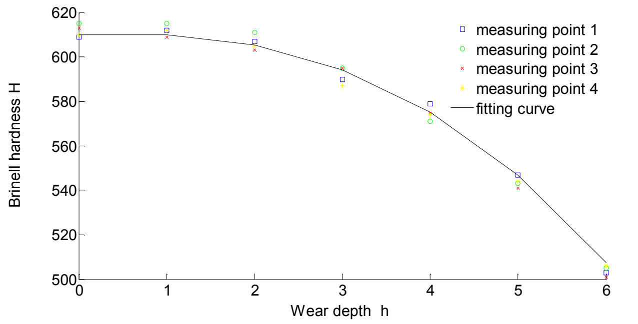

Figure 3. *Hardness and surface to effective hardened depth distance.*

The polynomial fitting relationship is:

$$H = -0.25h^3 - 1.6786h^2 + 2h + 596 \qquad (14)$$

Under the certain conditions of working environment, load and operational parameters, Eqs. (4) and Eqs. (13) show that gear rack wear depends on the contact slipping distance and contact stress. The wear of tooth profile is calculated by numerical simulation that is shown in Figure 4. Tooth crest and root are nearby double teeth meshing area. But at tooth crown area the composite motion of gear rack contact pair is mainly sliding. At the node 30, the influence of slip distance and contact stress is the largest, the wear is largest too. For the whole profile, slip distance of tooth root is greater than crown, so its wear is also larger.

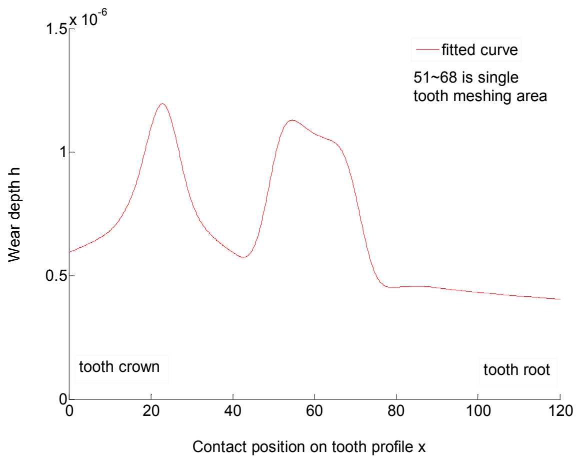

Figure 4. *The wear of rack along tooth profile.*

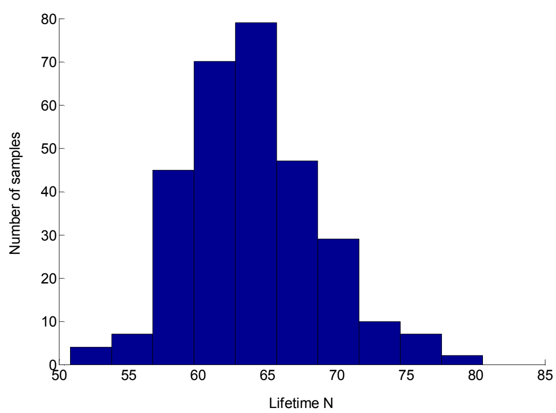

Figure 5. *The wear life histogram.*

The sample space capacity of the wear reliability analysis is selected as 1000. The random variable is the gear rack contact profile length that is the coincidence degree. Let it be distributed normally, the 1000 groups of wear life are calculated. Wear life distribution is shown in Figure 5, under the significant level of 0.05, distribution fit test results show that the wear life is distributed gamma.

So the wear life reliability is given in the following table 1. The reliability increase with the decrease of the predicted wear life.

Table 1. *Prediction and reliability of the wear life year.*

Predicted wear life *N*	Reliability *R*
75	0.0135
65	0.4054
60	0.7949
55	0.9743
40	0.9992

6. Conclusions

The wear of tooth profile is affected by the contact stress and the slip distance, and changed with the different meshing position. The wear of tooth root is greater than the tooth crown. The wear of maximum contact stress or slip distance is not largest, it is need to analysis the comprehensive effect.

The mathematical model can be applied to the wear simulation of all kinds gear rack, the data handling of wear condition monitoring and its prediction of reliability life. Study on the wear process of gear rack structure of the large heavy-load equipment, the research method of combining key measured data and numerical simulation analysis has a good prospect in engineering application.

The program is used to calculate reliability of large modulus gear rack wear probability life for different working performance requirements to establish the corresponding failure threshold. There is important reference value for the operation and maintenance of gear rack device.

The wear coefficient is suggested to obtain or correct through the working condition simulation test or the actual operation for that the accuracy of wear parameters and life are improved.

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