Using Fish-Market-Fishes to Demonstrate the Methodological Approach to Establish Mathematical Relations Between Body Size and Body Weight

Maria Pagano, Andrea Viggiano[*]

Department of Medicine, Surgery and Dentistry "Scuola Medica Salernitana", University of Salerno, Baronissi (SA), Italy

Email address:
aviggiano@unisa.it (A. Viggiano)

[*]Corresponding author

Abstract: Body mass index (BMI) is widely used to evaluate if a person has a normal body weight. This index may appear strange to a student because he could expect a cubic relation between body volume and any linear body dimension. The aim of the present experiment was to show the experimental approach to establish a mathematical relation between linear body dimensions and body weight by using a simple animal model. To this end, twelve sea bass and thirteen sea breams were obtained from a local fish-market. For each fish it was measured the body weight, the linear body dimensions, the body volume, the body surface area, and the visceral fat weight. The mathematical relations between all the experimental variables were evaluated pairwise, by plotting them on X-Y graphs and calculating the best fitting power-model. The results demonstrated that in fishes body weight fitted with any of the linear body dimensions raised to a power smaller than 2. The strongest of such correlations was between body weight and body length raised to a power of 1.5. Moreover, BMI did not correlate with visceral fat content. These results demonstrated that in fishes: 1) a non-linear correlation exists between body weight and linear body dimensions; 2) growth is allometric; 3) BMI is a fictitious index and does not describe a physiological phenomenon; 4) BMI is not predictive of visceral fat content; 5) other variables should be taken into account to obtain a more affordable mathematical model to describe the relation between body weight and linear body dimensions.

Keywords: Linear Body Dimensions, Body Surface Area, Body Weight, Visceral Fat, Body Mass Index

1. Introduction

Students could be confused when faced for the first time with the formula of body mass index BMI = weight/height^2. In fact, being the weight closely related to the volume, someone would expect to see the height raised to a power of 3.

The Quételet index, alias body mass index (BMI), is widely used as an index for overweight and obesity in humans [1]. BMI is still widely used [2] despite its well known limitations [3]. Apparently, the existence of this index postulates that the ratio between the weight and the height squared should be constant. The physiological meaning of this relation is expected to derive from the non-linear relation between body mass and height. In fact, the three linear dimensions (height, width, depth) of all segments of the body do not grow isometrically but allometrically (i.e. they are not in a constant proportion) [4]. The consequence of the allometric growth of the body is that even the body surface area is not proportional to the height squared [5].

Looking at human growth, it can be noted that the height (h) of a person is the most evident dimension that grows allometrically, while the ratio between the anterior-posterior (r_1) and the side-to-side (r_2) diameters of any body-segment changes to a lesser extent during growth. Thus, assuming that the last two dimensions grow quasi-isometrically, the following relations are expected:

$$r_1 \sim r_2 \tag{1}$$

$$V \sim r_1 \cdot r_2 \cdot h \sim r_1^2 \cdot h$$

$$S \sim r_1 \cdot h$$

where "$\sim$" means a proportional relation, V = body volume and S = body surface area. By algebraic passages it follows:

$$V \sim r_1^2 \cdot h$$

$$r_1 \sim V^{1/2} \cdot h^{-1/2}$$

$$S \sim V^{1/2} \cdot h^{-1/2} \cdot h \sim V^{1/2} \cdot h^{1/2}$$

Moreover, if body mass (M) can be considered proportional to body volume, then:

$$V \sim M \qquad (2)$$

$$S \sim V^{1/2} \cdot h^{1/2} \sim M^{1/2} \cdot h^{1/2} \qquad (3)$$

Last formula matches that proposed by Mosteller [6] and is one of the most commonly used formulas to estimate the body surface area. This coincidence suggests that the hypotheses (1) and (2) are, at least, acceptable approximations.

Because the amount of heat that is exchanged per unit of time between the body and the external ambient depends (also) on the dimension of the body surface area, a (roughly) linear relation is expected between basal metabolic rate (E) and body surface area. Thus the following relation is expected:

$$E \sim S \sim M^{1/2} \cdot h^{1/2} \qquad (4)$$

If body mass were related to height squared, as predicted by BMI, then:

$$M \sim h^2$$

$$h \sim M^{1/2} \qquad (5)$$

$$E \sim S \sim M^{1/2} \cdot h^{1/2} \sim M^{1/2} \cdot M^{1/4} \sim M^{3/4} \qquad (6)$$

Last formula matches that proposed by Kleiber [7], ones again strengthening the proposed assumptions.

There are, however, some weak points in the above arguments:

1) it has not been verified that $r_1 \sim r_2$;

2) the approximation M $\sim$ V could not be acceptable, particularly in obese subjects, because the density of the adipose tissue is lower than that of other tissues;

3) the rate of heat exchange does not depend only on the body surface area, but also on sweating, ventilation, amount of hairs, dressing, regulation of skin circulation and temperature difference between the skin and the external ambient.

Actually, the relation (6) described by Kleiber [7] emerged from data obtained from different animal species with extremely different body sizes, from mouse to elephant; but data restricted to some species can show other relations between basal metabolic rate and body weight, particularly in fishes.

Today there is still a great interest in evaluating anthropometric data as indexes of overweight-obesity and risk factors for health [8], but a mathematical model that fit all the experimental data of weight, height, body surface area and basal metabolic rate is still lacking. In clinical practice, reference values for the BMI have been established only through descriptive statistics of the population, and such reference values are not constant but changes depending on age.

Instead of just focusing the attention on humans, the idea underneath the present work was to obtain cues about the role of basal metabolic rate on the relations between body dimensions by looking at other a specie with a very different kind of metabolism. In fact, it has been described that in some fishes (salmon) the heart rate is nearly constant over different body masses [9], thus it could be argued that the basal metabolic rate in fishes should (roughly) linearly scale with body mass. This is partially supported by experimental data. In fact, despite the evaluation of the basal metabolic rate in fishes has some technical limitations, it has been reported that in fishes the metabolic rate scales with body mass raised to a power of 0.88 [10], which is closer to 1 than the value 0.75 proposed in (6) and by Kleiber [7]. Thus, if there is a quasi-linear relation between basal metabolic rate and body surface area, it could be argued that also body surface area should scale with body mass raised to a power close to 1. The present is a preliminary description of the methodological approach for the establishment of mathematical models of the physiological relations between linear bodydimensions, body surface area and body weight. For this reason a very simple and inexpensive model has been chosen: the evaluation of the relations between body mass, linear body dimensions and body surface area in fish-market sea bream and sea bass.

2. Materials and Methods

Twelve sea bass and thirteen sea breams, aged about two years, were obtained from a local fish market. All fishes were proved to be under fasting conditions because, after dissection, the digestive tube was found empty. For each fish the following measures were taken: 1) weight; 2) total length, from the tip of the jaw to the tip of the caudal fin; 3) maximal body depth, from the dorsal margin of the body to the ventral margin of the body; 4) maximal body width, from side to side; 5) body volume; 6) body surface area; 7) visceral fat weight.

The body volume of each fish was measured as following. First a jar was placed on a larger container. The jar was then completely filled with water; then the fish was placed into the jar and completely submerged into the water, thus causing a volume of water, equal to the volume of the fish, coming out of the jar and falling into the larger container. Finally, the water collected by the larger container was weighted, thus obtaining a measure of the fish's volume.

To measure body surface area the skin was dissected out, except for head and fins, then it was flattened over paper and the contour was reported. Then the area was calculated both by weighting the paper and by computer software using the image acquired through a scanner.

The mathematical relation between all the experimental variables were evaluated pairwise, by plotting them on X-Y graphs and calculating the best fitting power-model. The exponent of such fitting equation was finally considered to discuss whether there was a plausible linear of square

relation between each pair of variables. Moreover, the correlation coefficient of the fitting model was considered to discuss whether a simple power equation is a satisfying mathematical model of the relation between any of the paired experimental variables.

3. Results

The relation between any pairs of the linear dimensions was not linear (figure 1), meaning that even in fishes body growth is allometric.
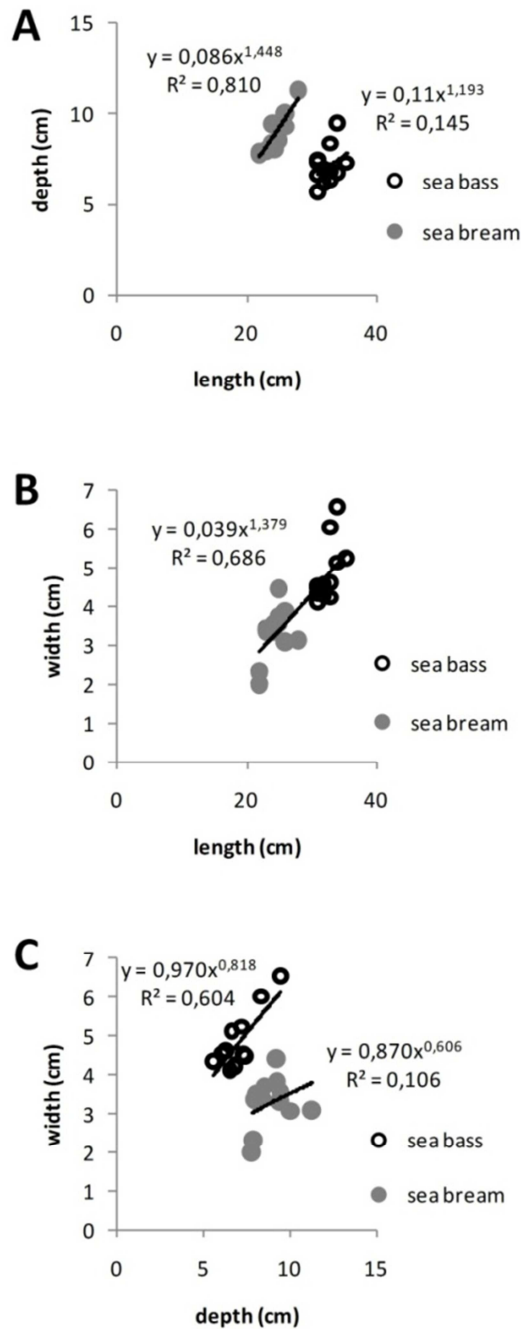

Figure 1. *Correlations between linear dimensions. The plots show that there was not a linear relation between any of the paired linear dimensions. The best fit power-model (y) and the correlation coefficient (R^2) are reported near each plot.*
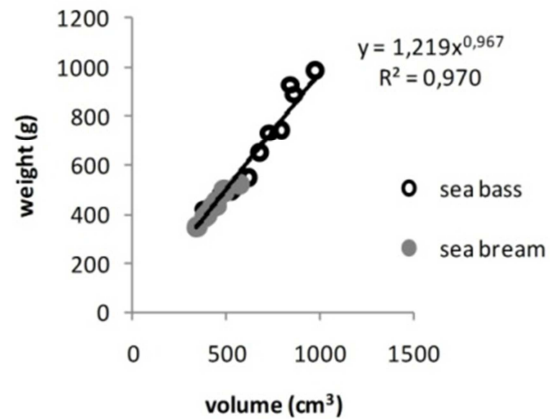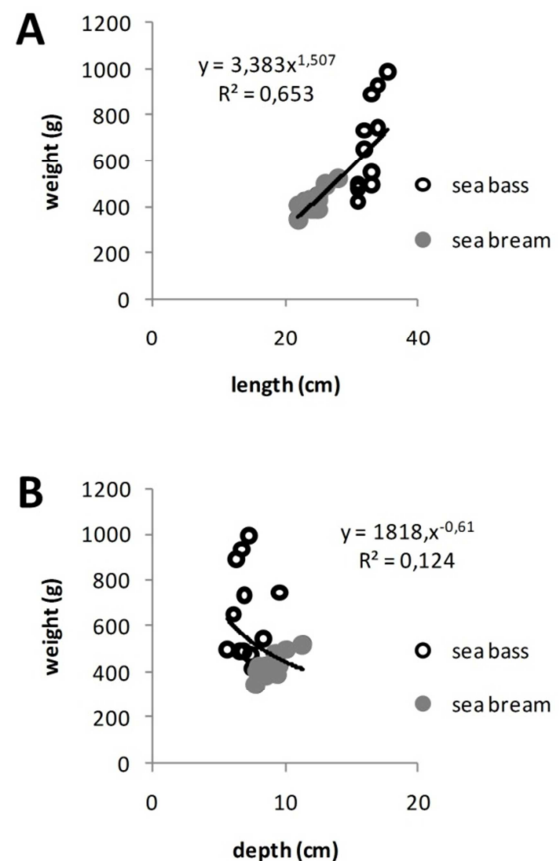

Figure 2. *Body weight against body volume. The best fit power-model (y) and the correlation coefficient (R^2) demonstrate a linear relation.*

The relation between body volume and body weight was quasi-linear. In fact, the best fit model had exponent 1 and a correlation coefficient equal to 1 (figure 2), meaning that assumption (2) was acceptable for the present data.

The relations between body weight and each of the linear body dimensions were not linear and the best fit models had exponents significantly different from 2 (figure 3). The greatest correlation coefficient was seen between body weight and body length, for which the best fit model had an exponent of 1.5 and a correlation coefficient of 0.6 (figure 3-A). This means that other variables affected body weight in addition to body length or other linear dimensions.
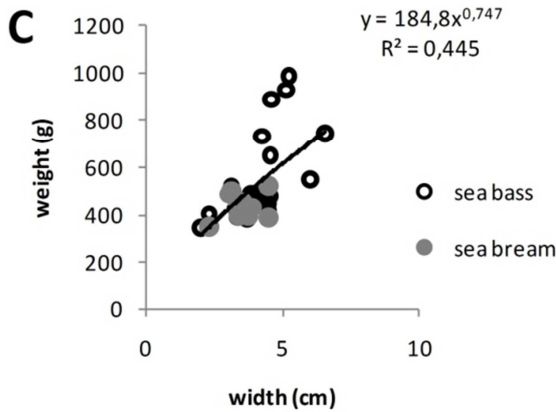
Figure 3. Weight plotted against each of the linear dimensions. The best fit power-model (y) and the correlation coefficient (R^2) are reported near each plot.
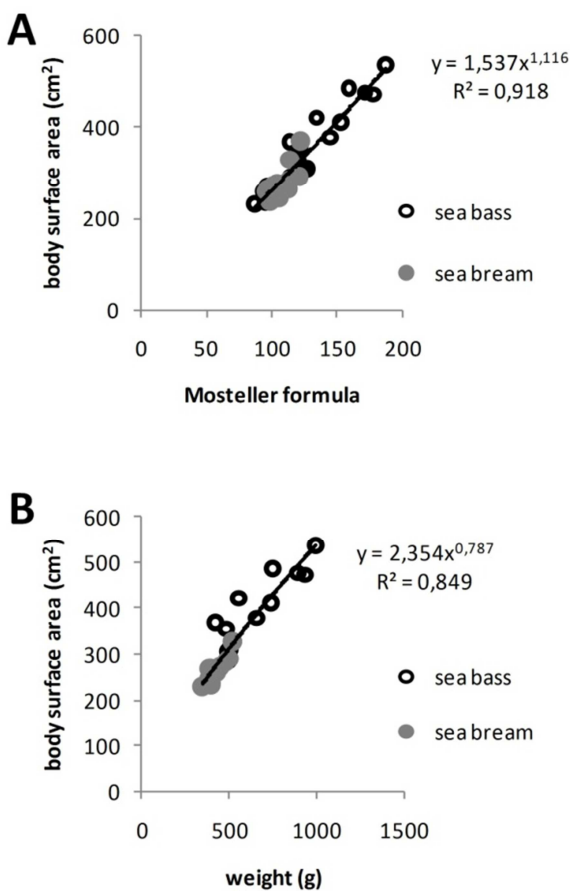

Figure 4. Body surface area. A: Mosteller's formula against actual body surface area. The plot shows a good agreement between the Mosteller's formula and the body surface area, but the best fit power-model (y) was not right linear. Mosteller's formula = M1/2 • h1/2, M = body mass, h = body length. B: body surface area against body weight. The best fit power-model (y) showed a power of 0.79, close to relation (6) and to that proposed by Kleiber (1932).

There was a strong correlation coefficient between the actual body surface area and the Mosteller's formula (3); however, the best fit model showed an exponent of 1.1, slightly greater than 1 (figure 4-A). Plotting body surface area against body weight, the best fit model showed

anexponent of 0.79 and a correlation coefficient of 0.85 (figure 4-B).

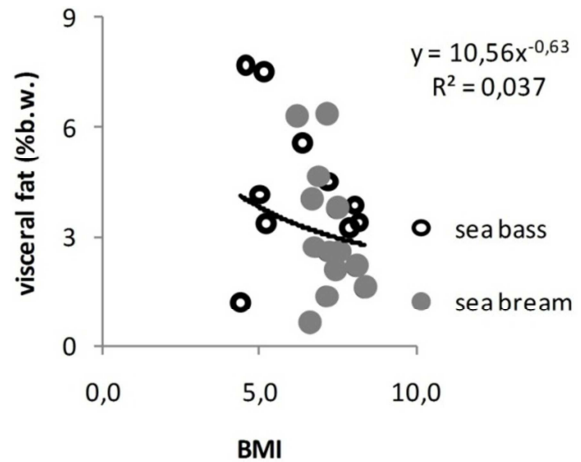The visceral fat content did not show any significant correlation with BMI (figure 5).

Figure 5. Visceral fat against BMI. The plot shows that there was not any significant relation between the two variables.

4. Discussion

The results of the present experiment confirmed that body growth in sea breams and in sea bass is allometric (figure 1). It has also been found that there is not a simple correlation between body weight and any of the linear body dimensions, because the correlation coefficients were small (figure 3). The strongest of such correlations was that between body weight and body length (figure 3-A), but the estimated exponent of this correlation was 1.5, significantly smaller than that predicted by BMI (i.e. exponent 2). The weakness of the BMI index as predictor of body fat content has also been demonstrated (figure 5).

The present data showed a strong correlation between body surface area and body mass with an exponent of 0.79 (figure 4-B). This exponent is slightly greater than 0.75, as proposed by Kleiber[7] for the relation between basal metabolic rate and body mass; this finding partially agrees with the greater exponent (0.88) reported by White [10] for the relation between basal metabolic rate and body mass in fishes. In other words, it appears that in fishes both the body surface area (observed in the present experiment) and the basal metabolic rate [10] are related to body mass with a power > 0.75. These findings support a correlation between basal metabolic rate and body surface area. Moreover, while the exponent 0.75 proposed by Kleiber [7] is expected to fit with the Mosteller's formula [6], as shown in (3)-(6), the present experimental data diverged from both the Kleiber's model [7] and the Mosteller's model [6]. In fact, despite there was a strong correlation between the Mosteller's model and the actual body surface area, the relation between them was not linear, with an exponent slightly greater than 1 (figure 4-A). In other words, in the fishes examined in the present experiment, body surface area grew with body mass and body length to a greater extent compared to that predicted by

the Mosteller's formula; accordingly, as described by White [10], also the basal metabolic rate grows with body mass to a greater extent compared to that predicted by the Kleiber's model [7].

The results of the present experiment demonstrated that BMI is a fictitious index and does not correlate with physiological phenomena, even in fishes. More accurate models should be considered for the physiological relations between linear dimensions and normal body weight.

Deviation from the expected body weight, moreover, is not predictive of visceral fat content, which is the most relevant risk factor for cardiovascular accidents. Thus, models for the visceral fat content should be studied, instead of models for the optimal body weight.

A mathematical relation between body dimensions, weight, body surface area, metabolic rate and visceral fat content is still supposed to exist, because they are obviously linked to each other. In fact, despite the simple power model appears unsatisfactory, a partial correlation has been observed between all the experimental variables, except for the visceral fat content. The parallel trend of the body surface area and the basal metabolic rate, with a power > 0.75 in fishes, suggests that these variables are related and must be taken into account. From a theoretical point of view, a mathematical model could link the basal metabolic rate (given a particular body temperature) to the body surface area if the body surface area were the main factor limiting heat exchange with the external ambient. However, this is clearly not the case. In fact, skin annexes, skin blood flow, ventilation and swelling (in mammals) greatly influence heat exchange. Thus future models should take into account these other variables, which can clearly differ between and within species.

5. Conclusion

The methodological approach to search for a mathematical model describing the relation between body weight and linear body dimensions has been described using a simple animal model easily accessible to everyone. The main conclusion from the present study is that body growth in sea breams and in sea bass is allometric but it does not fit with the power model predicted by BMI. A correlation between the body surface area and the basal metabolic rate appears to exist, but the relations between body mass and each of these two variables do not fit with the models proposed by Mosteller [6] and Kleiber [7]. Prospectively, other variables should be taken into account to obtain a more affordable mathematical model to describe the relation between body weight and linear body dimensions.

References

[1] Khosla, T., Lowe, C. R., (1967). Indices of obesity derived from body weight and height. Br J PrevSoc Med. 21, 122–128.

[2] Adab, P., Pallan, M., Whincup, P.H. (2018). Is BMI the best measure of obesity? BMJ 360, k1274.

[3] Gonzalez, M. C., Correia MITD, Heymsfield, S. B. (2017). A requiem for BMI in the clinical setting.CurrOpinClinNutrMetab Care.20, 314-321.

[4] Holford, N. H. G., Anderson, B. J. (2017). Allometric size: The scientific theory and extension to normal fat mass. Eur J Pharm Sci. 109S, S59-S64.

[5] Daniell, N., Olds, T., Tomkinson, G. (2012). Technical note: Criterion validity of whole body surface area equations: a comparison using 3D laser scanning. Am J PhysAnthropol. 148, 148-155.

[6] Mosteller, R. D., (1987).Simplified calculation of body-surface area.N Engl J Med. 317, 1098.

[7] Kleiber, M., (1932). Body size and metabolism.Hilgardia 6, 315-353.

[8] Krakauer, N. Y., Krakauer, J. C. (2018). Untangling waist circumference and hip circumference from body mass index with a body shape index, hip index, and anthropometric risk indicator. MetabSyndrRelatDisord. 16, 160-165.

[9] Clark, T. D., Farrell, A. P., (2011). Effects of body mass on physiological and anatomical parameters of mature salmon: evidence against a universal heart rate scaling exponent. J Exp Biol. 214, 887-193.

[10] White, C. R., Phillips, N. F., Seymour, R. S., (2006). The scaling and temperature dependence of vertebrate metabolism. BiolLett. 2, 125-127.