

# Globally Attractive of a Ratio-Dependent Lotka-Volterra Predator-Prey Model with Feedback Control

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**Abstract:** By constructing suitable Lyapunov function and developing some new analysis techniques, a Lotka-Volterra predator-prey system with ratio-dependent functional responses and feedback controls is studied and a sufficient condition which guarantees the globally attractive of positive solution for the predator-prey model is obtained. Moreover, the numerical simulation to the system is given to illustrate our results.

**Keywords:** Predator-Prey System, Feedback Control, Ratio-Dependent, Globally Attractive

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## 1. Introduction

As is well known, an ecosystem in the real world is continuously distributed by some forces, which can result in changes in the biological parameters such as survival rates. In ecology, we know that the practical question of interest is just whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables. This is very significant question in the control of ecology balance. One of

the methods for the realization of it is to alter the system structurally by introducing feedback control variables. The feedback control mechanism might be implemented by means of some biological control schemes or by harvesting procedure. In fact, during the last decade, the qualitative behaviour of the population dynamics with feedback control has been studied extensively. In [1], Yin and Li proposed the following single species model with feedback regulation and distributed time delay of the following form

$$\begin{cases} \dot{N}(t) = N(t)(a(t) - b(t) \int_0^\infty H(s) N(t-s) ds - c(t)u(t)), \\ \dot{u}(t) = -d(t)u(t) + e(t) \int_0^\infty H(s) N^2(t-s) ds, \end{cases} \quad (1)$$

where  $N(t)$  denotes the density of species at time  $t$  and  $u(t)$  is the regulator. By using the continuation theorem of coincidence degree theory, a criterion which guarantees the existence of positive periodic solution of system (1) is obtained. Furthermore, Chen [2] obtains sufficient condition which guarantees the global attractivity of the positive

solution of system (1) by constructing a suitable Lyapunov function.

In 2009, Nie et al [3] consider the following non-autonomous predator-prey Lotka-Volterra system with feedback controls

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t)[b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) + c_1(t)u_1(t)], \\ \frac{dx_2(t)}{dt} = x_2(t)[-b_2(t) + a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - c_2(t)u_2(t)], \\ \frac{du_1(t)}{dt} = f_1(t) - e_1(t)u_1(t) - d_1(t)x_1(t), \\ \frac{du_2(t)}{dt} = -e_2(t)u_2(t) + d_2(t)x_2(t). \end{cases} \quad (2)$$

where  $x_1(t)$  is the prey population density and  $x_2(t)$  is the predator population density,  $b_1(t)$ ,  $a_{11}(t)$ , are the intrinsic growth rate and density-dependent coefficient of the prey, respectively;  $b_2(t)$ ,  $a_{22}(t)$ , are the intrinsic growth rate and density-dependent coefficient of the predator, respectively;  $a_{12}(t)$  is the capturing rate of the predator and  $a_{21}(t)$  is the rate of conversion of nutrient into the reproduction of the predator;  $u_i(t)$  ( $i=1,2$ ) are control variables. Authors study whether or not the feedback controls have an influence on the permanence of a positive solution of the general non-autonomous predator-prey Lotka-Volterra type systems, and establish the general criteria on the permanence of system

(2), which is independent of some feedback controls. In additional, by constructing a suitable Lyapunov function, some sufficient conditions are obtained for the global stability of any positive solution to system (2). More work on feedback controls can be found in [4-14] and the references cited therein. However, as far as we know, there is very little result about the Lotka-Volterra system with ratio-dependent functional responses and feedback controls.

The objective of this paper is to investigate the asymptotic behavior of the time-dependent solution for a 3-species predator-prey system with feedback control. The systems of equations under consideration are of ratio-dependent Lotka-Volterra type and are described as follows

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - \frac{a_{13}(t)x_3(t)}{a_{13}(t)x_3(t) + x_1(t)} - d_1(t)u_1(t)], \\ \dot{x}_2(t) = x_2(t)[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - \frac{a_{23}(t)x_3(t)}{a_{23}(t)x_3(t) + x_2(t)} - d_2(t)u_2(t)], \\ \dot{x}_3(t) = x_3(t)[-r_3(t) + \frac{a_{31}(t)x_1(t)}{a_{13}(t)x_3(t) + x_1(t)} + \frac{a_{32}(t)x_2(t)}{a_{23}(t)x_3(t) + x_2(t)} + d_3(t)u_3(t)], \\ \dot{u}_1(t) = e_1(t) - f_1(t)u_1(t) + q_1(t)x_1(t), \\ \dot{u}_2(t) = e_2(t) - f_2(t)u_2(t) + q_2(t)x_2(t), \\ \dot{u}_3(t) = e_3(t) - f_3(t)u_3(t) - q_3(t)x_3(t), \end{cases} \quad (3)$$

where  $x_1(t)$ ,  $x_2(t)$ , and  $x_3(t)$  stand for the densities of the two preys and one predator, respectively, and  $u_i(t)$ ,  $i=1,2,3$ , are the indirect control variables. The given coefficients  $a_{ij}(t)$ ,  $d_i(t)$ ,  $e_i(t)$ ,  $f_i(t)$ ,  $q_i(t)$ , and  $r_i(t)$  are positive continuous bounded functions of  $t$  for  $i, j=1,2,3$ . Model (3) describes the interaction between prey and predator species which is based on a ratio-dependent functional response, that is, the rate at which an individual predator species consumes two individual prey species, and two preys compete for the food resource. Thus system (3) is the so-called food web model with one predator and two

competing preys. Here,  $r_1(t)$  and  $r_2(t)$  are the intrinsic growth rate in the absence of predator  $x_3(t)$  and the parameters  $a_{12}(t)$  and  $a_{23}(t)$  are the capturing (or catching efficiency) rates of the predator;  $a_{31}(t)$  and  $a_{32}(t)$  are the conversion rates (or maximum growth rates).  $r_3(t)$  is the death rate of the predator species  $x_3(t)$ . In [15], Wang et al establish the sufficient conditions which guarantee the permanence of systems (3) by developing a new analysis technique. When there are not feedback controls, the systems (3) are reduced to the following Lotka -Volterra model

$$\begin{cases} \dot{x}_1(t) = x_1(t)[a_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - \frac{a_{13}(t)x_3(t)}{a_{13}(t)x_3(t) + x_1(t)}], \\ \dot{x}_2(t) = x_2(t)[a_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - \frac{a_{23}(t)x_3(t)}{a_{23}(t)x_3(t) + x_2(t)}], \\ \dot{x}_3(t) = x_3(t)[-a_3(t) + \frac{a_{31}(t)x_1(t)}{a_{13}(t)x_3(t) + x_1(t)} + \frac{a_{32}(t)x_2(t)}{a_{23}(t)x_3(t) + x_2(t)}], \end{cases} \quad (4)$$

where the meaning of the parameters of systems (4) is as same as those of (3). Wang et al [16] show that this system is permanent and globally asymptotically stable under some appropriate conditions by constructing suitable Lyapunov function. Comparing the systems (3) and (4), one could see that we introduce the control variables  $u_i(t)$ ,  $i=1,2,3$  so as to implement a feedback control mechanism. Here, we will consider the Lotka-Volterra predator-prey system (3) with ratio-dependent functional responses and feedback controls. By constructing suitable Lyapunov function, the sufficient conditions are established for the globally attractive of positive solution for the model.

The plan of the paper is as follows: In Section 2, by constructing a non-negative Lyapunov function, we shall derive sufficient conditions for the globally attractive of positive solution for the predator-prey model (3). Some numerical simulations to the models are given in Section 3.

## 2. Globally Attractive

In order to establish a globally attractive result for the system (3), we need some preparations. Due to the biological interpretation of the model, it is reasonable to consider only positive solutions of (3), in other words, to take admissible initial conditions  $z(t_0)=z_0>0$ . Firstly, we introduce the following notations and definitions. Given a function  $g(t)$  defined on  $[t_0, +\infty)$ , we set

$$g^m = \sup\{g(t) | t_0 < t < +\infty\},$$

$$g^l = \inf\{g(t) | t_0 < t < +\infty\}.$$

**Definition 2.1.** System (3) is called permanent, if there exist positive constants  $M_i, N_i, m_i, n_i$ ,  $i=1,2,3$ , and  $T$ , such that  $m_i \leq x_i(t) \leq M_i$ ,  $n_i \leq u_i(t) \leq N_i$  for any positive solution  $Z(t)=(x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$  of (3) as  $t > T$ .

**Definition 2.2.** System (1.3) is said to be globally attractive, if there exists a positive solution  $X(t)=(x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$  of the system (3) such that

$$\lim_{t \rightarrow +\infty} |x_i(t) - y_i(t)| = 0, \quad \lim_{t \rightarrow +\infty} |u_i(t) - v_i(t)| = 0,$$

for any other positive solution  $Y(t)=(y_1(t), y_2(t), y_3(t), v_1(t), v_2(t), v_3(t))$  of the

system (3).

For the system (3), we let

$$M_1 = \frac{r_1^m}{a_{11}^l}, \quad M_2 = \frac{r_2^m}{a_{22}^l}, \quad N_3 = \frac{e_3^m}{f_3^l}, \quad N_1 = \frac{e_1^m + q_1^m M_1}{f_1^l},$$

$$N_2 = \frac{e_2^m + q_2^m M_2}{f_2^l},$$

$$M_3 = \frac{a_{31}^m + a_{32}^m - r_3^l + d_3^m N_3}{(r_3^l - d_3^m N_3) \min \left\{ \frac{a_{13}^l}{M_1}, \frac{a_{23}^l}{M_2} \right\}},$$

$$m_1 = \frac{r_1^l - a_{12}^m M_2 - a_{13}^m M_3 - d_1^m N_1}{a_{11}^m},$$

$$m_2 = \frac{r_2^l - a_{21}^m M_1 - a_{23}^m M_3 - d_2^m N_2}{a_{22}^m},$$

$$m_3 = \frac{a_{31}^l + a_{32}^l - r_3^m}{r_3^m \max \left\{ \frac{a_{13}^m}{m_1}, \frac{a_{23}^m}{m_2} \right\}}, \quad n_1 = \frac{e_1^l + q_1^l m_1}{f_1^m},$$

$$n_2 = \frac{e_2^l + q_2^l m_2}{f_2^m}, \quad n_3 = \frac{e_3^l - q_3^m M_3}{f_3^m}.$$

**Lemma 2.1** (See [15], Theorem 1). Assume that the system (3) satisfies the following conditions

$$(H_1) \quad r_1^l > a_{12}^m M_2 + a_{13}^m M_3 + d_1^m N_1,$$

$$(H_2) \quad r_2^l > a_{21}^m M_1 + a_{23}^m M_3 + d_2^m N_2,$$

$$(H_3) \quad a_{31}^m + a_{32}^m > r_3^l - d_3^m N_3 > 0, \quad (H_4) \quad a_{31}^l + a_{32}^l > r_3^m,$$

$$(H_5) \quad e_3^m > q_3^l m_3, \quad (H_6) \quad e_3^l > q_3^m M_3,$$

then the system (1.3) is permanent.

**Lemma 2.2** (See [17], Lemma 8.2). If the function  $f(t): R^+ \rightarrow R$  is uniformly continuous, and the limit

$$\lim_{t \rightarrow \infty} \int_0^t f(s) ds \text{ exists and is finite, then } \lim_{t \rightarrow +\infty} f(t) = 0.$$

**Theorem 2.1.** Assume that the system (3) satisfies  $(H_1)-(H_6)$  and the following conditions

$$(H_7) \quad a_{11}^l > a_{21}^m + \frac{(a_{13}^m + a_{13}^m a_{31}^m) M_3}{(a_{13}^l m_3 + m_1)^2} + q_1^m,$$

$$(H_8) \quad a_{22}^l > a_{12}^m + \frac{(a_{23}^m + a_{23}^m a_{32}^m)M_3}{(a_{23}^l m_3 + m_2)^2} + q_2^m,$$

$$(H_9) \quad \frac{a_{13}^l a_{31}^l m_1}{(a_{13}^m M_3 + M_1)^2} + \frac{a_{23}^l a_{32}^l m_2}{(a_{23}^m M_3 + M_2)^2} > \frac{a_{13}^m M_1}{(a_{13}^l m_3 + m_1)^2} + \frac{a_{23}^m M_2}{(a_{23}^l m_3 + m_2)^2} + q_3^m,$$

$$(H_{10}) \quad f_i^l > d_i^m, (i = 1, 2, 3),$$

then the system (1.3) is globally attractive.

Proof.

Let  $X(t) = (x_1(t), x_2(t), x_3(t), u_1(t), u_2(t), u_3(t))$  be a positive solution of the system (3) and  $Y(t) = (y_1(t), y_2(t), y_3(t), v_1(t), v_2(t), v_3(t))$  be any positive solution of the system (1.3) with initial conditions  $x_i(t_0) > 0, u_i(t_0) > 0, i = 1, 2, 3$ , then from Lemma 2.1, there exist positive constants  $M_i, N_i, m_i, n_i$  and  $T$ ,

such that  $m_i \leq x_i(t) \leq M_i, n_i \leq u_i(t) \leq N_i$  for all  $t > T$ .

Set Lyapunov function

$$V(t) = \sum_{i=1}^3 [|\ln x_i(t) - \ln y_i(t)| + |u_i(t) - v_i(t)|].$$

Computing the upper right derivative of  $V(t)$  along with the solution of the system (3), one can easily obtain

$$\begin{aligned} D^+V(t) &= \sum_{i=1}^3 D^+ [|\ln x_i(t) - \ln y_i(t)| + |u_i(t) - v_i(t)|] \\ &= \operatorname{sgn}\{x_1(t) - y_1(t)\} [-a_{11}(t)(x_1(t) - y_1(t)) - a_{12}(t)(x_2(t) - y_2(t)) \\ &\quad - (\frac{a_{13}(t)x_3(t)}{a_{13}(t)x_3(t) + x_1(t)} - \frac{a_{13}(t)y_3(t)}{a_{13}(t)y_3(t) + y_1(t)}) - d_1(t)(u_1(t) - v_1(t))] \\ &\quad + \operatorname{sgn}\{x_2(t) - y_2(t)\} [-a_{21}(t)(x_1(t) - y_1(t)) - a_{22}(t)(x_2(t) - y_2(t)) \\ &\quad - (\frac{a_{23}(t)x_3(t)}{a_{23}(t)x_3(t) + x_2(t)} - \frac{a_{23}(t)y_3(t)}{a_{23}(t)y_3(t) + y_2(t)}) - d_2(t)(u_2(t) - v_2(t))] \\ &\quad + \operatorname{sgn}\{x_3(t) - y_3(t)\} [\frac{a_{31}(t)x_1(t)}{a_{13}(t)x_3(t) + x_1(t)} - \frac{a_{31}(t)y_1(t)}{a_{13}(t)y_3(t) + y_1(t)} \\ &\quad + \frac{a_{32}(t)x_2(t)}{a_{23}(t)x_3(t) + x_2(t)} - \frac{a_{32}(t)y_2(t)}{a_{23}(t)y_3(t) + y_2(t)} + d_3(t)(u_3(t) - v_3(t))] \\ &\quad + \operatorname{sgn}\{u_1(t) - v_1(t)\} [-f_1(t)(u_1(t) - v_1(t)) + q_1(t)(x_1(t) - y_1(t))] \\ &\quad + \operatorname{sgn}\{u_2(t) - v_2(t)\} [-f_2(t)(u_2(t) - v_2(t)) + q_2(t)(x_2(t) - y_2(t))] \\ &\quad + \operatorname{sgn}\{u_3(t) - v_3(t)\} [-f_3(t)(u_3(t) - v_3(t)) + q_3(t)(x_3(t) - y_3(t))] \\ &\leq |x_1(t) - y_1(t)| [-a_{11}(t) + a_{21}(t) + \frac{a_{13}(t)y_3(t) + a_{13}(t)a_{31}(t)y_3(t)}{(a_{13}(t)x_3(t) + x_1(t))(a_{13}(t)y_3(t) + y_1(t))} + q_1(t)] \\ &\quad + |x_2(t) - y_2(t)| [a_{12}(t) - a_{22}(t) + \frac{a_{23}(t)y_3(t) + a_{23}(t)a_{32}(t)y_3(t)}{(a_{23}(t)x_3(t) + x_2(t))(a_{23}(t)y_3(t) + y_2(t))} \\ &\quad + q_2(t)] + |x_3(t) - y_3(t)| [\frac{a_{13}(t)y_1(t) - a_{13}(t)a_{31}(t)y_1(t)}{(a_{13}(t)x_3(t) + x_1(t))(a_{13}(t)y_3(t) + y_1(t))} \\ &\quad + \frac{a_{23}(t)y_2(t) - a_{23}(t)a_{32}(t)y_2(t)}{(a_{23}(t)x_3(t) + x_2(t))(a_{23}(t)y_3(t) + y_2(t))} + q_3(t)] + |u_1(t) - v_1(t)| [-f_1(t) \\ &\quad + d_1(t)] + |u_2(t) - v_2(t)| [-f_2(t) + d_2(t)] + |u_3(t) - v_3(t)| [-f_3(t) + d_3(t)] \end{aligned}$$

$$\begin{aligned}
&\leq -|x_1(t) - y_1(t)| \left[ a'_{11} - a^m_{21} - \frac{(a^m_{13} + a^m_{13}a^m_{31})M_3}{(a^l_{13}m_3 + m_1)^2} - q^m_1 \right] \\
&- |x_2(t) - y_2(t)| \left[ -a^m_{12} + a^l_{22} - \frac{(a^m_{23} + a^m_{23}a^m_{32})M_3}{(a^l_{23}m_3 + m_2)^2} - q^m_2 \right] \\
&- |x_3(t) - y_3(t)| \left[ \frac{a^l_{13}a^l_{31}m_1}{(a^m_{13}M_3 + M_1)^2} + \frac{a^l_{23}a^l_{32}m_2}{(a^m_{23}M_3 + M_2)^2} - \frac{a^m_{13}M_1}{(a^m_{13}M_3 + M_1)^2} - \frac{a^m_{23}M_2}{(a^m_{23}M_3 + M_2)^2} - q^m_3(t) \right] \\
&- |u_1(t) - v_1(t)| [f^l_1 - d^m_1] - |u_2(t) - v_2(t)| [f^l_2 - d^m_2] - |u_3(t) - v_3(t)| [f^l_3 - d^m_3].
\end{aligned}$$

In view of conditions  $(H_7) \sim (H_{10})$ , one has

$$\begin{aligned}
\alpha = \min \{ &a'_{11} - a^m_{21} - \frac{(a^m_{13} + b^m_{13}a^m_{31})M_3}{(b^l_{13}m_3 + m_1)^2} - q^m_1, -a^m_{12} + a^l_{22} - \frac{(a^m_{23} + b^m_{23}a^m_{32})M_3}{(b^l_{23}m_3 + m_2)^2} - q^m_2, \\
&\frac{b^l_{13}a^l_{31}m_1}{(b^m_{13}M_3 + M_1)^2} + \frac{b^l_{23}a^l_{32}m_2}{(b^m_{23}M_3 + M_2)^2} - \frac{a^m_{13}M_1}{(b^l_{13}m_3 + m_1)^2} - \frac{a^m_{23}M_2}{(b^l_{23}m_3 + m_2)^2} - q^m_3, \\
&f^l_1 - d^m_1, f^l_2 - d^m_2, f^l_3 - d^m_3 \} > 0.
\end{aligned}$$

Thus,

$$D^+V(t) \leq -\alpha \sum_{i=1}^3 [|x_i(t) - y_i(t)| + |u_i(t) - v_i(t)|]. \quad (5)$$

Integrating (5) from  $T$  to  $t$  ( $T \geq t_0$ ), one has

$$V(t) + \alpha \int_T^t \left\{ \sum_{i=1}^3 [|x_i(s) - y_i(s)| + |u_i(s) - v_i(s)|] \right\} ds \leq V(T) < +\infty.$$

Therefore,

$$\int_T^t \left\{ \sum_{i=1}^3 [|x_i(s) - y_i(s)| + |u_i(s) - v_i(s)|] \right\} ds \leq \frac{V(T)}{\alpha} < +\infty. \quad (6)$$

By (6), we have

$$\sum_{i=1}^3 [|x_i(t) - y_i(t)| + |u_i(t) - v_i(t)|] \in L^1(T, +\infty).$$

From the uniformity permanence of the systems (3),  $\sum_{i=1}^3 [|x_i(t) - y_i(t)| + |u_i(t) - v_i(t)|]$  is uniformly continuous. By Lemma 2.2, we can obtain that

$$\lim_{t \rightarrow +\infty} |x_i(t) - y_i(t)| = 0, \quad \lim_{t \rightarrow +\infty} |u_i(t) - v_i(t)| = 0, \quad (i = 1, 2, 3).$$

This ends the proof of Theorem 2.1.

### 3. Numerical Simulation

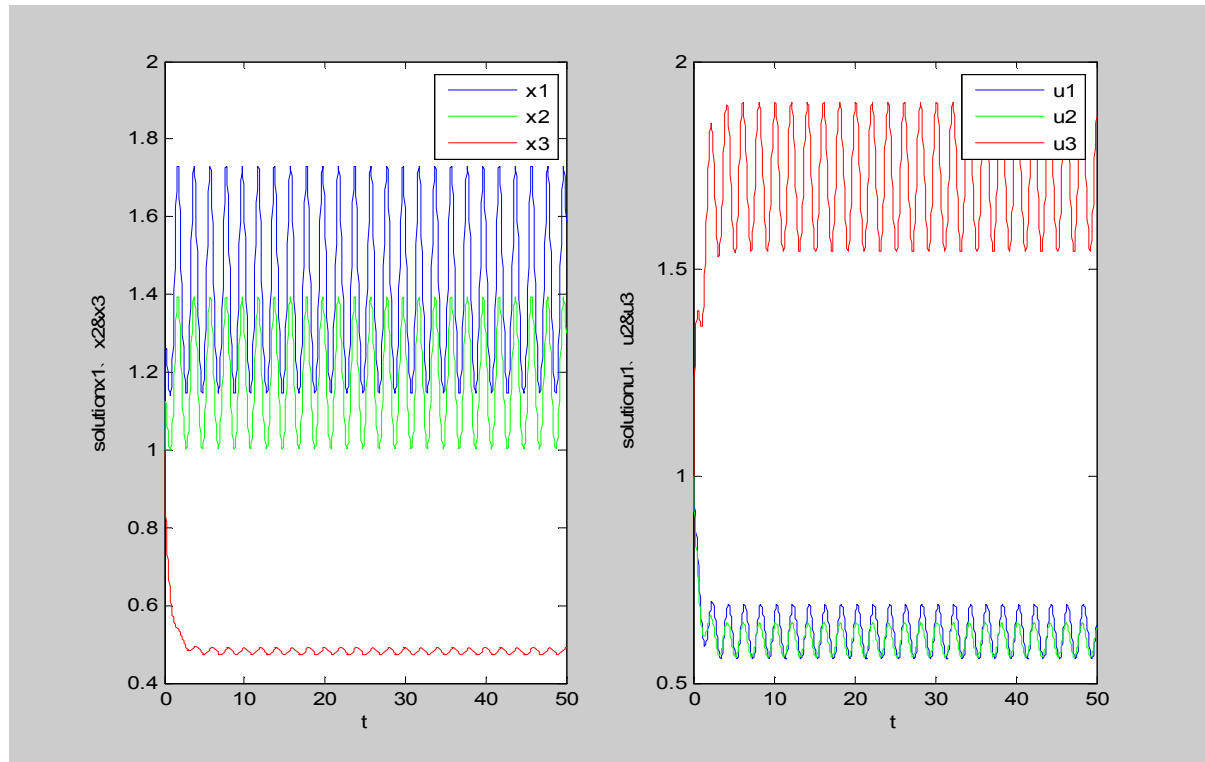
In this section, we give some numerical simulations supporting our theoretical analysis. As an example, we consider the following two-prey one-predator model with ratio-dependent functional responses and feedback controls.

$$\begin{cases}
\dot{x}_1(t) = x_1(t)[7.5 + 0.5 \cos \pi t - [5 + \sin \pi t]x_1(t) - [0.4 + 0.1 \sin \pi t]x_2(t) \\
\quad - \frac{[0.15 + 0.01 \sin \pi t]x_3(t)}{[0.15 + 0.01 \sin \pi t]x_3(t) + x_1(t)} - [(1.0 + 0.5 \sin \pi t)/10]u_1(t)], \\
\dot{x}_2(t) = x_2(t)[8.5 + 0.5 \cos \pi t - [7 + \sin \pi t]x_1(t) - [0.13 + 0.1 \sin \pi t]x_2(t) \\
\quad - \frac{[0.14 + 0.01 \sin \pi t]x_3(t)}{[0.14 + 0.01 \sin \pi t]x_3(t) + x_2(t)} - [(1.4 + \sin \pi t)/10]u_2(t)], \\
\dot{x}_3(t) = x_3(t)[-2.7 + 0.1 \cos \pi t] + \frac{[2.2 + 0.1 \sin \pi t]x_1(t)}{[0.15 + 0.01 \sin \pi t]x_3(t) + x_1(t)} \\
\quad + \frac{[2.1 + 0.1 \sin \pi t]x_2(t)}{[0.14 + 0.01 \sin \pi t]x_3(t) + x_2(t)} + [(0.3 + 0.1 \sin \pi t)/10]u_3(t), \\
\dot{u}_1(t) = [0.8 + 0.2 \cos \pi t] - [1.6 + 0.1 \sin \pi t]u_1(t) + [0.15 + 0.1 \sin \pi t]x_1(t), \\
\dot{u}_2(t) = [0.5 + 0.1 \cos \pi t] - [1.4 + 0.2 \sin \pi t]u_2(t) + [0.30 + 0.1 \sin \pi t]x_2(t), \\
\dot{u}_3(t) = [2 + 0.5 \cos \pi t] - [1.2 + 0.2 \sin \pi t]u_3(t) - [0.15 + 0.01 \sin \pi t]x_3(t).
\end{cases} \quad (7)$$

It is easy to show that the two-prey one-predator model (7) satisfy the conditions of Theorem 2.1. It follows from Theorem 2.1 that the predator-prey model (7) is permanent and globally attractive. By employing the software package MATLAB7.1, we can solve the numerical solutions of the equations (7) which are shown in Figures 1, Figures 2, Figures 3 and Figures 4.

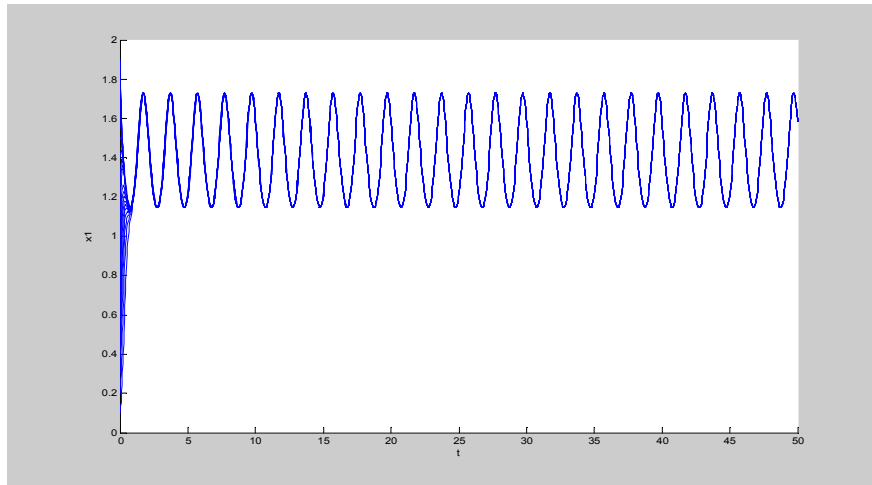
Figures 1 shows that the permanence of the systems (7) with the initial conditions  $x_1(t_0)=1, x_2(t_0)=1, x_3(t_0)=1, u_1(t_0)=1, u_2(t_0)=1, u_3(t_0)=1$ . It is not difficult to

find that  $0.4 < x_i(t) < 1.8$  ( $i=1,2,3$ ) and  $0.5 < u_i(t) < 2$  ( $i=1,2,3$ ) as  $t > 10$ . From Figures 2, Figures 3 and Figures 4, one can find that  $\lim_{t \rightarrow \infty} |x_i(t) - y_i(t)| = 0$  ( $i=1,2,3$ ) for any two solutions  $X(t)=(x_1(t), x_2(t), x_3(t))$  and  $Y(t)=(y_1(t), y_2(t), y_3(t))$  of the system (7) with the different initial conditions, which show that the predator-prey model (7) is globally attractive.

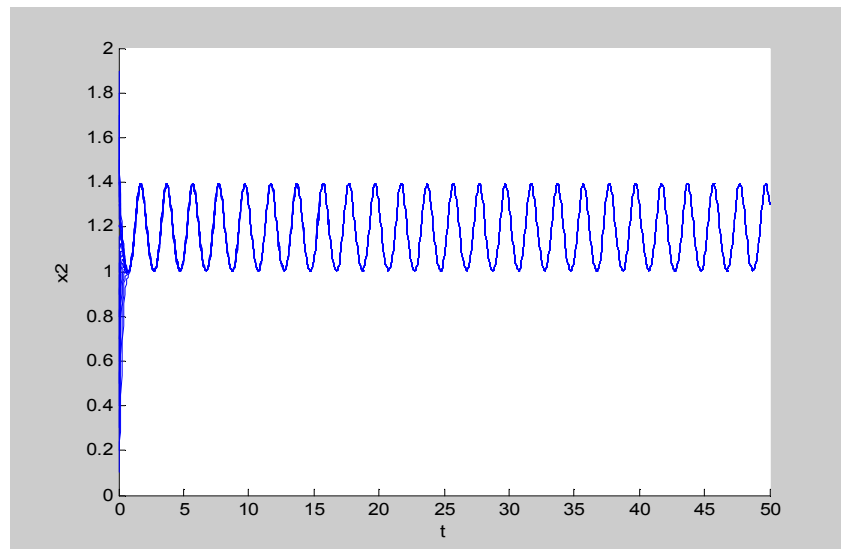


**Fig. 1.** The numerical solution of systems (7) with the initial conditions.

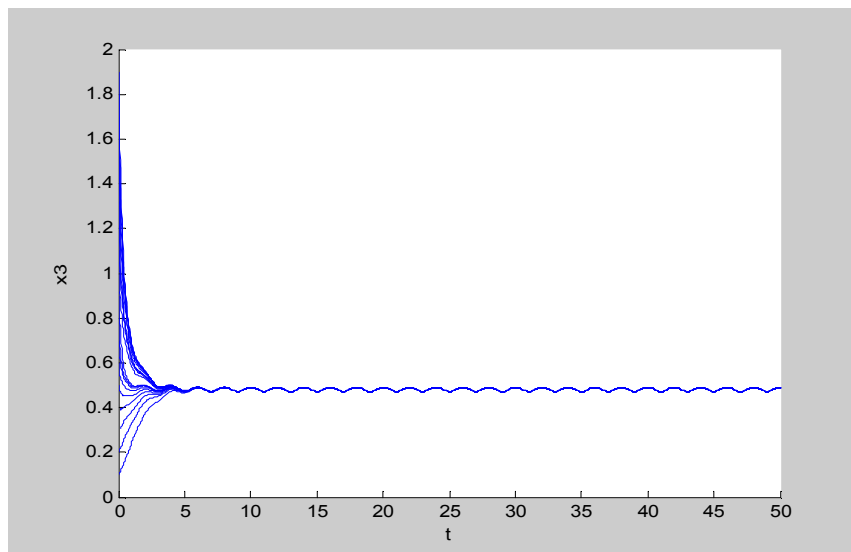
$$x_1(t_0)=1, x_2(t_0)=1, x_3(t_0)=1, u_1(t_0)=1, u_2(t_0)=1, u_3(t_0)=1.$$



**Fig. 2.** Dynamic behavior of the first component  $x_1(t)$  of the solutions to system (7) with the different initial conditions.



**Fig. 3.** Dynamic behavior of the first component  $x_2(t)$  of the solutions to system (7) with the different initial conditions.



**Fig. 4.** Dynamic behavior of the first component  $x_3(t)$  of the solutions to system (7) with the different initial conditions.

## 4. Conclusion

This paper presents the use of Lyapunov stability theorem for systems of nonlinear differential equations. This method is a powerful tool for solving nonlinear differential equations in mathematical physics, chemistry and engineering etc. The technique constructing a suitable Lyapunov function provides a new efficient method to handle the nonlinear structure.

We have dealt with the problem of positive solution for a Lotka-Volterra predator-prey system with ratio-dependent functional responses and feedback controls. The general sufficient conditions have been obtained to ensure the global attractive of positive solution for the predator-prey model. Moreover, some numerical simulations to the equation are given to illustrate our results. In particular, the sufficient conditions that we obtained are very simple, which provide flexibility for the application and analysis of the Lotka-Volterra predator-prey system.

Remark: Obviously, the models (3) is the extension of model (4). Adding delay term to the proposed model (3) is our next research work.

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